



## ROBUST LINEAR CONTROL APPLIED TO A CRANE

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**Abstract.** *Cranes are present in countless industries thus scientific research related to its control remains an active field. The solutions usually include solving an open loop minimum time optimal control implemented in closed loop. This paper discusses the design of a parametric robust controller to maintain the optimal trajectory in the presence of modeling errors. The structure used is a closed loop control system with a feed forward compensation of the optimal trajectory. The modeling errors are due to a deliberate parametric uncertainty caused by change of attitude during a maneuver when it is driven by an operator, situation that the load lifting is normally unknown a priori. For ensuring performance in the presence of this change, the crane can make moves with a load lifting even with a controller not designed for each different maneuver or the lifting being part of the control law. To illustrate the proposed methodology, it is considered an experimental application to a real system of a cart-pendulum system. It is shown that the controller designed ensures the design specifications so the real plant behavior stays very close to that obtained by numerical simulation.*

**Keywords:** *Robust Control, Crane.*

### 1. INTRODUCTION

The cranes are present in countless industries in materials handling activities, and thus the scientific research related to their control remains active.

From a practical standpoint, one of the main issues around crane control is allow it to perform its work in less time, so optimal control strategies are a natural approach. An example of this approach is the work of Auernig and Troger (1987), where is shown the analytical solution for a minimum time considering the appropriate restrictions on the Pontryagin minimum principle.

One difficulty associated to optimal control approaches was discussed by Blanchini (1994) on the need to resolve the problem with both ends constraints. The author also considered the problem uncertainties in his work and designed a control system that considers the restrictions characteristics. Bemporad et al. (2001) also discusses this difficulty in optimal control approaches. Pao and Singhose (1995) shows that in a general way, robust and optimal controllers can be equivalent to purely optimal controllers for some types of systems, demonstrating the importance of considering the uncertainties in the project. Bemporad et al. (2001) also discusses this difficulty in optimal control approaches. In the study of Lau and Pao (2001) are discussed the equivalence between optimal time control and command shaping for flexible systems, being both strategies relevant to the crane control problem.

An approach example of command shaping is done by Choi and Lee (2001), who developed a way to determine the crane trajectory based on Lyapunov stability theorem. The authors extended this work for several specific types of cranes. Another example is the work of Chen et al. (2007), where is proposed an open-loop control with the path set based on the acceleration compensation principle.

According to Sorensen et al. (2007) the wide variety of techniques and strategies developed in the literature in this line of research can be grouped into three categories: minimum time optimal control, command shaping and closed loop control.

In order to control a crane in an open loop the model must be accurate. The work of Da Cruz et al. (2008) shows the efficacy of linear programming technique to obtain an optimal trajectory with an open loop control for a ship unloader, using the maximum average speed as the objective function. The authors Puglia et al. (2011) follows the same line applying linear programming to determine a cart-pendulum system trajectory, but the objective function used is the control effort minimization while the minimum time is obtained sequentially reducing the overall maneuver time until some restriction is about to be violated.

Although the open-loop control is possible, to implement a control strategy as intrinsically dependent of the mathematical model accuracy adds some risks associated with the plant uncertainties and temporal variations. Therefore it is of practical interest to have a feedback control that can keep the system performance even under the inevitable plant modeling errors or temporal variations. For this reason persists the research line in feedback control for cranes,

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including robust techniques. An example is the work of Hičár and Ritók (2006) who apply the Ackermann pole placement method for a robust crane control.

In work of Da Cruz and Leonardi (2012), is showed how the load hoisting can be incorporated into the minimum time optimal control problem. However, the trajectory must be first known to the optimal control problem solution. This is a limitation, because many times a lifting is done by an operator, whose action is not known in advance.

This paper discusses the design of a control system to overcome the problem of the ignorance of the load lifting behavior. It is used a robust controller to the model parametric uncertainties since the lifting is reflected in a crane model parameter.

To illustrate the proposed method is used a pilot scale cart-pendulum system. It is assumed that the optimal control signal in open loop is available, allowing to control the cart-pendulum in a scenario without uncertainties. The feedback tends to maintain the optimal trajectory for cases where the uncertainties degrade the plant performance without, however, interfere unnecessarily in cases where the signal of optimal control has the designed performance. That is, the optimal control action is a feed-forward system of the closed loop minimum time optimal control trajectory.

The robust control is designed with the QFT (Quantitative Feedback Theory) technique, initially proposed by Horowitz (1986), which gives a degree of robustness appropriate when the uncertainties are parametric. In this application the main uncertainty is the effective pendulum length, which is an explicit parameter of the model. This uncertainty can be real or even intentional. This last case is, for example, a hoisting crane. If the performance can be guaranteed even in the presence of a length variation, the crane can maneuver with hoisting without the controller being designed for each maneuver or the lifting condition being part of the control law.

It is shown that the designed robust controller withstands a change in the pendulum effective length over 50% while maintaining the trajectory monitoring error less than 1% for frequencies ranging up to near the plant resonant frequency. That is, the control system can overcome the lack of previous load hoisting time function problem, since the pendulum length variation is within this limit.

## 2. METHOD

The proposed control system for a crane includes a closed loop robust controller to deal with parametric uncertainties. It uses the optimal control signal generated for the open loop as a feed forward action in order to reduce the control effort.

### 2.1 Control Problem

It is usually necessary to hoist the crane during the maneuver, but it was disregarded in the modeling and the pendulum length was transformed into a parametric uncertainty, which the robust controller should handle.

The system used for experimental implementation is shown in Figure 1.

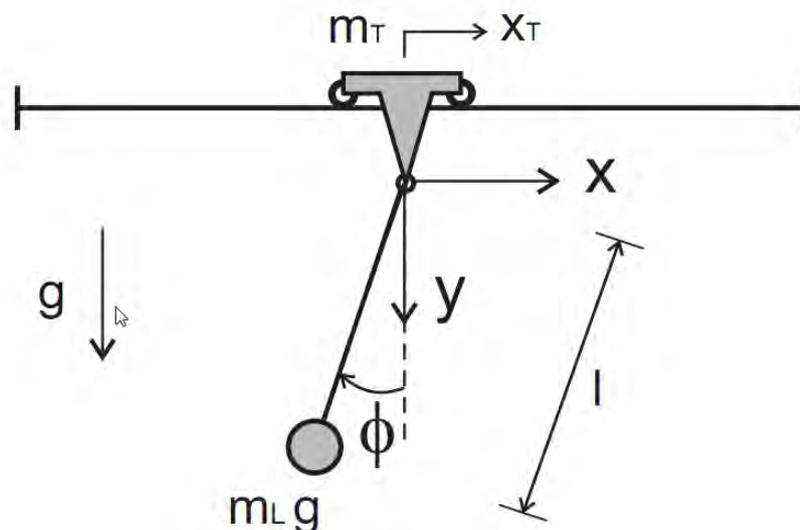


Figure 1 – Schematic of the Cart-pendulum System, Source: Puglia et al. (2011).

The equation

$$-\ddot{x}_T \cos \phi + \ddot{\phi} l = -g \cdot \text{sen} \phi - \frac{c \dot{\phi}}{m_L} \quad (1)$$

is the mathematical representation of the system dynamics. Considering that the angle of oscillation of the load should be small (smaller than 0.2 rad), it is reasonable to make the following simplification:  $\cos(\phi) \approx 1$ , and  $\sin(\phi) \approx \phi$ . Given these simplifications, the model dynamics reduces to

$$-\ddot{x}_T + \ddot{\phi} l = -g \phi - \frac{c \dot{\phi}}{m_L} \quad (2)$$

The cart drive has an integrated position control. Since this control system is fairly accurate in the frequency range relevant to this issue, its dynamics can be neglected. Thus, the cart position can be regarded as the manipulated variable of the system. Assuming the cart position as the manipulated variable and the load angle as the controlled variable, it is possible to represent the system dynamics in the transfer function form:

$$\frac{\phi(s)}{x_T(s)} = \frac{s^2}{l s^2 + \frac{c}{m_L} s + g} \quad (3)$$

For this particular application the damping is very small, thus  $c \approx 0$ . Given this simplification, the adopted model for the project is:

$$N(s) = \frac{s^2}{l s^2 + g} \quad (4)$$

The control structure used is shown in Figure 2. Note that when the switch is closed the feed-forward action is present in the control law. Otherwise, the control structure is a standard feedback control.

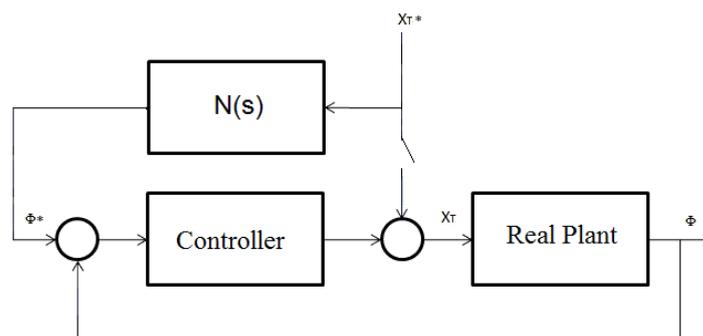


Figure 2 – Control System.

The cart position must be provided according to the optimal control generated to the plant in open loop. It can be seen that, in the case of the real plant being exactly the nominal plant, the error ( $e = \phi^* - \phi$ ) is null and the controller will not interfere with the feed-forward action (closed switch). However whenever the  $\phi$  trajectory tends to diverge from its optimum  $\phi^*$ , the robust controller must ensure that the trajectory remains close to it ( $\phi \approx \phi^*$ ).

The error signal in Figure 2 is

$$E_1(s) = \frac{N(s) - P(s)}{1 + F(s)P(s)} X_T^*(s) \quad (5)$$

where  $P(s)$  is as the real plant. However, when the switch is open the error is

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$$E_2(s) = \frac{N(s)}{1 + F(s)P(s)} X_T^*(s) \quad (6)$$

Note that if we find a controller  $F(s)$  able to reduce the error  $|e_2(j\omega)|$  in the frequency range that matters to the problems it will be easier (sufficient condition) to reduce the error  $|e_1(j\omega)|$  whenever the magnitude of the difference between the real plant and its nominal model is less than the magnitude of the real model, i.e.,

$$|N(j\omega) - P(j\omega)| \leq |N(j\omega)|. \quad (7)$$

The condition (7) immediately implies that the relative modeling error  $\varepsilon(j\omega)$  should be less than 100%, in that frequency range, thus

$$\varepsilon(j\omega) = \left| \frac{N(j\omega) - P(j\omega)}{N(j\omega)} \right| \leq 1. \quad (8)$$

The advantage of using this control strategy is precisely the fact that it is easier to reduce the tracking error of the reference when using the action forward. For this reason the design QFT was conducted with the switch of Figure 2 opened, although in the application it was kept closed.

## 2.2 Control Design

The robust controller is obtained using the QFT technique that allows the design, for a given performance with stability constraints, to be bounded in the Nichols Chart.

The plant variations are represented by templates of various plants generated from all parametric variations. The length from the load to the cart is the parameter that is considered here as the main uncertainty. Just for the sake of completeness small variations in gain are also considered. Ensuring performance in the presence of length changing, the crane can make precise moves with lifts.

As performance specification it is desired that the output of the control system follows the reference with an error less than 1% in the frequency range that goes near to the plant resonance. Additionally it is desired that such performance is kept even with a change in the pendulum length in the range from 0.25 m to 0.15 m. Additionally, in a maneuver in which the cart travels within the displacement limits, the control should not reach the driver saturation.

## 3. RESULTS

### 3.1 QFT Design

The controller design was performed considering the Bytronic1 cart-pendulum system whose characteristics are described in the manufacturer's manual. The nominal length  $l$  of the pendulum was set at 0.25m corresponding to the maximum possible length for this equipment. The parametric range considered was from 0.25 m to 0.15 m in length. However, to include additional robustness, it was assumed that the gain can vary in the range of 0.9 to 1.1.

The harmonic transfer function of the Plant ( $s = j\omega$ ) is given by

$$N(j\omega) = \left( \frac{\omega^2}{l\omega^2 - g} \right) + \left( 0 \right)j \quad (9)$$

Note that (9) always results a Real number for any value of  $\omega$ , and thus the parametric changes are represented on the Nichols chart just as gain variations. This feature makes the templates reshape the robustness barrier in a quite uniform way (see Figure 3).

The frequencies from 0.1 to 10 rad/s define the range of practical interest for this particular problem based on the frequency response of the plant. The chosen frequencies for the analysis are 0.1, 6.2, 6.27, 6.3 and 10 rad/s. The frequency 6.27 rad/s is the resonant frequency of the plant, where occurs an abrupt change of phase. Other frequencies are in the surroundings of the resonance frequency and complete the band of interest.

The stability condition requires that the absolute value of the closed loop transfer function

<sup>1</sup>Bytronic International LTD. "Bytronic Pendulum Control System - ver.2.1", England.

$$T(s) = \frac{F(s)P(s)}{1 + F(s)P(s)} \quad (10)$$

be limited ( $|T(j\omega)| < \infty$ ) for all  $\omega$  frequencies. However, in the QFT design only a few frequencies are considered. Because of this discretization and also to prevent highly undamped responses, this condition is replaced by  $|T(j\omega)| < M$ , where  $M$  must be a small value when the discretization is coarse. The area imposed by  $M$  defines the stability robustness region. In the case of this model it was empirically used  $M = 1.2$  dB. Inspecting the Nichols chart (see Figure 4) we can see that a controller with merely an integral action is able to make the transfer function of open loop (black) respect the barriers of robustness (color) for any value of its gain. The maximum gain was determined based on equipment limitations observed during the experiments.

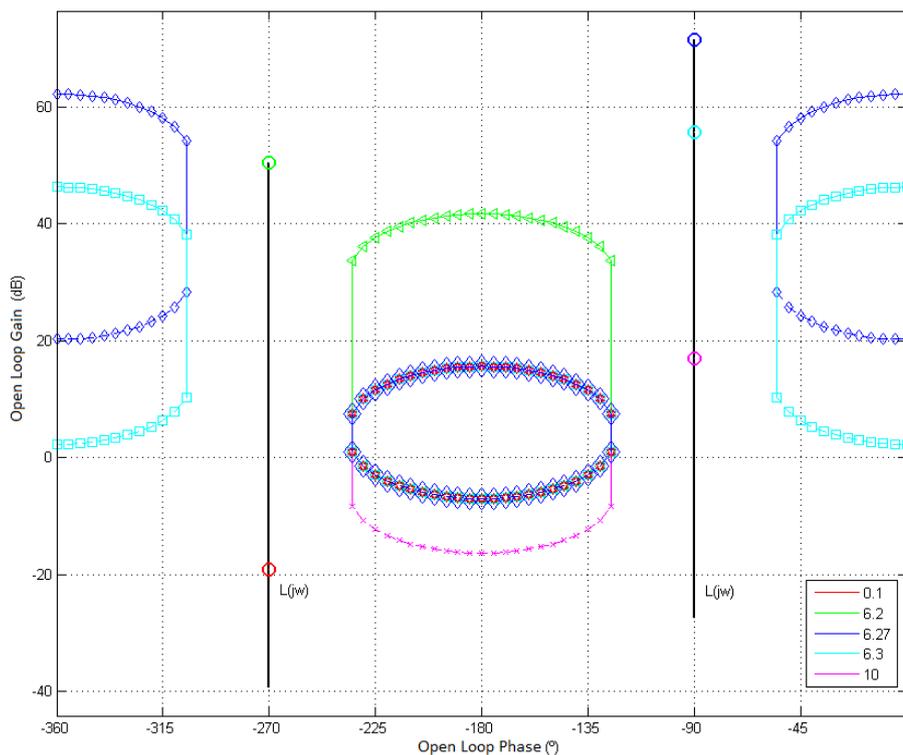


Figure 4. Loop Shaping in the Nichols chart.

### 3.2 Experimental and Simulated Results

The equipment used for the experimental tests can be seen in figure 5. The numerical simulations and their experimental tests were performed simultaneously in order to compare the presumed performance with the practical performance. The controller was tuned by means of a trial and error procedure, which is typical for the QFT design and the resulting transfer function and the corresponding a closed loop transfer function are, respectively

$$F(s) = \frac{12}{s} \quad (11)$$

$$T(s) = \frac{12s^2}{0,25s^3 + 9,81s} \quad (12)$$

The designed control system must be able to follow the solution of the optimization problem proposed by Puglia (2011). This problem involves minimizing the control effort and the maneuver duration time, generating the optimal signal control  $xT^*$  and the corresponding optimal trajectory  $\phi^*$  for  $l = 0.25$ m.

The plots in the Figure 7, Figure 8 and Figure 10, shows the performance obtained. The first plots (a) show the optimum position  $u^*$  and the experimental value of  $u$  position. Note that since the position is the control signal of the control system, deviations from  $u^*$  represent the required effort it needs to perform the control. The plots (b) show the experimental angle versus the reference angle. It is also shown the angle behavior of the system operating open loop.

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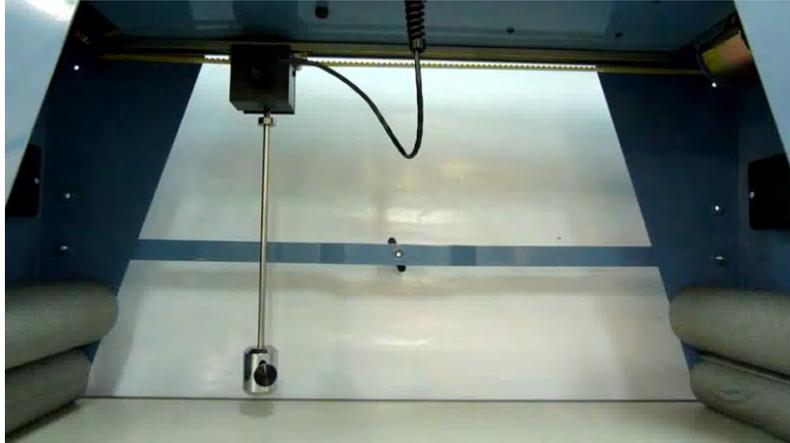


Figure 5. Byronic Cart-pendulum System.

In a first trial it was observed the controller performance to an actual length  $l = 0.25\text{m}$ , i.e. without error in this parameter, since the optimal control signal was obtained for the same  $l$ . Figure 6 shows the experimental results.

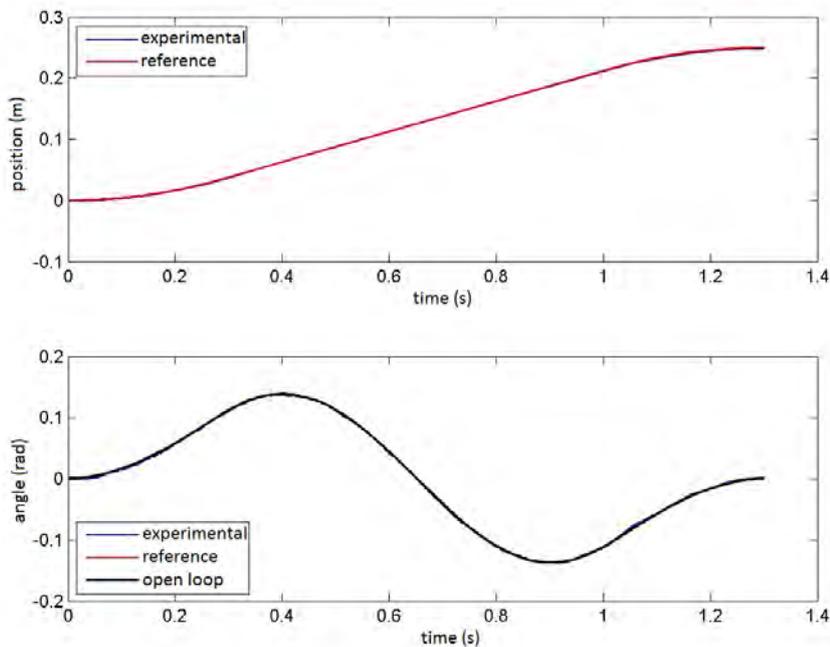


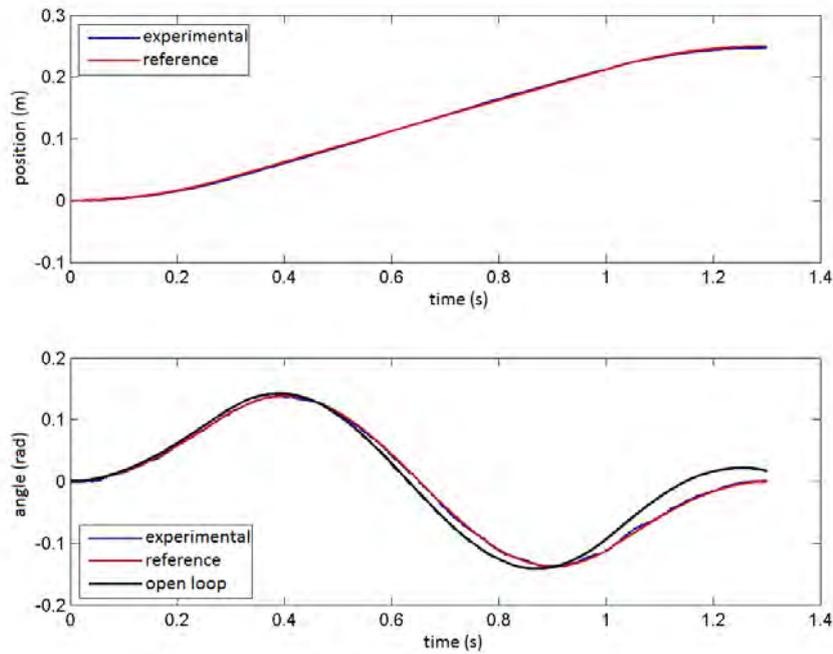
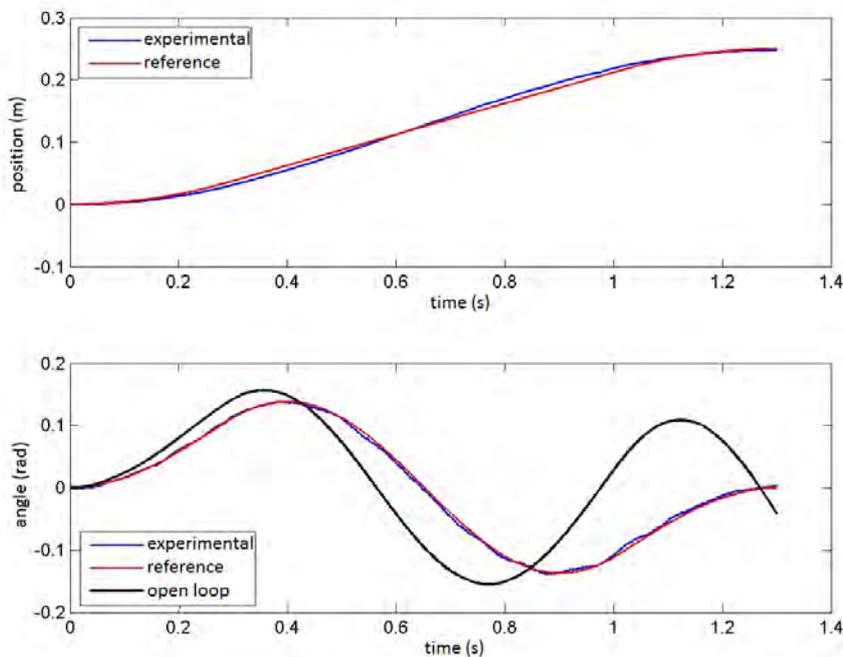
Figure 6. Controller Performance for  $l = 0.25\text{m}$ .

It can be seen that the optimal control signal (position) almost overlaps the simulated result in the same way as the output (angle) is roughly equal to the reference and the open loop. In this case the controller does not interfere significantly since the open-loop control law is yet suitable for the plant. Note that the plant reaches the final position of the maneuver at rest just as in the work of authors Puglia et al. (2011).

Figures 7 and 8 show the control performance in face of the parametric variation of the real plant. The worst case refers to Figure 8 which corresponds to the highest modeling error (about 50% of the parameter  $l$ ).

It can be seen that when the plant differs from the nominal plant, the response without the robust feedback does not reach the final position at rest thus losing the optimality of the kinematic response.

It is also possible to verify that in these cases the robust controller causes the actual response to be substantially close to that of the reference plant.

Figure 7. Controller Performance for  $l = 0,20\text{m}$ .Figure 8. Controller Performance for  $l = 0,15\text{m}$ .

In order to test the performance of the controller in face of a dynamic change of the plant during a maneuver we have simulated the system while varying the  $l$  parameter over time. This test was performed only by simulation due to limitations of the equipment since it is not capable of hoisting vertically. The variation of  $l$  over time and the comparison of the controlled system against the open loop are presented in Figure 9 and 10, respectively:

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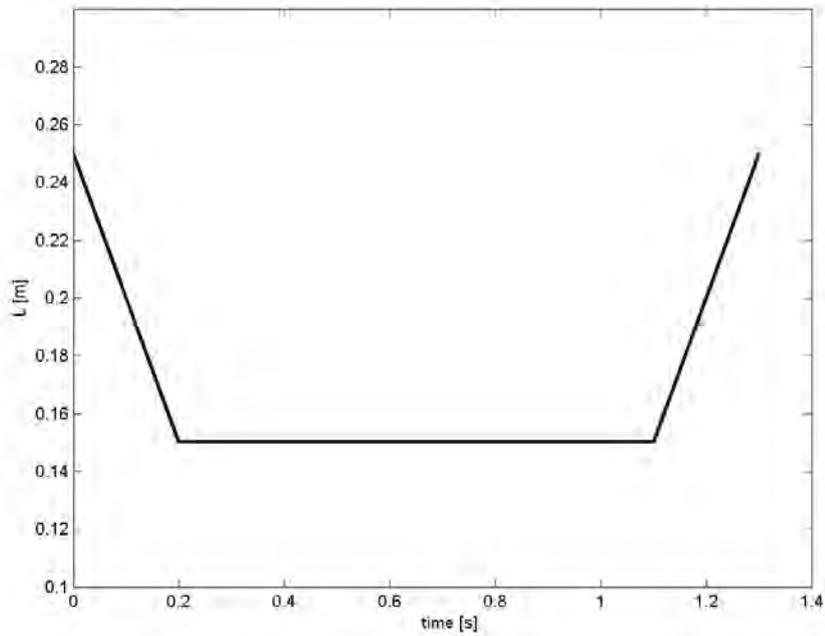


Figure 9. Hoisting trajectory.

The trajectory imposed to the control system is the same as in the previous experiments, but here it includes a lift at the beginning and at the end of movement. It can be seen that the controller can keep the performance very close to the optimum even with a huge dynamic parametric variation during the maneuver.

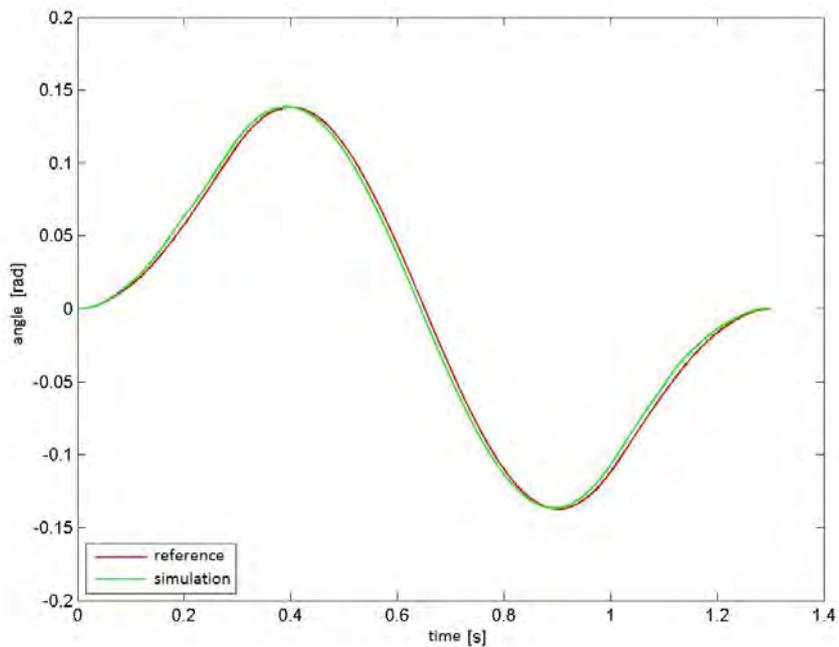


Figure 10. Performance with L varying over time.

#### 4. CONCLUSIONS

This paper discussed the design and application of a robust control system with parametric uncertainties of the model of a cart-pendulum system.

The robust control was designed via the QFT (Quantitative Feedback Theory) technique and the main parametric uncertainty considered was the effective length of the pendulum. This parameter was intentionally considered unknown so that performance could be guaranteed even in the presence of its change. Thus the crane can hoist loads during the maneuver even not designing the controller for that nor requiring that the lifting trajectory being part of the control law.

It was shown that a controller with a merely integral action is robust enough to tolerate a change up to 50% in the effective length of the pendulum while keeping the theoretical error lower than 1% for the frequencies range that goes up to near the resonant frequency of the plant.

The limitations associated to the construction of the equipment do not allow testing a change in length during movement of the cart. However, the experiment with a fixed length has been shown that the performance is very close to the simulated behavior.

As a proposal for continuity, it is suggested to apply the proposed methodology to a real crane or overhead crane.

#### 5. ACKNOWLEDGEMENTS

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