ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

DYNAMIC ANALYSIS AND OPTIMIZATION OF VIBRATIONS CONTROL BY LINEAR QUADRATIC REGULATOR ALGORITHM ON A COMPLETE VEHICLE MODEL

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Abstract. *This work aims to optimize the gains of an active control with linear quadratic regulator (LQR), applied on a complete vehicle model under a sinusoidal road surface profile, for reduction of RMS accelerations transmitted to the driver's seat and the vehicle body. Since the gain of LQR control is formulated from the matrices Q and R, the procedure determines the optimal control matrices that minimize the transmitted RMS accelerations. The model is analyzed in the time domain through the state-space formulation, and the optimization process is implemented with the method of genetic algorithms. The parameters Q and R, which provided the best gain for minimizing the optimization problem, were able to reduce the RMS accelerations more than 90% when compared to passive situations or similar active control from the literature. Finally, we analyze the influence on the other degrees of freedom and the necessary forces for the obtained results.*

Keywords: *vehicle dynamics, LQR control, parameters Q and R*

1. INTRODUCTION

The design of an active control system can be done basically in two ways: by feed forward and by feedback. The feed-forward control uses an input signal to generate the actuator force in the sense of reducing the disturbance of the system. The feedback control system uses the measured signals to generate a signal that attenuates the disturbances (Hansen and Snyder, 1997). The feed-forward control shows difficulties regarding the availability of signals to generate forces. This makes the feedback control more used in many applications.

According to Savaresi and Taneli, 2010, the first active control to be introduced in the automotive industry was the braking control. The anti-lock braking system (ABS) has become a basic requirement in modern cars. This technology arose from the industry requirement in creating control equipment smaller and more efficient, with quick response and low transmission error after measuring the involved parameters.

Active controls in vehicles work independently, because there is no interaction between the control systems applied to suspension, steering and braking; then working with decoupled systems makes the control action easier to be implemented. However, recent research is being developed to ensure that systems interact, through the Global Chassis Control (GCC), where the complete vehicle is a single object, so the full control can be achieved (Preumont, 2011).

In the classical suspension model, spring and damper are the components that regulate energy of the system, storing and dissipating it, respectively. Therefore, the control should act similar to the spring and damper actions, regulating its forces or defining a design that provides both forces.

According to Isermann, 2003, the classification of the controller can be related to input power and the bandwidth of the actuator. Three key features should be noted: the range of control, that is, the range of force provided by the actuator, the bandwidth that the controller works, that is, how fast or slow can be the employment of force, the force required which comprises the control action and the bandwidth of the actuator.

Due to the distinct characteristics of controls, the ability of operation of each one is different, since each inertial component of the vehicle moves with different frequencies according to the degree of freedom considered. The vehicle has two bands of working, the frequency of the car body (1-5 Hz) and of the wheel (15 - 20 Hz).

Although there are two design variables for controlling, available technologies for control are usually designed as shock absorbers, i.e, electronic dampers. However, some recent research have sought to regulate the stiffness of the spring in control design (Du and Zhang, 2011).

Works of Zago (2010) and Shirahatt (2008), to name a few, after applying control by linear quadratic regulator (*LQR*) yielded significant gains in vehicle models compared with systems without control. In other way, works of Robandi et. al. (2000) and Ghoreishi et. al. (2011) suggest that the search for elements of the matrices **Q** and **R** can improve the dynamic response of the analyzed systems. These studies have motivated this work to find elements of the matrices Q and R that can improve the response of a vehicle model in a specific road.

ISSN 2176-5480

Thus, the overall goal of the work is to implement an active control *LQR* for attenuation of accelerations in a passenger vehicle with 8 GDL under a sinusoidal road profile. Since gains provided by the solution of the *LQR* depend on the matrices **Q** and **R**, we find the elements of these matrices, through the genetic algorithm, so the acceleration of the vehicle can be as small as possible.

2. COMPLETE VEICULAR MODEL WITH ACTIVE CONTROL OF SUSPENSION

Many models are available in the literature to represent the movements of vehicles, involving rolling around the *x*-axis, pitching around the *y*-axis and vertical displacements. For obtaining the influence of the unsprung mass over the vertical movement of the body, a model with 2 degrees of freedom (DOF) would be sufficient (Wong, 2001). For vibration analysis in the body and in the driver's seat, we need a more complete model having a degree of freedom added to the 7 DOF usually proposed as a full vehicle.

There are basically two ways to provide better comfort to passengers: choice of optimal parameters of suspensions or design of a control system of vibrations. The actuators responsible for generating the control force can be positioned according to the designer's desire; however, in general they are placed between the body and suspension.

In this work, the suspensions are considered independent. This is generally ensured by torsion bars, which are stabilizing components, and their function is to increase the stiffness of the shaft and reduce the tendency for body roll, guaranteeing greater stability of the vehicle in curves (Leal *et al*, 2012). The model also considers the body's center of gravity shifted to the rear, configuration previously adopted in other works (Drehmer, 2012; Shirahatt, 2008). The tire is represented by a spring without damping, and having point contact with the ground. The camber effect is ignored.

2.1 Suspension model of complete vehicle of 8 DOF

Equations (1) to (8) describe the dynamic equations of the 8 DOF complete vehicle model, as shown in Figure 1. In this work, values of stiffness, mass and damping are provided by Drehmer (2012), who used processes of heuristic optimization to find the best values for these parameters for a vehicle submitted to different track conditions, in order to provide the passenger the smallest possible vertical acceleration. Thus, the problem in analysis starts from a deep analysis of possibilities of passive vibration control.

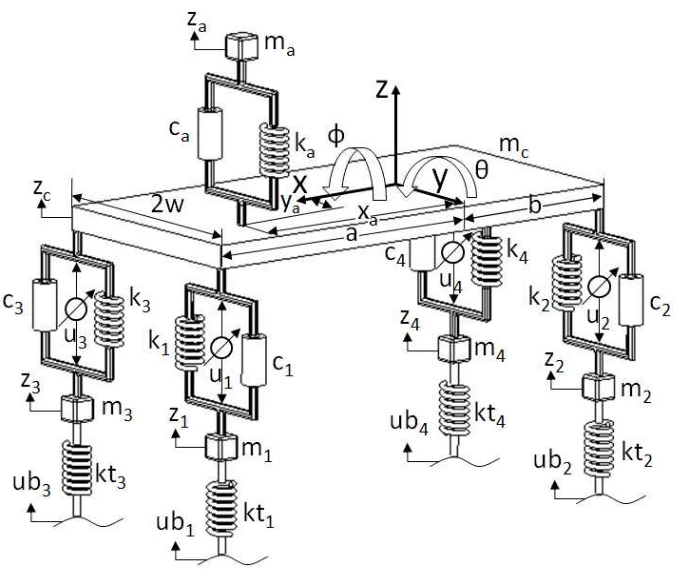

Figure 1. Vehicle model of eight degrees of freedom.

Where m_a, c_a, k_a are parameters of mass, damping coefficient and stiffness for the driver's seat, and x_a, y_a are distances from the seat to the vehicle's center of gravity, which is considered coincident with the origin of the coordinate system. a and b are distances from the front and rear part of the vehicle, respectively, to its CG. The distance $2w$ is the vehicle's width, and parameters m_1, m_2, m_3, m_4 are masses, c_1, c_2, c_3, c_4 the damping coefficients and k_1, k_2, k_3, k_4 the spring stiffness in the left front (1), rear left (2), right front (3) and rear right (4) suspensions.

DOF: vertical displacement of seat, z_a :

$$m_a \ddot{z}_a = k_a(z_c + z_a + \phi y_a - \theta x_a) + c_a(\dot{z}_c - \dot{z}_a + \dot{\phi} y_a - \dot{\theta} x_a) \quad (1)$$

DOF: vertical displacement of carbody, z_c:

$$\begin{aligned} m_c \ddot{z}_c &= k_a(-z_c + z_a - \phi y_a + \theta x_a) + c_a(-\dot{z}_c + \dot{z}_a - \dot{\phi} y_a + \dot{\theta} x_a) + \ldots \\ &+ k_1(-z_c + z_1 - \phi w + \theta a) + c_1(-\dot{z}_c + \dot{z}_1 - \dot{\phi} w + \dot{\theta} a) + \ldots \\ &+ k_2(-z_c + z_2 - \phi w - \theta b) + c_2(-\dot{z}_c + \dot{z}_2 - \dot{\phi} w - \dot{\theta} b) + \ldots \\ &+ k_3(-z_c + z_3 + \phi w + \theta a) + c_3(-\dot{z}_c + \dot{z}_3 + \dot{\phi} w + \dot{\theta} a) + \ldots \\ &+ k_4(-z_c + z_4 + \phi w - \theta b) + c_4(-\dot{z}_c + \dot{z}_4 + \dot{\phi} w - \dot{\theta} b) + \ldots \\ &+ u_1 + u_2 + u_3 + u_4 \end{aligned} \quad (2)$$

DOF: carbody roll, ϕ:

$$\begin{aligned} I_x \ddot{\phi} &= k_a y_a(-z_c + z_a - \phi y_a + \theta x_a) + c_a y_a(-\dot{z}_c + \dot{z}_a - \dot{\phi} y_a + \dot{\theta} x_a) + \ldots \\ & k_1 w(-z_c + z_1 - \phi w + \theta a) + c_1 w(-\dot{z}_c + \dot{z}_1 - \dot{\phi} w + \dot{\theta} a) + \ldots \\ & k_2 w(-z_c + z_2 - \phi w - \theta b) + c_2 w(-\dot{z}_c + \dot{z}_2 - \dot{\phi} w - \dot{\theta} b) + \ldots \\ & k_3 w(z_c - z_3 - \phi w - \theta a) + c_3 w(\dot{z}_c - \dot{z}_3 - \dot{\phi} w - \dot{\theta} a) + \ldots \\ & k_4 w(z_c - z_4 - \phi w + \theta b) + c_4 w(\dot{z}_c - \dot{z}_4 - \dot{\phi} w + \dot{\theta} b) + \ldots \\ & + u_1 w + u_2 - u_3 w - u_4 w \end{aligned} \quad (3)$$

DOF: carbody pitch, θ:

$$\begin{aligned} I_y \ddot{\theta} &= k_a x_a(z_c - z_a + \phi y_a - \theta x_a) + c_a x_a(\dot{z}_c - \dot{z}_a + \dot{\phi} y_a - \dot{\theta} x_a) + \ldots \\ &+ k_1 a(z_c - z_1 + w\phi - \theta a) + c_1 a(\dot{z}_c - \dot{z}_1 + w\dot{\phi} - \dot{\theta} a) + \ldots \\ &+ k_2 b(-z_c + z_2 - w\phi - \theta b) + c_2 b(-\dot{z}_c + \dot{z}_2 - w\dot{\phi} - \dot{\theta} b) + \ldots \\ &+ k_3 a(z_c - z_3 - w\phi - \theta a) + c_3 a(\dot{z}_c - \dot{z}_3 - w\dot{\phi} - \dot{\theta} a) + \ldots \\ &+ k_4 b(-z_c + z_4 + w\phi - \theta b) + c_4 b(-\dot{z}_c + \dot{z}_4 + w\dot{\phi} - \dot{\theta} b) + \ldots \\ &- u_1 a + u_2 b - u_3 a + u_4 b \end{aligned} \quad (4)$$

DOF: vertical displacement of left front suspension, z_1:

$$m_1 \ddot{z}_1 = k_1(z_c - z_1 + \phi w - \theta a) + c_1(\dot{z}_c - \dot{z}_1 + \dot{\phi} w - \dot{\theta} a) - u_1 + kt_1(ub_1 - z_1) \quad (5)$$

DOF: vertical displacement of left rear suspension, z_2:

$$m_2 \ddot{z}_2 = k_2(z_c - z_2 + \phi w + \theta b) + c_2(\dot{z}_c - \dot{z}_2 + \dot{\phi} w + \dot{\theta} b) - u_2 + kt_2(ub_2 - z_2) \quad (6)$$

DOF: vertical displacement of right front suspension, z_3:

$$m_3 \ddot{z}_3 = k_3(z_c - z_3 - \phi w - \theta a) + c_3(\dot{z}_c - \dot{z}_3 - \dot{\phi} w - \dot{\theta} a) - u_3 + kt_3(ub_3 - z_3) \quad (7)$$

DOF: vertical displacement of right left suspension, z_4:

$$m_4 \ddot{z}_4 = k_4(z_c - z_4 - \phi w + \theta b) + c_4(\dot{z}_c - \dot{z}_4 - \dot{\phi} w + \dot{\theta} b) - u_4 + k_{41}(ub_4 - z_4) \quad (8)$$

Equations (1) to (8) are displayed as a matrix equation according to Equation (9).

$$[\mathbf{M}]_{8x8}\{\ddot{\mathbf{q}}(t)\}_{8x1} + [\mathbf{C}]_{8x8}\{\dot{\mathbf{q}}(t)\}_{8x1} + [\mathbf{K}]_{8x8}\{\mathbf{q}(t)\}_{8x1} = [\mathbf{F}]_{8x4}\{\mathbf{u}(t)\}_{4x1} + [\mathbf{K_e}]_{8x4}\{\mathbf{ub}(t)\}_{4x1} \quad (9)$$

Where the degrees of freedom are exposed as the vector $\{\mathbf{q}\}$ in Equation (10):

$$\{\mathbf{q}\} = [z_a \quad z_c \quad \phi \quad \theta \quad z_1 \quad z_2 \quad z_3 \quad z_4]^T \quad (10)$$

ISSN 2176-5480

2.2 Model of vibration control system

The control assumes that all state variables can be read in real time and the actuator applies necessary forces instantaneously. Given the form of a second-order system shown in Eq. (9), this may be reduced to the first order, as described below. First of all, the variables are changed as shown in Eq. (11):

$$\{\mathbf{x}\} = \begin{Bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{Bmatrix} \tag{11}$$

It follows the established relation of Eq (11), but now described in Eq. (12):

$$\begin{aligned} \{\dot{\mathbf{x}}\} &= \frac{d}{dt}\begin{Bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{Bmatrix} = \begin{Bmatrix} \dot{\mathbf{q}} \\ -[\mathbf{M}]^{-1}[\mathbf{C}]\dot{\mathbf{q}} - [\mathbf{M}]^{-1}[\mathbf{K}]\mathbf{q} + [\mathbf{M}]^{-1}[\mathbf{F}]\mathbf{u} + [\mathbf{M}]^{-1}[\mathbf{Ke}]\mathbf{ub} \end{Bmatrix} \\ &= \begin{bmatrix} 0 & \mathbf{I} \\ -[\mathbf{M}]^{-1}[\mathbf{K}] & -[\mathbf{M}]^{-1}[\mathbf{C}] \end{bmatrix}\begin{Bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{Bmatrix} + \begin{bmatrix} 0 \\ [\mathbf{M}]^{-1}[\mathbf{F}] \end{bmatrix}\mathbf{u} + \begin{bmatrix} 0 \\ [\mathbf{M}]^{-1}[\mathbf{Ke}] \end{bmatrix}\mathbf{ub} \\ &\equiv [\mathbf{A}]\{\mathbf{x}\} + [\mathbf{B}]\{\mathbf{u}\} + [\mathbf{Bl}]\{\mathbf{ub}\} \end{aligned} \tag{12}$$

In this model there is the continuous influence of disturbances of the road $\mathbf{ub}$, whose incidence matrix $\mathbf{Ke}$ considers the linear behavior of tires to translate disturbances in input force to the system. The incidence matrix $\mathbf{F}$ incorporates the positions of the actuators that influence the system's response to actuation of the control. The equation for numerical integration, where *T* is the time step and *k* the index for discretization becomes:

$$\{\mathbf{x}(k+1)\} = e^{[\mathbf{A}]T}\{\mathbf{x}(k)\} + [\mathbf{A}]^{-1}\left[e^{[\mathbf{A}]T} - \mathbf{I}\right][\mathbf{B}]\mathbf{u}_k + [\mathbf{A}]^{-1}\left[e^{[\mathbf{A}]T} - \mathbf{I}\right][\mathbf{Bl}]\mathbf{ub} \tag{13}$$

The *LQR* control is a regulator which provides gain values to the system, which converted in forces, will tend to minimize the system energy. The control formulation seeks to reduce the system energy through minimization of a quadratic functional, shown in Eq. (14), which provides information of displacements and velocities of the system, beyond the acting forces.

$$J = \int_0^\infty \left(\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u}\right)dt \tag{14}$$

The functional is regulated by the matrices **Q** and **R**, where the increase in **Q** values decreases the state variables, and the decrease of the **R** values increases the control forces imposed to the system. However, little is known about the analytical correlations between the system to be controlled and the values assigned to these matrices, and each assignment of values to the system provides different gains.

Matrices **Q** and **R** are weights that influence directly the solution of the problem and consequent gain calculation, which takes place through the solution of Riccati equations for continuous systems. According to Kwon and Bang (1997), the formulation of the functional *J* can be based on the use of Lagrange multipliers to find the optimum, which is called the Hamiltonian of the control problem according to Eq (15).

$$H = \frac{1}{2}\left(\{\mathbf{x}\}^T\mathbf{Q}\{\mathbf{x}\} + \{\mathbf{u}\}^T\mathbf{R}\{\mathbf{u}\}\right) + \{\lambda\}^T\left([\mathbf{A}]\{\mathbf{x}\} + [\mathbf{B}]\{\mathbf{u}\}\right) \tag{15}$$

And its derivative with respect to the state variables and control forces gives Eq. (16) for the control $\mathbf{u}(t)$:

$$\mathbf{u}(t) = -[\mathbf{R}]^{-1}[\mathbf{B}]^T\{\lambda\} \tag{16}$$

Also , according to Kwon and Bang (1997), the solution through Lagrange multipliers is difficult to compute. A widely used method for solution considers the Lagrange multipliers as being proportionals to state variables, becoming:

$$\{\lambda\} = [\mathbf{S}]\{\mathbf{x}\} \tag{17}$$

ISSN 2176-5480

where $[\mathbf{S}]$ is the Ricatti matrix. Consequently, control forces also will have linearly proportional relation with the state variables, as given below:

$$\mathbf{u}(t) = -[\mathbf{G}]\{\mathbf{x}\} \tag{18}$$

Where the gain matrix $[\mathbf{G}]$ is provided by the control to the state variables. After mathematical manipulations, the solution of Riccati for gain calculation is obtained by the following algebraic equation:

$$0 = [\mathbf{S}][\mathbf{A}] + [\mathbf{A}]^T[\mathbf{S}] - [\mathbf{S}][\mathbf{B}][\mathbf{R}]^{-1}[\mathbf{B}]^T[\mathbf{S}] + [\mathbf{Q}] \tag{19}$$

After founding the $[\mathbf{S}]$ value, it will be replaced in Eqs. (17) and (16) to obtain forces as function of time.

3. CONSTRAINTS ON THE OPTIMIZATION PROBLEM

The simulation in vehicle dynamics needs some constraints in order to be close to actual situation; in this context this work show some constraints, one relative to travel limits of suspensions and another respect to permanent contact between tire and track. The control system design of a linear quadratic regulator involves the minimization of the quadratic cost functional:

$$J = \int_{t_0}^{t_f} \left(\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u} \right) dt$$
$$\text{subject to constraints: } \begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{Bl}\,\mathbf{ub} \\ \mathbf{x}(t_f) = \mathbf{x}_f \quad \text{e} \quad \mathbf{x}(t_0) = \mathbf{x}_0 \\ \mathbf{y} = \mathbf{C}\mathbf{x} \end{cases} \tag{20}$$

where the matrix **Q** (matrix of penalties of the state variables) is definite non-negative and **R** (penalty matrix of control actions) is definite positive. A widespread form is to establish that the matrices **Q** and **R** are diagonal and to choose large values for the variables you want to be small in the time domain (Gabasa, 2009). As shown above, the *LQR* control has dependence on the defining parameters, thus we look for parameters that minimize the functional J while satisfying constraints inherent to the problem.

According to Gabasa, 2009, the optimization of the objective function J does not guarantee that other requirements of the system are met, and to solve them penalties are created each time the system violates some constraint. Thus, the constrained optimization with penalties becomes an unconstrained optimization problem. The first constraint considers that the vehicle should always have the four tires in contact with the ground, and the constraint equation is given by:

$$z_i - u_{bi} < |\delta| \tag{21}$$

where δ expresses the initial displacement of the suspensions after static equilibrium of the vehicle due to the action of gravity. That is, the equation shows that during simulation, where gravitational forces are neglected, if the displacement between the suspension and the floor is higher than the initial displacement caused by the gravitational force, the tire loses contact with the floor.

The second constraint regards the limit of suspension travel, where only the upper limit was considered to hit the stop, being determined by the variable $displ\_\max$.

$$\begin{aligned} z_{s1} &< displ\_\max \\ z_{s2} &< displ\_\max \\ z_{s3} &< displ\_\max \\ z_{s4} &< displ\_\max \end{aligned} \tag{22}$$

According to Gillespie (1992), the displacement of the suspension is basically determined by the weight of the body on the suspension springs, and considering they are in parallel it would be only:

$$displacement\ of\ suspension = \frac{g * m_s}{K_{total\ suspension}} \quad \text{, where} \quad K_{total\ suspension} = \sum_{n=1}^{4} K_n \tag{23}$$

However, this is true when there is no roll and pitch simultaneously with the vertical movement. In this case, the interaction follows Eq. (24).

According to Gillespie (1992), the upper limit found in vehicles must be at least 0.127 m to absorb at least 0.5 g without hitting any stop. The author mentions that most passenger vehicles have the upper limit around 0.177 to 0.2032 m, being this latter value of maximum displacement (*displ_max*) considered in this work.

$$\begin{cases} z_{s1} = z_c - z_1 + \phi w - \theta a \\ z_{s2} = z_c - z_2 + \phi w + \theta b \\ z_{s3} = z_c - z_3 - \phi w - \theta a \\ z_{s4} = z_c - z_4 - \phi w + \theta b \end{cases} \tag{24}$$

Finally, the optimization problem with constraints becomes:

$$\begin{array}{l} \text{Find parameters } \mathbf{Q} \text{ and } \mathbf{R} \text{ to minimize} \\ F_{objective} = \dfrac{RMS\ \ddot{z}_c + RMS\ \ddot{z}_a}{2} \\ \text{under} \\ z_i - u_b < |\delta| \quad , i = 1,2,3,4 \\ z_{si} < desloc\_max \end{array} \tag{25}$$

The penalty increases as the constraints are violated and according to the degree of violation occurred. The designer also has the option of applying an additional weight ε to the penalty. In equation (26) is described the way in which the constraints are applied in the optimization problem.

$$\begin{array}{l} \text{given that} \\ R_1 = z_i - u_b - |\delta|, \\ R_2 = z_{si} - displac\_max. \\ \text{if} \quad R_1 > 0, \quad penalty = penalty + R_1; \\ \text{if} \quad R_2 > 0, \quad penalty = penalty + R_2; \\ f(x) = f(x) * (1 + \varepsilon * penalty) \end{array} \tag{26}$$

4. RESULTS AND DISCUSSION

The simulation of the complete model of 8 DOF proposed by Shirahatt *et al.* (2008) is subject to some limitations due to the lack of enough information about the matrices **Q** and **R** to determine the gain, and also they don't mention how to search them. Another problem was relative to some mistake to satisfy the right-hand rule in their orthogonal axes *XYZ*, which influences the formalism of equations that describe the dynamic behavior of the system. However, this paper was the closest to be used to make a direct comparison with our results, since the input data is adequate for a complete model vehicle

In this work we use the same data given by Shirahatt article (2008) to simulate the passive system and the active system controlled by *LQR*, according to Table 1:

The disturbance used is similar to sinusoid proposed by Zago *et al*. (2010), but for this case there is a lag between the input excitation of front and rear wheels. According to Shirahatt *et al*. (2008), there were also imposed lags between the left and right tires, as the sinusoids were tilted for a time of 0,2 s.

The function used to determine the profile is given by :

$$ub_{1,3}(t) = \begin{cases} \dfrac{h}{2}\left(1 - cos(\omega t)\right), & \text{for} \quad 0 \le t \le \dfrac{2\lambda}{v} \\ 0, & \text{for} \quad t \ge \dfrac{2\lambda}{v} \end{cases} \tag{27}$$

ISSN 2176-5480

$$ub_{2,4}(t) = \begin{cases} \frac{h}{2}(1 - cos(\omega(t - tau))), & \text{for} \quad \tau \le t \le \tau + \frac{2\lambda}{v} \\ 0, & \text{for} \quad t \ge \tau + \frac{2\lambda}{v} \end{cases} \tag{28}$$

Table 1. Data used for model simulation.

Variable	Description	Unit	Optimal value	
			Passive	Active
k_a	Stiffness of the seat	N/m	98935	95161
k_1	Stiffness of the left front suspension	N/m	96861	78158
k_2	Stiffness of the left rear suspension	N/m	52310	41731
k_3	Stiffness of the right front suspension	N/m	96861	78158
k_4	Stiffness of the right rear suspension	N/m	52310	41731
c_a	Seat damping	N s/m	615	415
c_1	Damping of the left front suspension	N s/m	2460	2012
c_2	Damping of the left rear suspension	N s/m	2281	1848
c_3	Damping of the right front suspension	N s/m	2460	2012
c_4	Damping of the right rear suspension	N s/m	2281	1848
k_t	Stiffness of front and rear tires	N/m	200000	200000
m_a	Mass of seat	kg	100	100
m_c	Sprung mass	kg	2160	2160
m_1	Left front unsprung mass	kg	85	85
m_2	Right front unsprung mass	kg	60	60
m_3	Left rear unsprung mass	kg	85	85
m_4	Right rear unsprung mass	kg	60	60
I_x	Inertia mass moment for rolling axis	$kg \cdot m^2$	946	946
I_y	Inertia mass moment for pitch axis	$kg \cdot m^2$	4140	4140
a	Distance between the front of the vehicle and the center of gravity of the sprung mass	m	1.524	1.524
b	Distance between the rear of the vehicle and the center of gravity of the sprung mass	m	1.156	1.156
w	Half width of the sprung mass	m	0.725	0.725
x_a	Distance *X* from seat to CG	m	0.234	0.234
y_a	Distance *Y* from seat to CG	m	0.375	0.375

Source: Shirahatt *et al.* (2008)

Being $ub_{1,3}$ the excitations on the front tires and $ub_{2,4}$ the excitations on the rear tires; the height of the wave $h = 0{,}05$ m, the wavelength $\lambda = 20$ m and the vehicle velocity $v = 20$ m/s. The lag between front and left tires is given by the variable $\tau = \left(\frac{a+b}{v}\right)$, and the angular frequency is defined by $\omega = \left(\frac{2\pi v}{\lambda}\right)$.

The determination of matrices **Q** and **R** can also be made by the optimization process, in this work is performed by GA. To check the influence of the modified parameters on the dynamic response, the first search using GA modifies equally all terms of matrices **Q** and **R**, as provided below.

$$\mathbf{Q} = \alpha \mathbf{I}_{16x16}$$
$$\mathbf{R} = \beta \mathbf{I}_{4x4}$$

The formulation of the active control by *LQR* enables different forms and responses of the objective function. The objective function composed by the RMS value of accelerations of the truck and the driver's seat was the best one for reducing vibrations throughout the vehicle, as shown below.

ISSN 2176-5480

Find parameters α *e* β to minimize

$$F_{objective} = \frac{\left(RMS\ \ddot{z}_c + RMS\ \ddot{z}_a\right)}{2}$$

under

$$z_i - u_b > |\delta| \quad , i = 1,2,3,4$$

$$z < displ\_max$$

$$1 \le \alpha \le 10^{16}$$

$$10^{-8} \le \beta \le 1$$

Another possibility of composing the matrix **Q** is through variations of diagonal values referring to displacements, *i.e.*, the first eight positions, and sets the remaining ones with unit values. The procedure is similar to the scheme of trial and error but it leaves free for finding the diagonal values of matrix **R**.

$$\mathbf{Q} = \text{diagonal}\begin{bmatrix}\alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & \alpha_6 & \alpha_7 & \alpha_8 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{bmatrix}$$
$$\mathbf{R} = \text{diagonal}\begin{bmatrix}\beta_1 & \beta_2 & \beta_3 & \beta_4\end{bmatrix}$$

The limits used for the matrix elements were set from the concept that high **Q** values tend to decrease the values of the state variables, and low R values raise the intensity of control forces. Since the simulations on models proposed by Zago *et al*. (2010) and Shirahatt *et al*. (2008) showed values for **Q** variables in the order of up to 10^{12}, it was decided to leave the search space of this parameter for high values and to enable the algorithm test small values for the elements of **R**. It follows the second proposal for the optimization problem.

Find parameters $\alpha_1 \ldots \alpha_8$ and $\beta_1 \ldots \beta_4$ to minimize

$$F_{objective} = \frac{\left(RMS\ \ddot{z}_c + RMS\ \ddot{z}_a\right)}{2}$$

under

$$z_i - u_b > |\delta| \quad , i = 1,2,3,4$$

$$z < displac\_max$$

$$10^5 \le \alpha \le 10^{16}$$

$$10^{-6} \le \beta \le 1$$

Table 2 shows the parameters used in the optimization procedure by GA for both proposals. The higher number of individuals for the second proposal was necessary to obtain a lower coefficient of variation estimated in Eq. (29).

Table 2. Determination of GA for the optimization process.

Parameter	**Proposal 1**	**Proposal 2**
Number of variables	2	12
Population size	20 individuals	30 individuals
Objective function order	Lower value of the function	Lower value of the function
Selection	Roulette	Roulette
Mutation	Uniform	Uniform
Crossover	Scattered	Scattered

The analysis of these responses is performed according to RMS values of acceleration of the body and seat, and from the maximum displacements and accelerations of other degrees of freedom. Table 3 lists values of responses for both proposals and its comparative with results obtained by Shirahatt *et al*. (2008).

Table 3. Optimal results obtained for the two proposals.

Parameter	Passive value Shirahatt *et al.* (2008)	Active value Shirahatt *et al.* (2008)	Active value Proposal 1	Active value Proposal 2
Objective function	$0.2933\,\mathrm{m \cdot s^{-2}}$	$0.0513\,\mathrm{m \cdot s^{-2}}$	$0.3970\,\mathrm{m \cdot s^{-2}}$	$1.3117 \cdot 10^{-3}\,\mathrm{m \cdot s^{-2}}$
Rms $\ddot{z}_a$ (seat's acceleration)	$0.3032\,\mathrm{m \cdot s^{-2}}$	$0.0534\,\mathrm{m \cdot s^{-2}}$	$0.3786\,\mathrm{m \cdot s^{-2}}$	$9.0050 \cdot 10^{-4}\,\mathrm{m \cdot s^{-2}}$
Rms $\ddot{z}_c$ (body's acceleration)	$0.2834\,\mathrm{m \cdot s^{-2}}$	$0.0492\,\mathrm{m \cdot s^{-2}}$	$0.4154\,\mathrm{m \cdot s^{-2}}$	$1.7229 \cdot 10^{-3}\,\mathrm{m \cdot s^{-2}}$
Max $\ddot{z}_a$ (seat's acceleration)	$2.0849\,\mathrm{m \cdot s^{-2}}$	$0.2350\,\mathrm{m \cdot s^{-2}}$	$1.3640\,\mathrm{m \cdot s^{-2}}$	$3.5938 \cdot 10^{-3}\,\mathrm{m \cdot s^{-2}}$
Max $\ddot{z}_c$ (body's acceleration)	$1.9172\,\mathrm{m \cdot s^{-2}}$	$0.2268\,\mathrm{m \cdot s^{-2}}$	$1.4939\,\mathrm{m \cdot s^{-2}}$	$7.8870 \cdot 10^{-3}\,\mathrm{m \cdot s^{-2}}$
Max z_a (seat's displacement)	$0.0725\,\mathrm{m}$	$0.0187\,\mathrm{m}$	$0.0560\,\mathrm{m}$	$1.1017 \cdot 10^{-4}\,\mathrm{m}$
Max z_c (body's displacement)	$0.0690\,\mathrm{m}$	$0.0181\,\mathrm{m}$	$0.0615\,\mathrm{m}$	$2.1577 \cdot 10^{-4}\,\mathrm{m}$
Max ϕ (rolling angle)	$0.0096\,°$	$0.0029\,°$	$2.0854\,°$	$0.0203\,°$
Max $\ddot{\phi}$ (roll acceleration)	$0.5041\,\mathrm{rad \cdot s^{-2}}$	$0.0888\,\mathrm{rad \cdot s^{-2}}$	$1.8133\,\mathrm{rad \cdot s^{-2}}$	$0.0180\,\mathrm{rad \cdot s^{-2}}$
Max θ (pitch angle)	$0.0222\,°$	$0.0025\,°$	$1.2884\,°$	$0.0156\,°$
Max $\ddot{\theta}$ (pitch acceleration)	$1.1700\,\mathrm{rad \cdot s^{-2}}$	$0.0582\,\mathrm{rad \cdot s^{-2}}$	$0.9654\,\mathrm{rad \cdot s^{-2}}$	$0.0164\,\mathrm{rad \cdot s^{-2}}$
Max z_1 (front left displacement)	$0.0383\,\mathrm{m}$	$0.0320\,\mathrm{m}$	$0.0569\,\mathrm{m}$	$0.0581\,\mathrm{m}$
Max z_2 (rear left displacement)	$0.0122\,\mathrm{m}$	$0.0305\,\mathrm{m}$	$0.0513\,\mathrm{m}$	$0.0419\,\mathrm{m}$
Max z_3 (front right displacement)	$0.0290\,\mathrm{m}$	$0.0293\,\mathrm{m}$	$0.0522\,\mathrm{m}$	$0.0431\,\mathrm{m}$
Max z_4 (rear right displacement)	$0.0125\,\mathrm{m}$	$0.0288\,\mathrm{m}$	$0.0595\,\mathrm{m}$	$0.0547\,\mathrm{m}$

The optimal control of the proposal 2 allowed to decrease the RMS value of acceleration in the driver's seat by 99% compared to the situation without control, and 98% for the control proposed by Shirahatt *et al*. (2008). The RMS value of the body acceleration showed a decrease of 99% compared to the situation without control, and 96% compared to the control proposed by the same author. The maximum values of accelerations in the seat and body followed the same percentage gain, the same occurs for the maximum angular accelerations and displacements of seat and body. However, the maximum angular displacement of pitch and roll provides a 111% increase and a 30% decrease, respectively, with respect to the situation without control.

The objective function for the second optimization proposal showed an interval of variations for a sequence of thirty evaluations, and the coefficient of variation of the response was found according to Eq. (29) where σ is the standard deviation and $\overline{X}$ the mean. The coefficient varies between zero and one.

$$CV = \frac{\sigma}{\overline{X}} \qquad CV = 0{,}45 \tag{29}$$

Table 3 lists values for the second proposal with reference to the best founded value of the objective function, and matrices **Q** and **R** show components as follows:

$$\alpha_1 = 8.9757 \cdot 10^{12} \quad \alpha_2 = 7.8936 \cdot 10^{12} \quad \alpha_3 = 9.3776 \cdot 10^{12} \quad \alpha_4 = 4.0346 \cdot 10 \quad \alpha_5 = 10^8 \quad \alpha_6 = 10^8$$
$$\alpha_7 = 10^8 \quad \alpha_8 = 10^8 \quad \beta_1 = 0.0656 \quad \beta_2 = 0.1638 \quad \beta_3 = 0.3677 \quad \beta_4 = 0.0587$$

Figure 2 to Figure 5 depict the dynamic behavior of the vehicle for the best value of the objective function of the proposal 2, where the suspension displacement is calculated according to Eq. (24).

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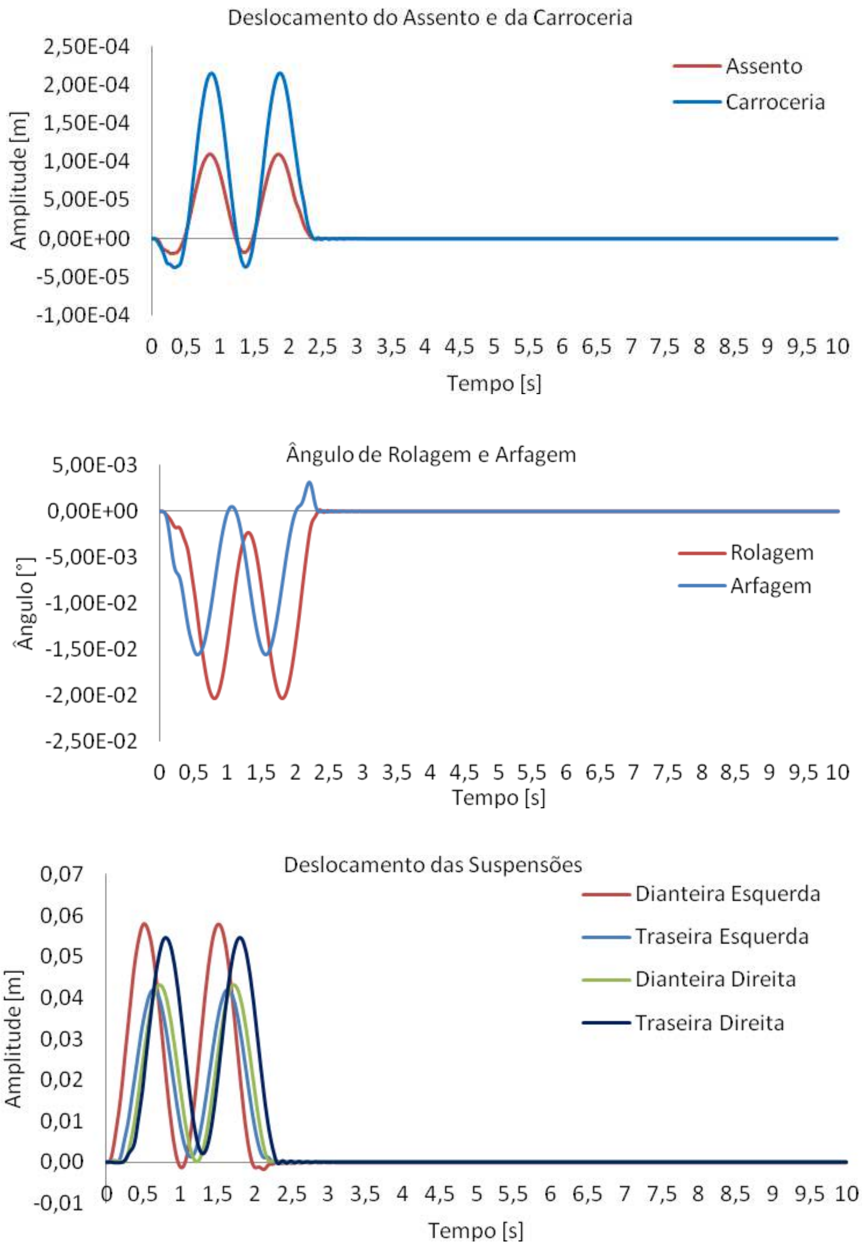

Figure 2. Displacement of the eight degrees of freedom

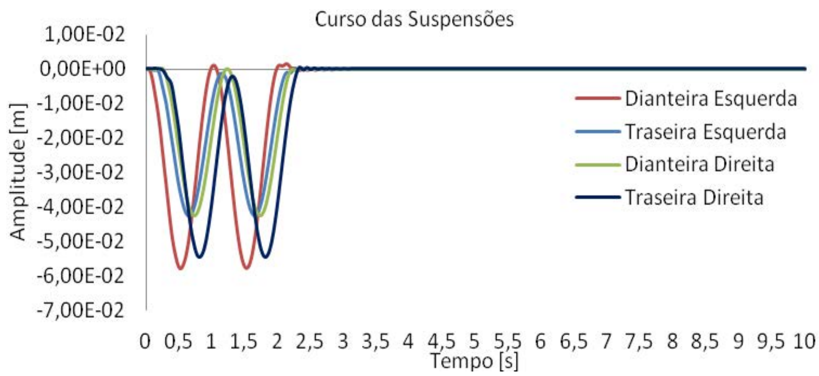

Figure 3. Suspension travel

As provided in section 3 and according to Gillespie (1992), the maximum travel of the suspension for passenger cars is between $0{,}17$ m and $0{,}20$ m, limits considered in this work, as verified in Figure 3.

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Figure 4 and Figure 5 show the seat and body accelerations and the active control forces for the second proposal.

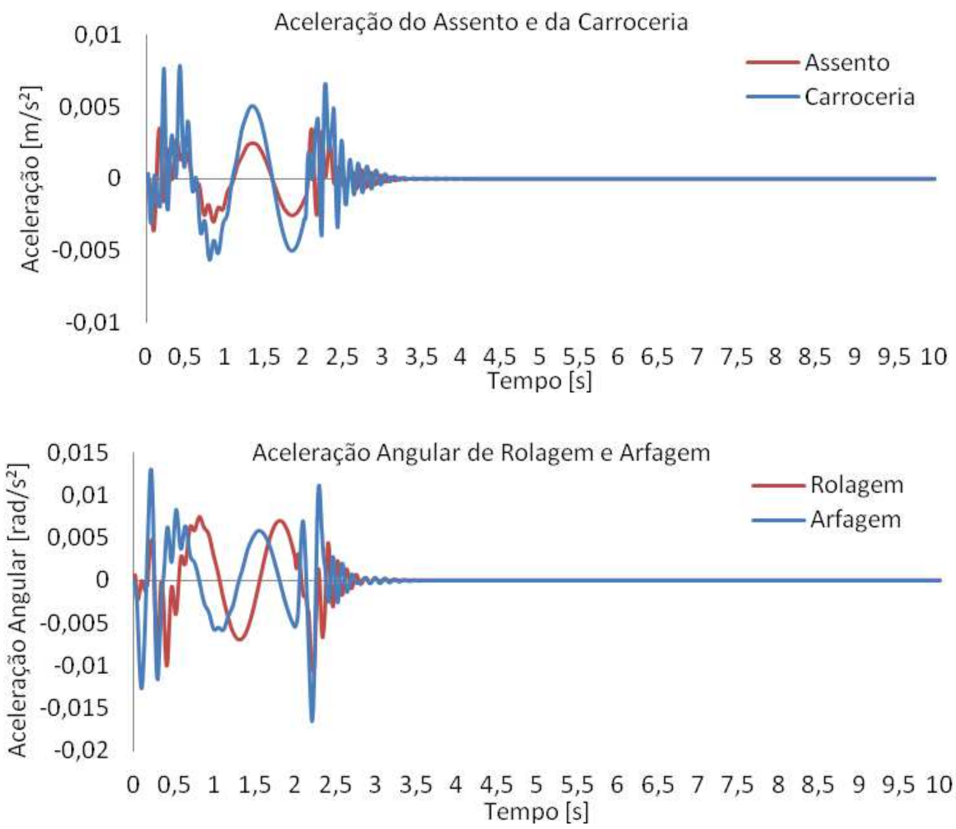

Figure 4. Accelerations on body and seat

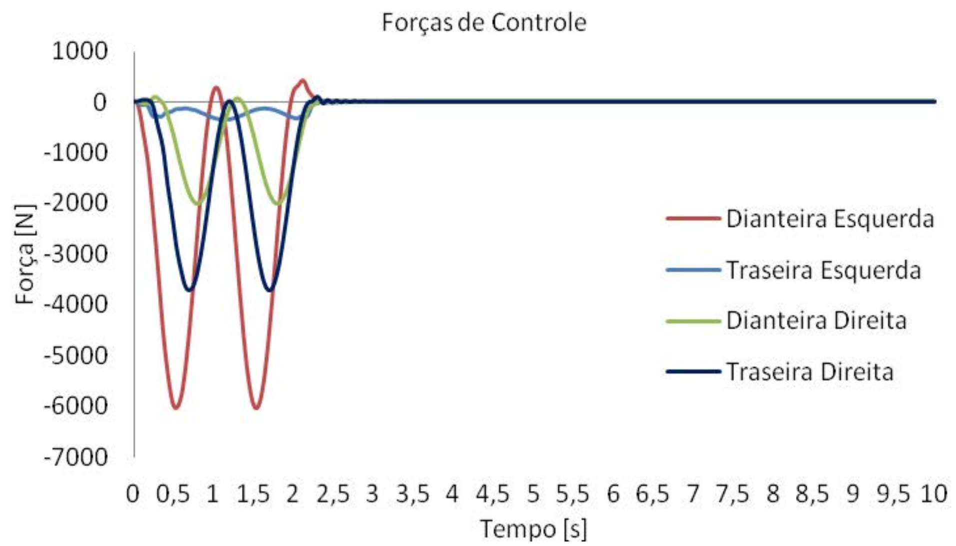

Figure 5. Forces imposed on the system

Despite the reduction in RMS of displacements and accelerations, forces imposed on the system, relative to the solution proposed by Shirahatt *et al*. (2008) and simulated in this study, increased 100% in the left front suspension, 15% in the right front suspension, 130% in the rear right suspension and decreased 13% in the left rear suspension.

ISSN 2176-5480

5. CONCLUSSIONS

The choice of an objective function with information of acceleration of the body and the driver's seat was the best one which provided sufficient information about the dynamic behavior in displacements and accelerations.

The proposed optimization about elements of matrices **Q** and **R** for one system with eight degrees of freedom and subject to one sinusoidal excitation showed significant gains when compared to results obtained by Shirahatt et. al. (2008). Most accelerations and displacements exhibited a reduction over 90%, despite of the involved forces increased over 100% relative to the reference.

In future studies it will be evaluated the possibility of optimizing all elements of matrices Q and R, which could generate control signals with lower forces, but with a significant improvement of the vehicle vibration. It is also suggested the construction of an experimental prototype with the proposed LQR control.

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