



ON THE EFFECT OF A MAGNETO RHEOLOGICAL DAMPER IN DYNAMICAL JUMPS IN A NONLINEAR VIBRATING SYSTEM EXCITED BY A NONIDEAL MOTOR

Vinicius Piccirillo¹

Angelo Marcelo Tusset²

UTFPR – Federal Technological University of Paraná, Department of Mathematics, Av. Monteiro Lobato, Km 04, s/nº. CP 20, 84016 – 210, Ponta Grossa – PR, Brazil

¹piccirillo@utfpr.edu.br, ²a.m.tusset@gmail.com

José Manoel Balthazar

UNESP – São Paulo State University, Department of Statistics, Applied Mathematical and Computation, CP 178, 13500-230, Rio Claro, Brazil.

jmbaltha@rc.unesp.br

Abstract. *The dynamical response of systems with magneto-rheological damper (MRD) presents a different behavior due to their nonlinear characteristic. MRD nonlinear response is associated with adaptive dissipation related to their hysteretic behavior. This property is very attractive in engineering field. This paper discusses the attenuation of the Sommerfeld effect (jump phenomena) in a nonlinear dynamics MRD-nonideal mechanical oscillator connected with an unbalanced motor excitation with limited power supply. The Bouc-Wen mathematical model is used to represent the MRD behavior. Numerical simulations show different aspects about Sommerfeld effect, illustrating the influence of the different electric current applied in MRD to control the force developed by this damper.*

Keywords: *Magneto rheological damper, Sommerfeld effect, Nonlinear Dynamics*

1. INTRODUCTION

The study of problems that involves the coupling of several systems, has been investigated in last years, in function of the change of constructive characteristics of the machines and structures. In this way, some phenomena are observed in composed dynamic systems supporting structure and rotating machines, where are verified that the unbalancing of the rotating parts is the greatest causer of vibrations. In the study of these systems, for a more realistic formulation is to consider the action of an energy source with limited power (non-ideal), that is, to consider the influence of the oscillatory system on the driving force and vice versa. Recently a number of works have been done, in order to investigate the resonant conditions of non-ideal vibrating oscillator systems (Balthazar, *et al.*, 2003), and a number of several of non-ideal vibrating systems has been studied, for some examples, (Piccirillo, *et al.* 2009; Tusset, *et al.*, 2013a), undeserved others.

Sommerfeld effect is a kind of problem that occurs in non-ideal systems near resonance frequencies. The jump phenomena in the vibration amplitude and the increase of the power required by the source to operate next to the system resonance are both manifestations of this non-ideal problem. This phenomenon suggests that the vibratory response of the non-ideal system emulates an “energy sink” in the regions next to the system resonance, by transferring the power from the source to vibrations of the support structure, instead of the speeding up the driver machine (Castão, *et al.*, 2010). In other words, one of the problems confronted by mechanical engineers is how to drive a system through a system resonance and avoid this “disappearance of energy” as originally described by Sommerfeld (Nayfeh and Mook, 1979).

Palacios *et al.* 2009a, presented a research which contained an analysis of Lugre friction in elimination of Sommerfeld effect for a non-ideal structural system (NIS). The authors observed significant reductions on the resonance capture phenomenon when this friction law is considered in NIS and consequently, the Sommerfeld effect is then eliminated. The analysis of the Sommerfeld effect of a Duffing–Rayleigh oscillator under a non-ideal excitation (unbalanced motor with limited power supply) using the method of averaging and numerical computation was investigated by Felix and Balthazar, 2009b. Furthermore for the reduction of the Sommerfeld effect, jump phenomenon and resonance capture were used shape memory alloy spring. According to Belato (1998), the jump phenomenon related to the Sommerfeld Effect is associated with a cyclic saddle-node bifurcation, with the system losing stability in the point where the jump occurs.

In this work, we use the semi-active approach to reduce the resonance vibrations of a non-ideal structure (Sommerfeld Effect) in nonideal system by applying a nonlinear damping with magnetorheological fluids. The mechanism of the damper (MR) is similar to the mechanism of hydraulic dampers in which the force is obtained by passage of fluid through an orifice. This variable resistance to fluid flow allows using fluid viscous in MR dampers and

other devices electrically controllable. Thus, the magnetic properties of the fluid allow its use as a damper controlled by an electric voltage (V) or an electric current (A) (Tusset, *et al.*, 2009).

These magnetic properties permit its use as a damper, controlled by an electrical current (Tusset *et al.*, 2009). The use of MR damper control in the suppression of unwanted oscillations variable is done by the electrical current or voltage which changes the viscosity of the fluid's internal damper. The damping force will depend on the velocity of the piston of the damper and the density of internal fluid.

2. MAGNETO RHEOLOGICAL DAMPER MODEL WITH HYSTERESIS

A large number of analytical models based on different descriptions have been done with the objective of describing the nonlinear properties of MR dampers. The force–velocity characteristics of magnetorheological damper measured after various excitations and electric currents, indicates a nonlinear behavior such as hysteresis (Ma, *et al.*, 2003).

The velocity of the piston also has an important influence on the dynamic properties of the magnetorheological damper fluid. If the velocity is high, the duration in which the particles are in the magnetic field is short. This results in saturation of the damping force to the upper velocity's ± 0.4 m/s. Saturation can also occur in relation to the applied electric current in the coil being used electric currents between 0 and 1.5 A. (McManus, *et al.*, 2002)

2.1 Bouc-Wen Model to MR Damper

A model that can be solved numerically and has been used extensively to model systems with hysteresis is the Bouc–Wen model. The Bouc–Wen model is considered to be extremely versatile and can display a wide variety of hysteresis behavior. Figure 1 show the layout of the Bouc–Wen model (Dominguez, *et al.*, 2006).

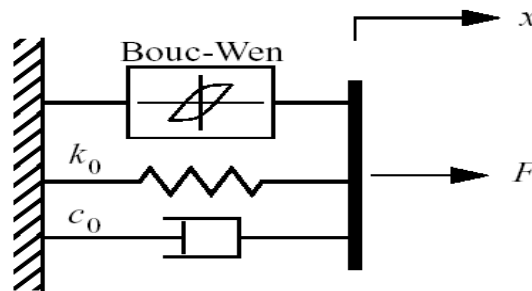


Figure 1: Bouc-Wen model for MR damper

The force F of the system is determined by

$$F = c_0 \dot{x} + k_0 x + \alpha z \quad (1)$$

and z is obtained from the equation:

$$\dot{z} = -\zeta |\dot{x}| z |z|^{n-1} - \xi \dot{x} |z|^n + \Lambda \dot{x} \quad (2)$$

This model can incorporate the force f_0 MR damper accumulator as an initial displacement x_0 and a coefficient of stiffness k_0 . As it can be seen in Eqs. (1) and (2) the control variable (i) does not appear explicitly. Since the goal is to control the strength of the damper using the electric current, we use an approximation of the function (1) that depends explicitly on the electric current (Tusset, *et al.*, 2013b)

$$F = \frac{3.2}{(3e^{-3.4i}) + 1} \dot{x} + k_0 x + \frac{8.5}{(1.28e^{-3.9i}) + 1} z \quad (3)$$

The electrical current to be applied can be determined by solving numerically the following function:

$$C(i) = \frac{3.2}{(3e^{-3.4i}) + 1} \dot{x} + k_0 x + \frac{8.5}{(1.28e^{-3.9i}) + 1} z - F \quad (4)$$

3. NON-IDEAL EXCITATION

Let us consider a vibrating system (NIS), which includes a direct current (DC) motor with limited power supply, operating on a structure (Figure 2). The excitation of the system is limited by a characteristic of the energy source (nonideal energy source). Then, the coupling of the vibrating oscillator and the DC motor takes place. As the vibration of the mechanical system depends on the DC motor, also the motion of energy source depends on vibrations of the system. Hence, it is important to analyze what happens to the motor, as the response of the system changes.

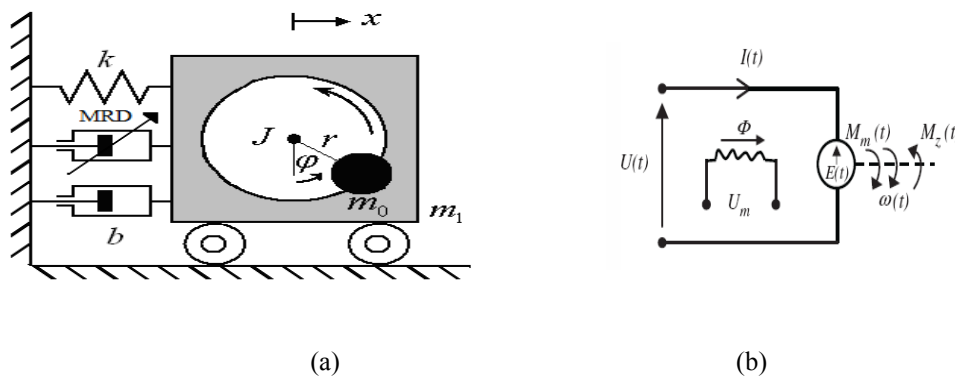


Figure 2: (a) Non-ideal mechanical system (NIS) and (b) the electrical schematic representation of the DC motor

The considered vibrating system consists of a mass m_1 , a linear damping with viscous damping coefficient b . A non-ideal DC motor, with a driving rotor of a moment of inertia J and r is the eccentricity of the unbalanced mass.

The electrical scheme of the DC motor representation is presented in Fig. 3 (b). The equations governing the motion of the DC motor are typically written in the form (Warminski and Balthazar, 2003);

$$J \frac{d^2 \varphi}{dt^2} = M_m(t) - M_z(t) - H(\varphi) \quad (5)$$

$$U(t) = R_t I(t) + L_t \frac{dI(t)}{dt} + E(t) \quad (6)$$

where time functions $U(t)$ and $I(t)$ are the voltage and the current in the armature, R_t and L_t is resistance and inductance of the armature, $E(t)$ is the internally generated voltage, $M_z(t)$ is an external torque applied to the motor drive shaft, $H(\varphi)$ is a frictional torque and $M_m(t)$ denotes the torque generated by the motor. The torque $M_m(t)$ and internal generated voltage $E(t)$ can be expressed as

$$M_m(t) = c_M \Phi I(t) \quad (7)$$

$$E(t) = c_E \Phi \omega(t) \quad (8)$$

where c_M , c_E are mechanical and electrical constants and Φ is the magnetic flux. Let us assume that external exciting current I_m and voltage U_m are constant and then the magnetic flux Φ is also constant in the considered model. Taking into account Eqs. (5) - (8), we can write the differential equations of the complete electro-mechanical system presented in Fig. 1 as follows:

$$Mx'' + bx' + F + k_1 x + k_{nl} x^3 - m_0 r (\varphi'^2 \sin \varphi + \varphi'' \cos \varphi) = 0 \quad (9)$$

$$(J_M + m_0 r^2) \varphi'' = c_M \Phi \tilde{I}(t) - \tilde{H}(\varphi') + m_0 r x'' \cos \varphi \quad (10)$$

$$\frac{d\tilde{I}(t)}{dt} = -\frac{R_t}{L_t} \tilde{I}(t) - \frac{c_E \Phi}{L_t} \varphi' + \frac{\tilde{U}(t)}{L_t} \quad (11)$$

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where a prime denotes a derivative with respect to dimensional time and $M = m_1 + m_0$, and F is the MRD restoring force. It is convenient to work with dimensionless position and time, in such a way that Equation 9 - 11 is rewritten in the following form

$$\ddot{u} + \zeta \dot{u} + F_{MRD} + u + \gamma u^3 - w_1 (\dot{\varphi}^2 \sin \varphi + \ddot{\varphi} \cos \varphi) = 0 \quad (12)$$

$$\ddot{\varphi} = p_3 I(\tau) + w_2 \ddot{u} \cos \varphi - H(\varphi) \quad (13)$$

$$\dot{I} = U(\tau) - p_1 I(\tau) - p_2 \dot{\varphi} \quad (14)$$

where

$$\omega_0^2 = \frac{k_1}{M}, \quad \zeta = \frac{b}{M\omega_0}, \quad \gamma = \frac{k_{nl}}{k_1} x_{st}^2, \quad I = \frac{\tilde{I}}{I_r}, \quad w_1 = \frac{m_0 r}{M x_{st}}, \quad w_2 = \frac{m_0 r x_{st}}{(J + m_0 r^2)}, \quad p_1 = \frac{R_t}{L_t \omega_0}, \quad p_2 = \frac{c_E \Phi}{L_t I_r},$$

$$p_3 = \frac{c_M \Phi I_r}{(J + m_0 r^2) \omega_0^2}, \quad U(\tau) = \frac{\tilde{U}(\tau)}{L_t I_r \omega_0}, \quad H(\varphi) = \frac{\tilde{H}(\varphi')}{(J + m_0 r^2) \omega_0}, \quad \tau = \omega_0 t, \quad u = \frac{x}{x_{st}}, \quad F_{MRD} = \frac{F}{M \omega_0^2 x_{st}}$$

and x_{st} means a static displacement of the system, τ is the dimensionless time and I_r is a rated current in the armature and dots indicating differentiations with respect to dimensionless time, the function $H(\varphi)$ is the resistive torque applied to the motor and in this work $H(\varphi)$ will be neglected ($H(\varphi) = 0$).

4. NUMERICAL SIMULATION RESULTS

This section considers numerical simulations that are design to illustrate the magneto rheological damper influence in jump phenomena in a complete electro-mechanical system. The DC motor and mechanical parameters used in numerical simulations are given in table 1.

In non-ideal mechanical systems the oscillator cannot be driven by systems, whose amplitude and frequency are arbitrarily chosen, once the forcing source has a limited available energy supply. For this kind of oscillator, the driven system cannot be considered as given a priori, but it must be taken as a consequence of the dynamics of the whole system (oscillator and motor). Therefore, a non-ideal oscillator is, in fact, the combined dynamical system resulting from the coupling of a passive and an active oscillators which serves as the driving source for the first one. The resulting motion will be thus the outcome of the dynamics for the combined systems. The dimensionless voltage applied across the armature U is the control parameter in non-ideal system. For each value of U , the non-ideal system present one frequency and amplitude behavior.

Table 1. System parameter used in simulation (Warminski and Balthazar, 2003).

w_1	w_2	p_1	p_2	p_3	ζ	γ
0.2	0.3	0.3	3	0.15	0.1	0.2

It is known that the dynamics of a system close to the fundamental resonance region may be analyze through a frequency-response diagram, which is obtained plotting the amplitude of the oscillating system versus the frequency of the excitation term. For the complete electro-mechanical system, this graph is estimated by numerical simulation defining the amplitude as the maximum value of the amplitude of the mechanical oscillation (denoted by A), and the frequency as the mean value of the rotational speed of the $\dot{\varphi}$ (denoted by ω).

Fig. 3 represents the resonance curve without MRD device when the mean frequency ω is slowly increased. The curve was calculated using an incremente $\Delta U = 0.01$ as the variation of the control parameter U . The transient response is also considered in the computation because its evolution inside the state space determines the occurrence of the jump phenomenon, during the passage through resonance region ($\omega \approx 1$). In Fig. 3(a), can be seen that, when the value of the control parameter is $U \approx 3.1$, the resonance region was reached, so is generating large amplitude vibrations of the system, shown in Fig. 3(b), in other words, the system reaches the maximum amplitude of displacement. Note that close to resonance ($\omega \approx 1$), the power it is supplied to the DC motor to initiate the jump increase (see Fig. 3(c) and

(d)). Then the operating frequency increases and thus the system amplitude decreases, resulting in lower power consumption by the DC motor.

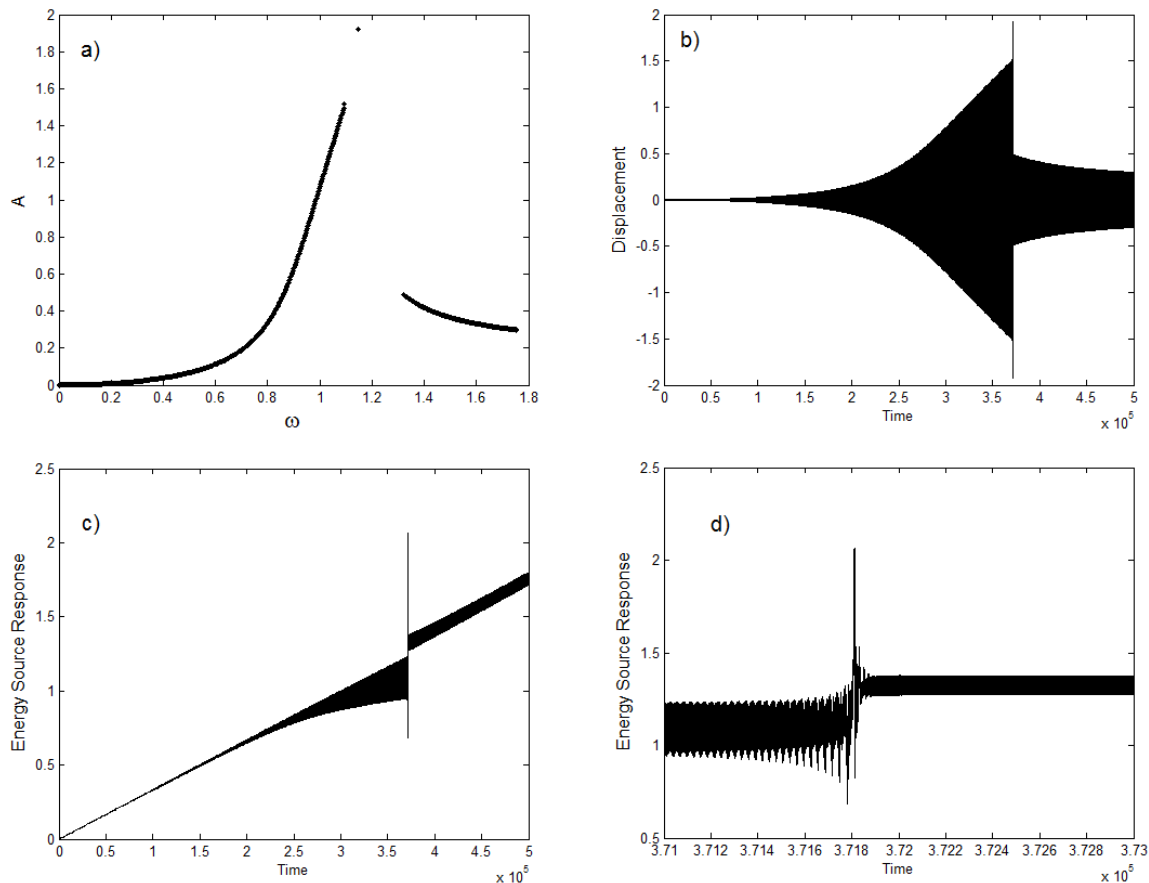


Figure 3: Jump phenomenon observed when the mean frequency ω is slowly increased: (a) Frequency-response diagram without MRD, (b) Time response, (c) Energy source and (d) Zoom of figure (c).

The jump phenomena is characterized by sudden amplitude transition, as indicated in figure 3(a). This happens because there is not enough damping in the system to stop the DC motor from transmitting large amounts of energy to the nonlinear oscillator. Now, the same non-ideal system is investigated, however, the magneto rheological damper (MRD) is introduced in this system, as observed in Fig. 2. The Sommerfeld effect and MRD parameter in the non-ideal system, will be verified too. These results are compared with non-ideal system without MRD. Due to MRD nonlinear characteristic and dissipative behavior the jump response tends to present small vibrations amplitude.

When the MRD is introduced in non-ideal system, it can be observed that for $i = 0A$, that is, the current electric applied in MRD is zero, the reduction of amplitude in non-ideal system is observed, as shown in figures 4(a) and (b). Nevertheless, the Sommerfeld effect still happens because the damping introduced by MRD, in this case, it is not efficient to suppress or inhibit the energy transfer from DC motor to nonlinear oscillator. Moreover, when compared to the situation with non-ideal without MRD, the MRD non-ideal oscillator presents a smaller amplitude response.

Figure 5 shows the behavior of non-ideal systems with $i = 0.5A$. With this increase in the applied electric current the characteristics of the variation of damping force change and the Sommerfeld effect starts to be controlled, for this reason the MRD damper dissipate more vibrational energy now that previous one, therefore, the amplitude of motion decreases too. Figures 5(c) and (d) shows that the energy transferred from DC motor to the structure began to be controlled and this means that the MRD energy dissipation improve (or decrease) the jump.

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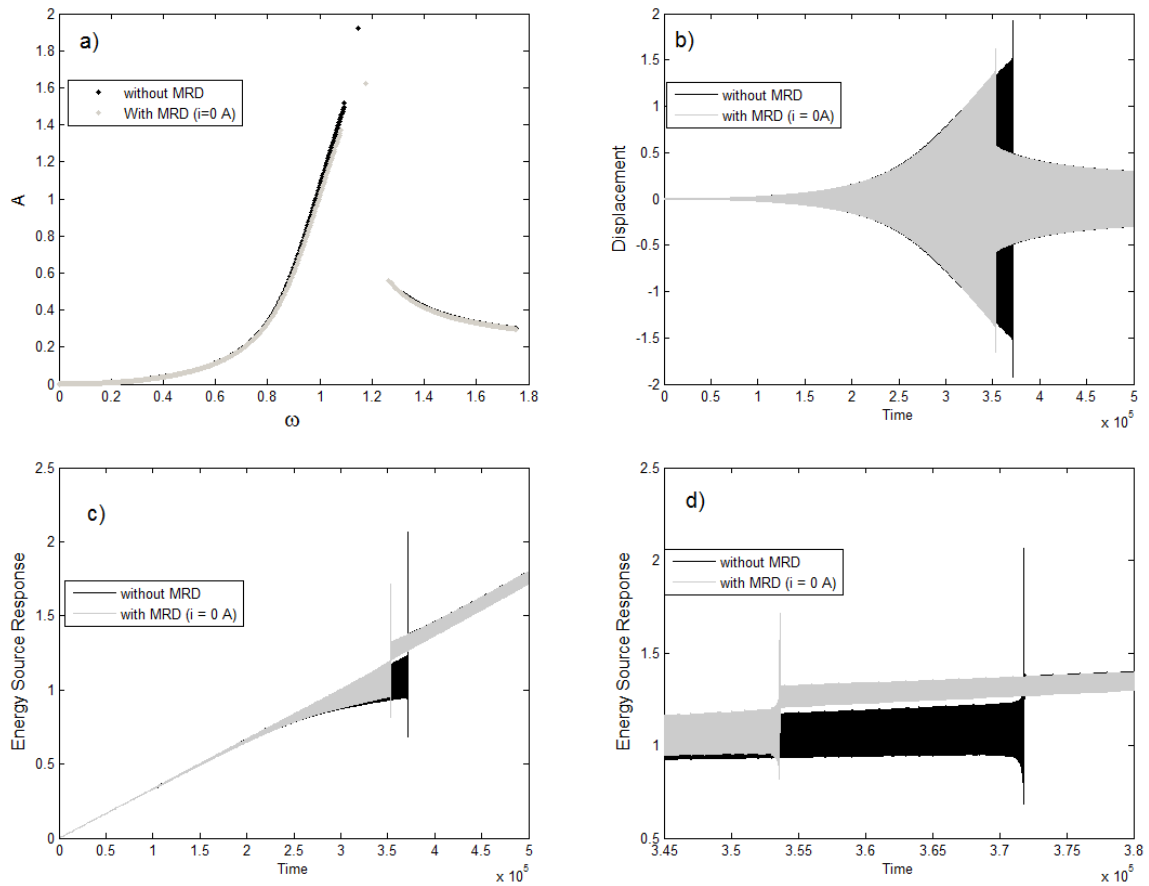
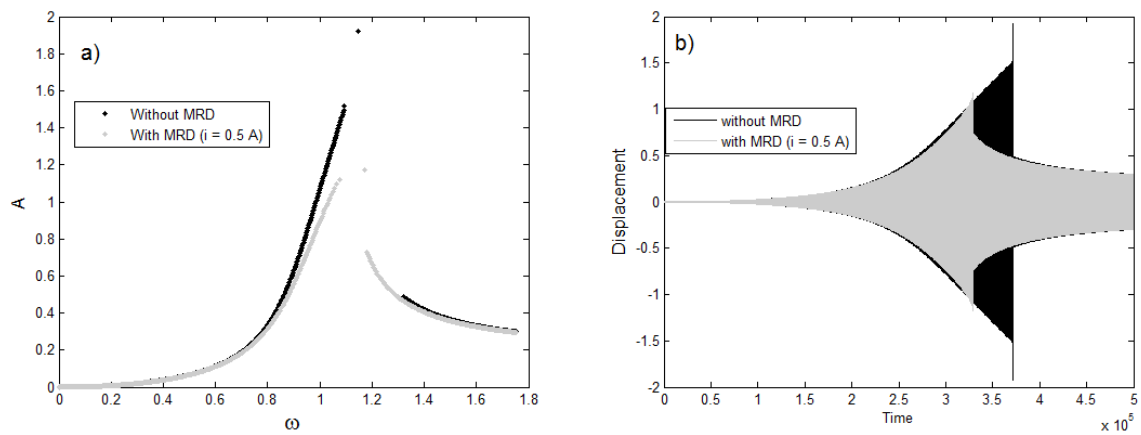


Figure 4: Jump phenomenon observed when the mean frequency ω is slowly increased: (a) Frequency-response diagram with MRD ($i = 0$ A), (b) Time response, (c) Energy source and (d) Zoom of figure (c).



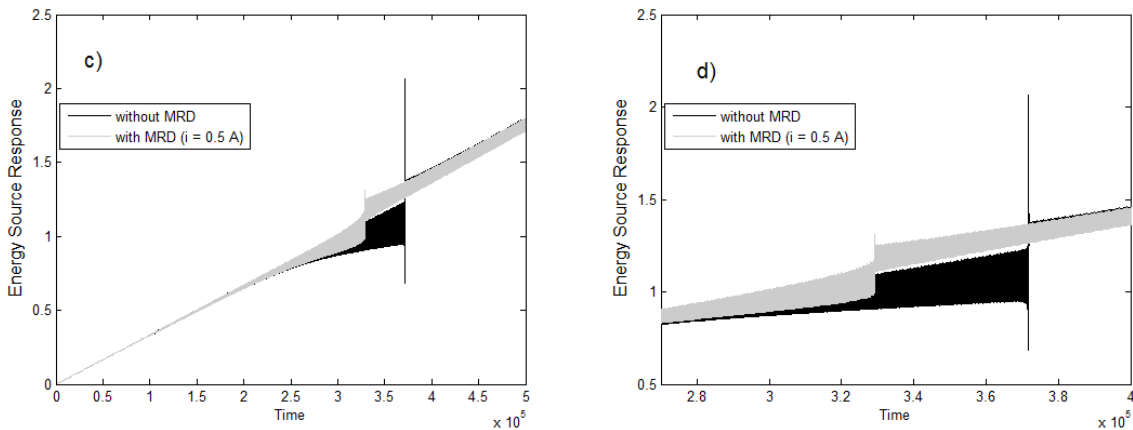


Figure 5: Jump phenomenon observed when the mean frequency ω is slowly increased: (a) Frequency-response diagram with MRD ($i = 0.5$ A) and (b) Time response.

Figure 6 and 7 shows the system, considering, that the current is increased to a level of 1 A and 1.5 A, respectively. In both cases, the Sommerfeld effect is completely put down because the damping force is more greater than the previous cases and the effect of the MRD results in a suppression of the jump phenomena. Note that the increase in electric current causes the decrease of the amplitude of the motion in the resonance region, eliminating possible jumps. In this case the energy transferred from DC motor to the oscillator was controlled by means of MRD.

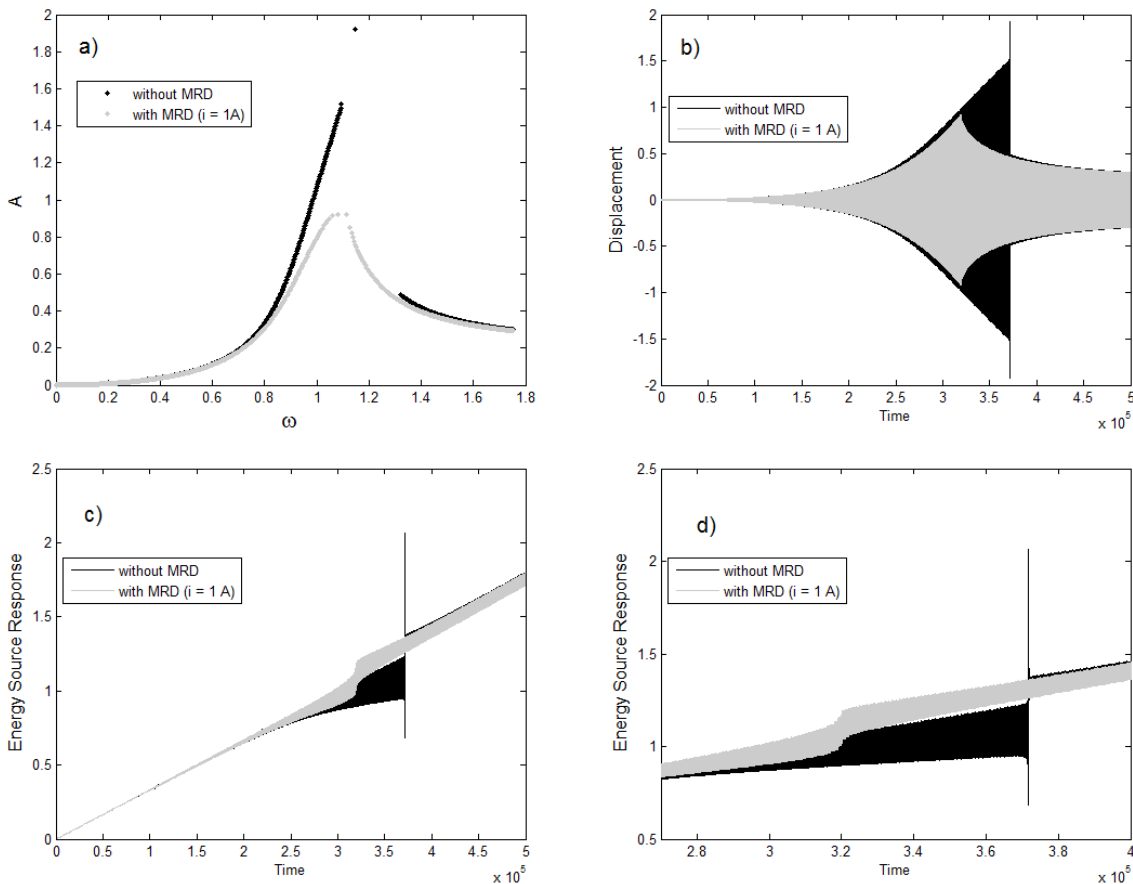


Figure 6: Jump phenomenon observed when the mean frequency ω is slowly increased: (a) Frequency-response diagram with MRD ($i = 1$ A) and (b) Time response.

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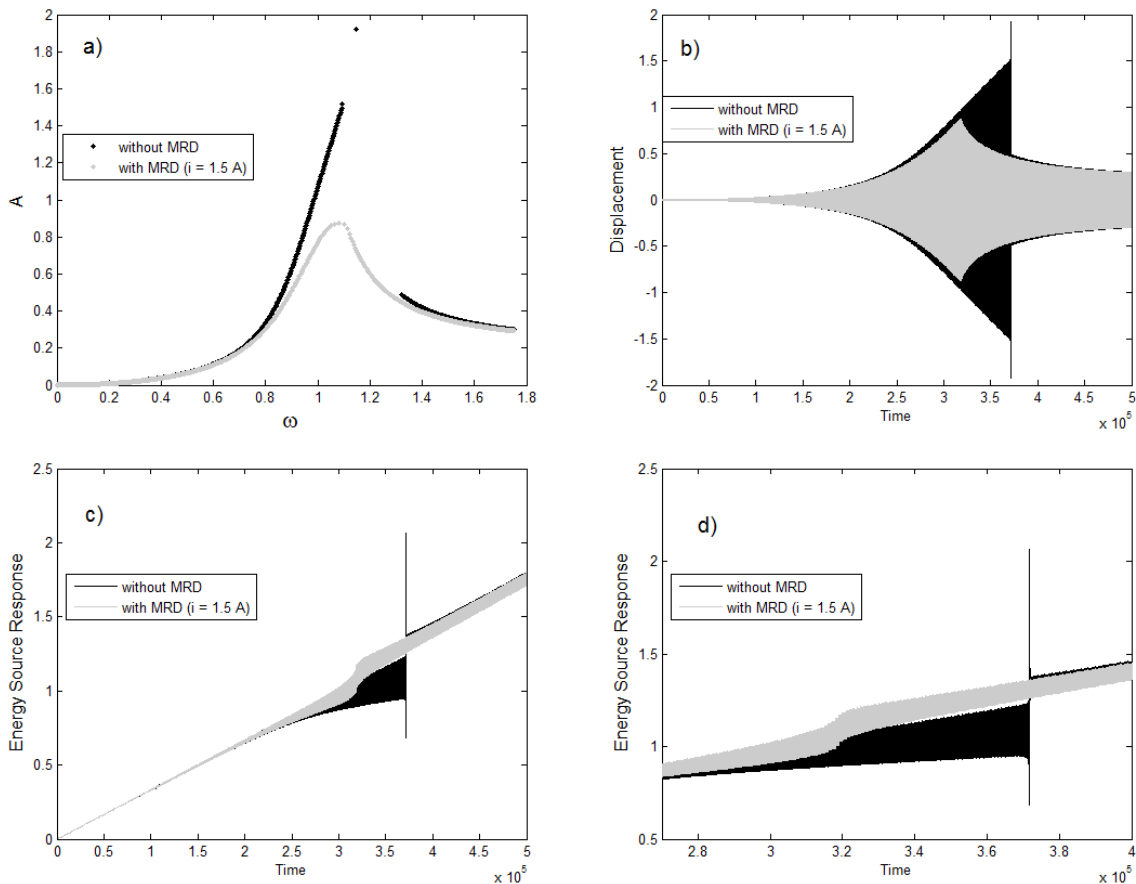


Figure 7: Jump phenomenon observed when the mean frequency ω is slowly increased: (a) Frequency-response diagram with MRD ($i = 1.5$ A) and (b) Time response.

Figure 5 shows the Sommerfeld effect and its suppression for different values of electric current. The equivalent non-ideal response is compared with the MRD non-ideal systems allow to verify the MRD effect in this dynamical system. It might be observed that the amplitude reduction can be achieved for different electric current.

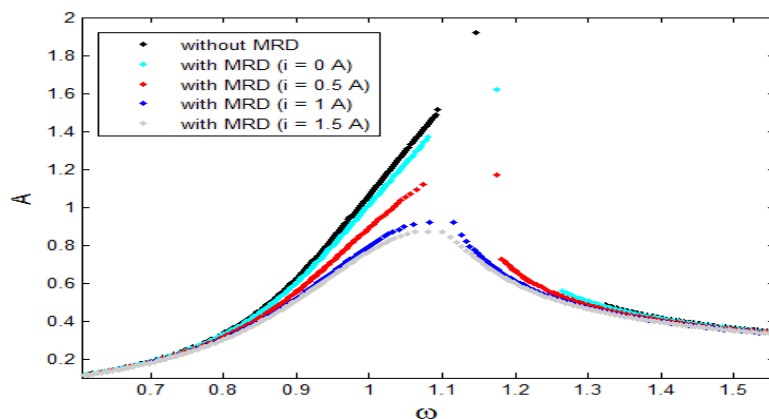


Figure 8: Jump phenomenon for different values of electric current.

5. CONCLUSION

In this paper, the attenuation and suppression of the Sommerfeld effect of a non-ideal vibrating system, using Magneto rheological damper (MRD) was presented. This happens as a consequence of the damper properties variation due to different electric current introduced in MR damper. For this reason, if the values of electric current in MRD is increased, the damping force increase as well, and therefore, the Sommerfeld effect and the amplitude in resonance regions is avoided. The analysis of this systems shows that the introduction of MR damper, in this case, it is efficient to suppress or inhibit the energy transfer from DC motor to nonlinear oscillator in resonance cases.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

- Balthazar, J. M., Mook, D. T., Weber, H. I., Brasil, R. M. F. L. R. F., Fenili, A., Belato, D. and Felix, J. L. P., 2003, "An overview on non-ideal vibrations", *Meccanica*, Vol. 38, p. 613 – 621.
- Belato, D., 1998, Não-linearidades no Eletro Pêndulo. MSc Dissertation. State University of Campinas, Brazil.
- Castão, K.A.L., Góes, L. C. S., Balthazar, J. M., 2010, "A note on the attenuation of the Sommerfeld effect of a non-ideal system taking into account MR damper and the complete model of a DC motor", *Journal of Vibration and Control*, Vol. 17, p. 1112-1118.
- Dominguez, A., Sedaghati, R. and Stiharu, I., 2006, "A new dynamic hysteresis model for magnetorheological dampers". *Smart Material and Structures*, Vol. 15, p. 1179–1189.
- Ma, X.Q., Wang, E.R., Rakheja, S. and Su, C.Y., 2003, "Evaluation of modified hysteresis models for magnetorheological fluid dampers". In: *The Fourth International Conference on Control and Automation (ICCA '03)*. Montreal, Canada.
- McManus, S. J., St. Clair, A., Boileau, P. E., Boutin, J. and Rakheja, S., 2002, "Evaluation of vibration and shock attenuation performance of a suspension seat with a semi-active magnetorheological fluid damper". *Journal of Sound and Vibration*, Vol. 253, p. 313–327.
- Nayfeh, A.H. and Mook, D.T., 1979. *Nonlinear Oscillations*. New York: Wiley-Interscience.
- Felix, J. L. P., Balthazar, J. M., Brasil, R. M. L. R. F.; Pontes Jr, B. R., 2009a, "On Lugre friction model to mitigate nonideal vibrations". *Journal of Computation and Nonlinear Dynamics*, Vol. 4, p. 034503-1-034503-5.
- Felix, J. L.P, Balthazar, J. M. and Brasil, R. M. L. R. F., 2009b, "Comments on nonlinear dynamics of non-ideal Duffing–Rayleigh oscillator: numerical and analytical approaches". *Journal of Sound and Vibration*, Vol. 319, p. 1136–1149.
- Piccirillo, V., Balthazar, J. M., Pontes Jr., B. R. and Felix, J. L. P. , 2009, "Chaos control of a nonlinear oscillator with shape memory alloy using an optimal linear control: Part II: Non-ideal energy source, *Nonlinear Dynamics*, Vol. 55, p. 139-149.
- Piccirillo, V., Balthazar, J. M., Pontes Jr, B. R. and Felix, J. L. P., 2008, "On a nonlinear and chaotic non-ideal vibrating system with shape memory alloy (SMA)". *Journal of Theoretical and Applied Mechanics*, Vol. 46, p. 597–620.
- Tusset, A. M., Rafikov, M. and Balthazar, J. M., 2009, "An intelligent controller design for magnetorheological damper based on quarter-car model", *Journal of Vibration and Control*, Vol. 12, p. 1907–1920.
- Tusset, A. M. and Balthazar, J. M., 2013a, "On the Chaotic Suppression of Both Ideal and Non-ideal Duffing Based Vibrating Systems, Using a Magneto rheological Damper". *Differential Equations and Dynamical Systems*, Vol. 21, p. 105-121.
- Tusset, A. M., Balthazar, J. M. and Felix, J. L. P., 2013b, "On elimination of chaotic behavior in a non-ideal portal frame structural system, using both passive and active controls". *Journal of Vibration and Control*, Vol. 19, p. 803-813.
- Warminski, J., and Balthazar, J. M., 2003, "Vibrations of a Parametrically and Self-Excited System with Ideal and Nonideal Energy Source", *RBCM – J. of the Brazilian Soc. Mechanical Sciences*, Vol. 25, p. 413 – 420.

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