ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

VARIABILITY STUDY IN TOPOLOGY OPTIMIZATION USING STATISTICAL TOOLS

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Abstract. *This paper intends to evaluate the impacts caused by inherent variabilities in optimal topologies of statically loaded structures. The selected topology optimization methodology is the Solid Isotropic Material with Penalization (SIMP) approach and the objective is to minimize the compliance of the structure. The uncertainty of five parameters has been evaluated: volume fraction, Young modulus, Poisson coefficient, boundary conditions and load. In order to perform this study, the authors have employed components of variation and design of experiments as statistical tools. It can be concluded that significant variations can be caused by the load and the volume fraction in the optimal topology. At the end, a robust topology for normal and uniform parameter distribution is proposed. The comparison between robust and deterministic topology is performed graphically.*

Keywords: *topological optimization, robust design, variability, six-sigma methodology*

1. INTRODUCTION

Topology Optimization aims to identify optimal material layouts, in other words, determines whether there is material at a certain location or not. Loads and boundary conditions are not restrictions to the optimization algorithm. Usually the constraints are given in terms of deformation, stress, natural frequency, and volume. It is usually employed during the conceptual design phase yielding a proposal that can be further adjusted for performance and manufacturability. Topology optimization requires a structural analysis that is usually performed using finite element methods and an optimization algorithm to minimize a cost function subject to some restrictions. Diverse optimization techniques such as genetic algorithms, optimality criteria method, among others, can be employed.

Additionally, the assessment and measurement of the uncertainty/variability during experiments is highly recommended. Topology optimization problems are no exception. For instance, Chen *et al.* (2009) have exemplified these variations in topology optimization by projects called Robust Shape and Topology Optimization (RSTO), in which random field uncertainties are considered and a robust design is implemented knowing these variations. Their study has shown that the difference between deterministic design and robust design is notable. However, their study has been focused on the robust shape design under fixed variations; there is no assessment of the uncertainties. In Asadpoure *et al.* (2011), a robust topology for structures under uncertainties has been provided as well. Nevertheless, in this article the variability have been also fixed and set only in the stiffness matrix, without further analysis of other possible variations. Tootkaboni *et al.* (2012) have studied topology optimization under uncertainties, by using a polynomial chaos approach. With pre-fixed parameters variability, a fairly difficult mathematical method to define the robust design is proposed.

This paper proposes the study of uncertainty in a topology optimization experiment. Further, in the article, a robust shape topology is also found. Differently, the authors of this paper propose a study of the uncertainty before the robust design is modeled. Most importantly, the method proposed to the analysis is fairly simple and easy to be implemented. In addition, a simple method for modeling the robust design is employed.

In order to measure these possible variations, caused by known uncertainty, and determine which parameters may cause the most variance in the final topology, statistical tools are used in this paper. As suggested by Yaman (2012), the method of Design of Experiments (DOE) is used, instead of the One Factor at Time (OFAT). In this way, it is possible to measure not only the variation in each parameter but also the variation caused by the interaction within variables. However, to utilize the best of the DOE, it is necessary to make a preliminary parameter analysis, the Component of Variation (COV) study. Both methods are described further in Section 2. The selected topology optimization problem is the compliance minimization of statically loaded structures implemented by Sigmund (2001) in Matlab®. The topology optimization routine is based on the Solid Isotropic Material with Penalization (SIMP) Method and is briefly explained in Section 2. The uncertainty of five parameters has been evaluated: volume fraction, Young modulus, Poisson coefficient, boundary conditions and load. Under these variabilities, different configurations can be found by the topology optimization routine. The comparison between two different configurations, described by matrices, is performed by modifying these matrices into vectors and comparing these vectors using the MAC (Modal Assurance Criterion) number. This MAC number is explained in Section 2. The numerical results are shown in Section 3 and conclusions are drawn in Section 4.

ISSN 2176-5480

2. METHODOLOGY

This section presents the employed methodology in this manuscript. Firstly, the topology optimization problem is posed and the methodology to solve it is briefly explained. Afterwards the statistical tools (COV and DOE) are presented. At last, the use of the MAC number is explained.

2.1 Topology Optimization

To illustrate the proposed method for uncertainty analysis, the well-established topology optimization concept is employed. This concept is explained by Santos and Trevisan (2005) as a complete optimal geometrical definition, defined by the knowing of external actions as well as mechanical and geometrical restrictions. Sigmund (2001) proposes a MATLAB® code that implements a topology optimization routine based on the Solid Isotropic Material with Penalization (SIMP) Method. The method is explained in Rozvany *et al.* (1992), in which the idea is penalize porous regions, because of its high production cost, and therefore produce a solid that is mostly composed by solid parts and empty spaces.

The SIMP Method fixates the material properties; variability is then imposed by the different relative densities in the discretization methods. The domain is set to be rectangular, in which square finite elements are used. Nelx is the number of elements in the x axis and Nely is the number of elements in the y axis. The method, then, has the objective of optimizing the Eq. 1.

$$\begin{aligned} min_x \ &: c(x) = \boldsymbol{U^T K U} = \sum_{\boldsymbol{e=1}}^{\boldsymbol{N}} (x_e)^p {u_e}^T k_0 u_e \\ subject\ to\ &: \frac{V(x)}{V_0} = f \\ &: \boldsymbol{KU} = \boldsymbol{F} \\ &: 0 < x_{min} \leq x \leq 1 \end{aligned} \qquad (1)$$

U is the global displacement vector, **F** is the force vector, **K** is the global stiffness matrix, u_e and k_e are the elements of **U** and **K**, x is the density vector, x_{min} is the vector of minimum relative densities, N = (nelx * nely) is the element number, p is the penalization factor (in this article p=3), V(x) is the material volume, V_0 is the calculated volume and f(volfrac) is the function relating both volumes.

A simple OC (optimality criteria) iterative method is used for this optimization. As suggested by Bendsøe (1995), Eq. 2 refers the optimization criteria.

$$x_e^{new} = \left\{ \begin{array}{ll} \max(x_{min}, x_e - m) & if\ \ x_e B_e^{\eta} \leq \max(x_{min}, x_e - m), \\ x_e B_e^{\eta} & if\ \ \max(x_{min}, x_e - m) < \ x_e B_e^{\eta} < \min(1, x_e + m), \\ \min(1, x_e + m) & if \min(1, x_e + m) \ \leq \ x_e B_e^{\eta} \end{array} \right\} \qquad (2)$$

Here, m is a positive limitation, η= ½ is a numerical dumping coefficient and B_e is calculated following Eq. 3.

$$B_e = \frac{-\frac{\partial c}{\partial x_e}}{\lambda \frac{\partial V}{\partial x_e}} \qquad (3)$$

Using finite elements, Eq. 1, 2 and 3 are solved. A convergence coefficient between x_e^{new} and x_e is adopted as being 1%. By this topology optimization, it is intended to realize an experiment to measure the variation in the final topology, depending on the variation in the initial parameters. The MATLAB® code is available in Sigmund (2001).

2.2 Component of Variation (COV)

Ross *et al.* (1995) defines the COV as a mean to plan adequate samples and therefore run the DOE. In order to plan a COV and a posterior DOE, it is first needed a sampling tree. The tree is a mere graphic or tabled representation of the variables and the way these variables interact with one another. It is a tool to represent the set of variables for each measurement during the experiment. The measures can then be obtained and analyzed using pertinent graphs. The variability chart shows which variable leads to the most variation in the measure, by illustrating the measurement and clearly identifying the variables with which this specific measurement have been made.

Another important graphic to be analyzed is the Xbar/R. The graphic R determines if the measurement is under control. Only after verifying the control of the experiment the results may be considered into analysis. To the experiment to be under control, the points in the R graphic must be within the lines of limit.

ISSN 2176-5480

The graphic Xbar illustrates if the major cause of variation is within the subgroup of analysis or outside. The subgroup analyzed is always the last level of the tree. If the points are outside the limits in the Xbar graphic, the major cause of variation is outside the subgroup.

A well based decision to whether the variable is important to the uncertainty of the experiment can be made after analyzing both graphs. It is possible to refine the experiment, by withdrawing the effects of the subgroup in analysis, taking the average of measurements on that subgroup. That way, escalating to the top of the tree, all the important variables can be studied.

2.3 Design of Experiments (DOE)

Noesis Solutions (2011) defines DOE as a systematic mean to perform experiments, leading to a maximum gain of knowledge with less experimentation. Kaminari (2002) establishes that the method is used in scientific research to evaluate multiple parameters and their interaction.

Multiple methods can be used to perform a DOE. For the purpose of this work, a well-established orthogonal method known as Full Factorial is used. In orthogonal methods, the parameters are independent of each other. Consequently, at each interaction new information is generated. The two level full factorial method is used to approximate linear functions and it is vastly used because it is rapid and efficient. The levels are set as '+' and '-'.

Figure 1 illustrates the interactions between parameters in a two level full factorial DOE with 3 parameters.

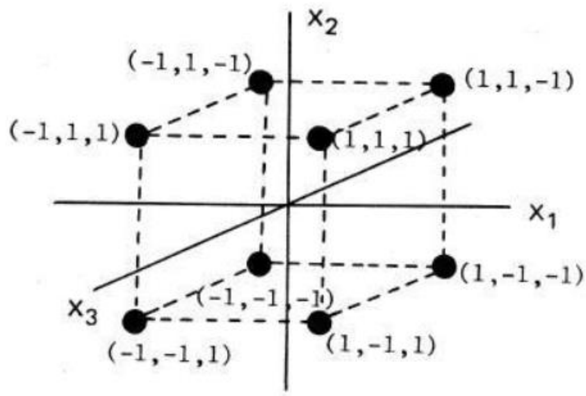

Figure 1 – Schematic drawing – interactions two level full factorial DOE with 3 parameters. Font: Noesis Solutions (2011)

DOE analysis is made through calculation of effects. Effect is a mean of the variation in results for '+' and '-' levels for each variable and its interactions. Table 1 presents an example to the calculation of effects.

Table 1 – Example table for the calculation of effects. Font: Ross *et al.* (1995)

A	B	AB	Y
+	+	+	94
+	-	-	82
-	+	-	78
-	-	+	98

The effect of B is calculated as shown in Eq. 4.

$$Effect_B = \frac{B_{+\text{mean}} - B_{-mean}}{2} = \frac{94+78}{2} - \frac{98+82}{2} = -4 \tag{4}$$

The effects can be plotted and compared, in order to visualize the parameters that cause most variance in the final result. The normal probability graph can be used to see the most relevant variables. Knowing that effects must follow a straight line, the variables or interactions that distances from the line are the ones relevant for the uncertainty of the solution.

All the graphs for the COV and DOE analysis were plotted using the JMP®10 software, which has a free trial license, available for 30 days.

2.4 The MAC number

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The number MAC is a statistic indicator illustrated by Allemang (2003). In this paper, it is used as a form to measure the variation between two vectorized matrices. The number is formulated as seen in Eq. 5.

$$MAC_{cdr} = \frac{\psi_{dr}{}^{H}\psi_{cr}\psi_{cr}{}^{H}\psi_{dr}}{\psi_{dr}{}^{H}\psi_{dr}\psi_{cr}{}^{H}\psi_{cr}} \tag{5}$$

Where ψ_{dr} is the 'd' vector, mode 'r', ψ_{cr} is the 'c' vector, mode 'r' and ψ^{H} is the complex transpose conjugate (hermitian conjugate) of ψ. The vectors must be used in columns and for real numbers we have $\psi^{H} = \psi'$.

The MAC number hence measures the variation between these two vectors 'd' and 'c'. It assumes values between 0 and 1, the closer to 1, the more similar are the analyzed vectors. For the purpose of this paper, a number which the more different the vector, the closer to 1 is needed. So Y = 1 - MAC is defined.

3. RESULTS AND DISCUSSION

This section presents to the reader the use of statistical tools mentioned in the previous section. Using the topology optimization MATLAB code presented by Sigmund (2001), a full COV is elaborated. Relevant parameters extracted from the COV are then used for the DOE analysis. At last, a robust topology is proposed.

3.1 Experiment Method

As previewed shown in the article, the statistical tools used by the authors are the COV and DOE, comparing the number Y=1-MAC for each interaction. The sampling tree is shown in Tab. 3, that can be seen further in the paper.

Relevant parameters, which noticeable cause major variations in the final topology, were chosen to make the sampling tree. The parameter '*f*' is the volume fraction, E is the Young module and nu is the Poisson coefficient. Nelx and Nely define the chosen domain and therefore are fixed. Load and Boundary conditions are inputed in the simulation using a user friendly interface designed by Zhao (2003). The domain is set to be $\{1 \le x \le 61; 1 \le y \le 21\}$. Load and boundary conditions are represented in the Tab. 2.

Table 2 – Boundary conditions and load application points

Situation	1	2	3
Boundary condition – Y restriction	(1,21)	(1,19)	-
Boundary condition – X and Y restrictions	(61,21)	(61,21)	-
Load Application Points	(28,1)	(31,1)	(34,1)

Every Fy are considered to be 10N.

3.2 COV analyzes

First, the sampling tree must be fixed. Table 3 presents the tree used in this experiment. In order to compare the different topologies using the MAC number and run COV interactions, it is necessary to set a model optimal topology. Table 4 shows the defined parameters to this model optimization. The parameters are set in the mean point of the tree. Figure 2(a) illustrates the topology optimization result.

Table 4 – Model optimization parameters

Nelx	Nely	*F*	E	Nu	Boundary condition	Load
60	20	0,45	0,9	0,4	1	2

Using the parameters defined in Tab. 4, it is possible to run different topology optimizations. Figures 2(b), 2(c) and 2(d) illustrate the variation depending on the input parameters.

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Table 3 – Model optimization parameters

Nelx	Nely	*F*	E	Nu	Boundary condition	Load	Measurement
60	20	0,4	1	0,3	1	1	1
60	20	0,4	1	0,3	1	2	2
60	20	0,4	1	0,3	1	3	3
60	20	0,4	1	0,3	2	1	4
60	20	0,4	1	0,3	2	2	5
60	20	0,4	1	0,3	2	3	6
60	20	0,4	1	0,5	1	1	7
60	20	0,4	1	0,5	1	2	8
60	20	0,4	1	0,5	1	3	9
60	20	0,4	1	0,5	2	1	10
60	20	0,4	1	0,5	2	2	11
60	20	0,4	1	0,5	2	3	12
60	20	0,4	0,8	0,3	1	1	13
60	20	0,4	0,8	0,3	1	2	14
60	20	0,4	0,8	0,3	1	3	15
60	20	0,4	0,8	0,3	2	1	16
60	20	0,4	0,8	0,3	2	2	17
60	20	0,4	0,8	0,3	2	3	18
60	20	0,4	0,8	0,5	1	1	19
60	20	0,4	0,8	0,5	1	2	20
60	20	0,4	0,8	0,5	1	3	21
60	20	0,4	0,8	0,5	2	1	22
60	20	0,4	0,8	0,5	2	2	23
60	20	0,4	0,8	0,5	2	3	24
60	20	0,5	1	0,3	1	1	25
60	20	0,5	1	0,3	1	2	26
60	20	0,5	1	0,3	1	3	27
60	20	0,5	1	0,3	2	1	28
60	20	0,5	1	0,3	2	2	29
60	20	0,5	1	0,3	2	3	30
60	20	0,5	1	0,5	1	1	31
60	20	0,5	1	0,5	1	2	32
60	20	0,5	1	0,5	1	3	33
60	20	0,5	1	0,5	2	1	34
60	20	0,5	1	0,5	2	2	35
60	20	0,5	1	0,5	2	3	36
60	20	0,5	0,8	0,3	1	1	37
60	20	0,5	0,8	0,3	1	2	38
60	20	0,5	0,8	0,3	1	3	39
60	20	0,5	0,8	0,3	2	1	40
60	20	0,5	0,8	0,3	2	2	41
60	20	0,5	0,8	0,3	2	3	42
60	20	0,5	0,8	0,5	1	1	43
60	20	0,5	0,8	0,5	1	2	44
60	20	0,5	0,8	0,5	1	3	45
60	20	0,5	0,8	0,5	2	1	46
60	20	0,5	0,8	0,5	2	2	47
60	20	0,5	0,8	0,5	2	3	48

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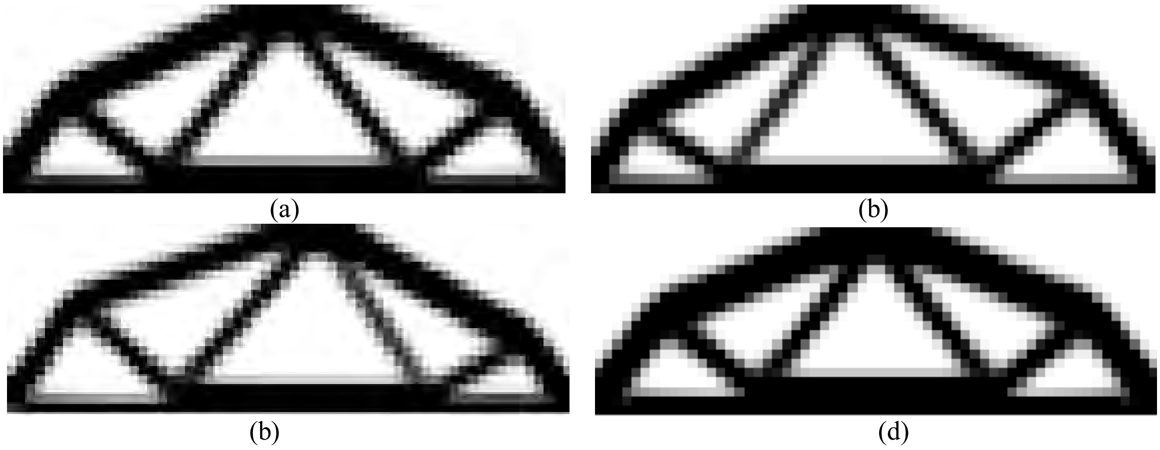

Figure 2 – (a) Model optimal topology, (b) Topology for the 4th Tree branch, (c) Topology for the 6th Tree branch and (d) Topology for the 26th Tree branch

After running the simulation for all sampling tree branches, the number MAC must be calculated for each interaction. These results can be statistically analyzed using the variability chart and Xbar/R graphics. Figure 3 shows the variability chart for this first interaction.

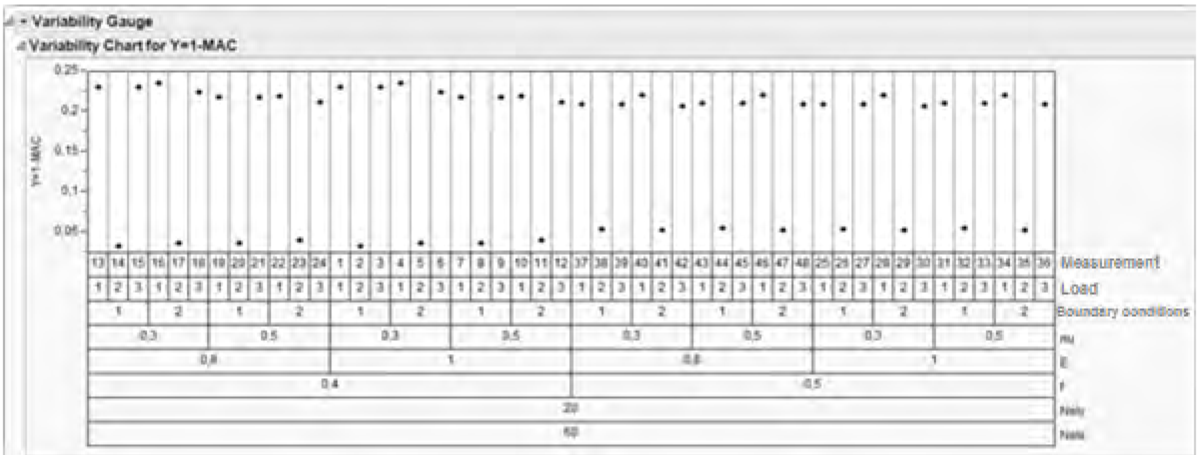

Figure 3 – Variability chart – 1st interaction

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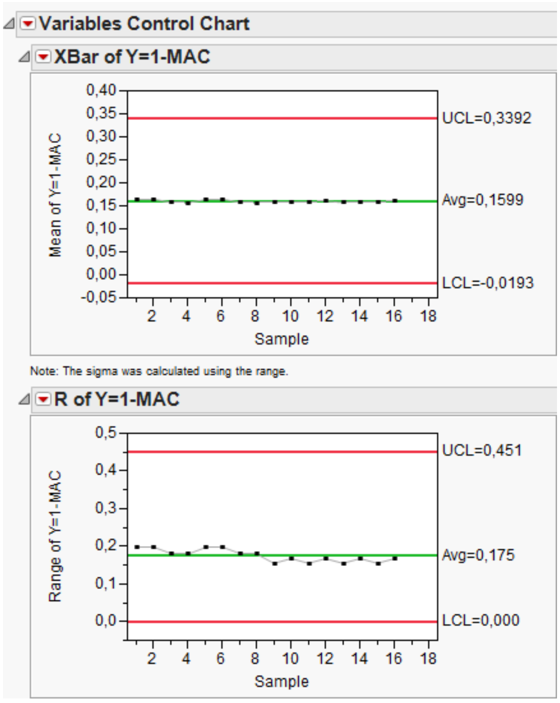
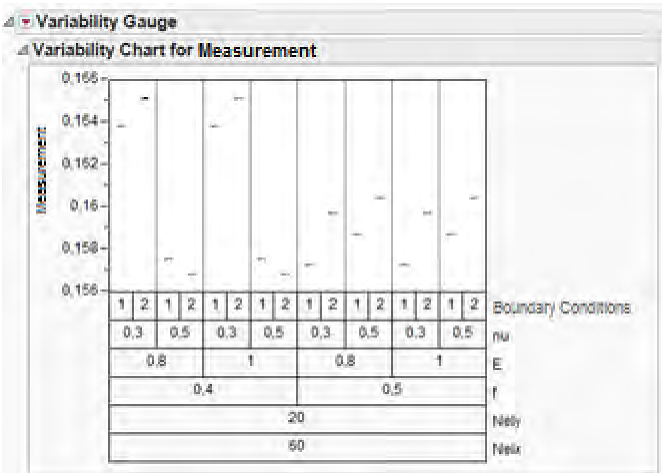


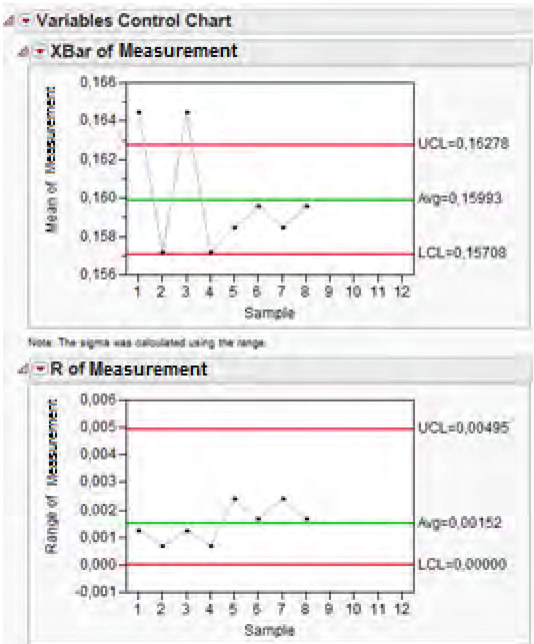
Figura 4 - Xbar/R graphic – 1st interaction

Clearly, the font of variation is within the subgroup load. Figure 4 confirms this statement with the Xbar/R Graphic.

These figures demonstrate that the main cause of variation in the final topology is the parameter 'load'. However, it is intended to discover other possible causes that may be substantial. To make further assessments with the COV it is necessary to withdraw the effect of the 'load' parameter. By ascending in the sampling tree, using the result means to diminish a level, new results can be plotted and be analyzed in Fig. 5(a) and Fig. 5(b).



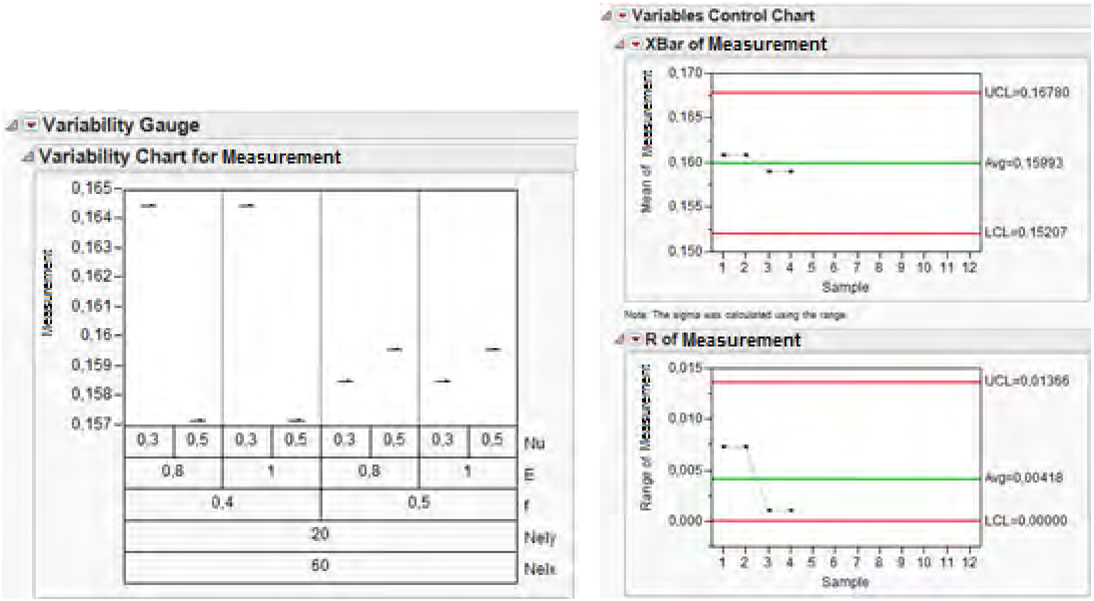
(a)



(b)

Figure 5 – (a) Variability chart of 2nd COV, (b) Xbar/R graphics – 2nd COV

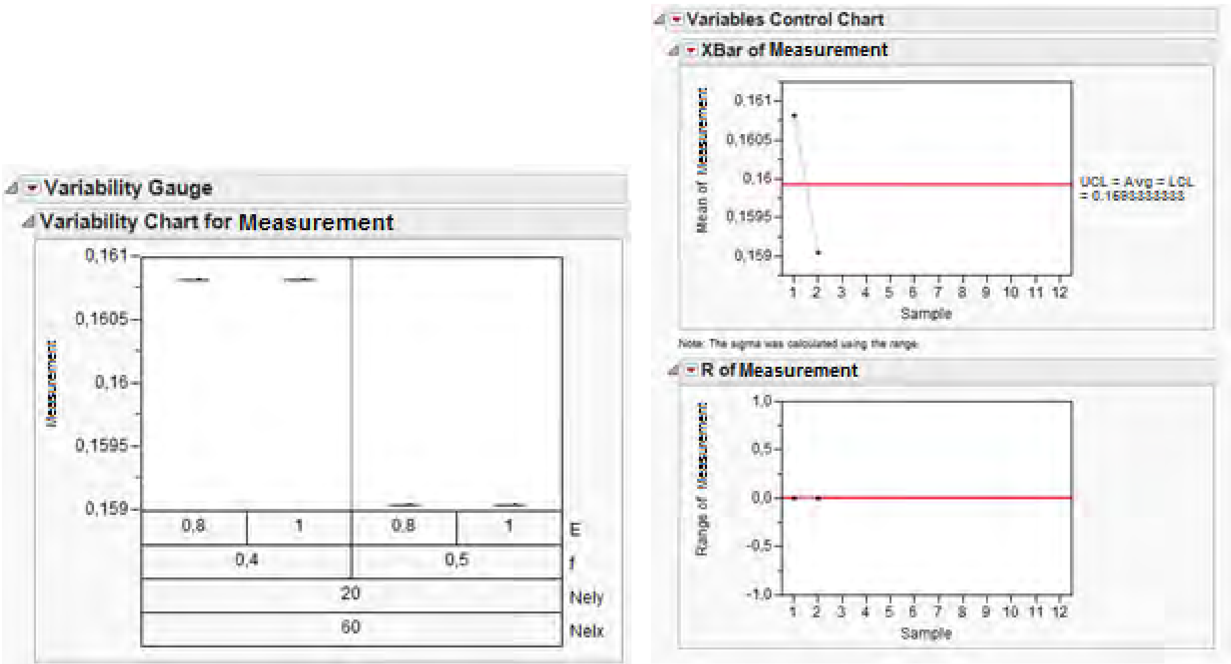
The major variation font is outside the observation group 'Boundary conditions'. Another interaction must be made. Ascending in the sampling tree one more time, withdrawing the effects of the subgroup 'Boundary conditions', Fig. 6(a) and Fig 6(b) can be studied.



(a) (b)

Figure 6 – (a) Variability Chart for 3rd COV, (b) Xbar/R graphics for 3rd COV

By analyzing these graphics it is possible to conclude that the major font of variation is within the analyzed subgroup ‘nu’. To obtain a complete parameter analyses, another COV is made. Figure 7(a) and 7(b) show the results.



(a) (b)

Figure 7 – (a) Variability chart for 4th COV, (b) Xbar/R graphics for 4th COV

Studying figures 7(a) and 7(b), it is possible to conclude that the major cause of variation is outside the subgroup studied and hence the fraction volume ‘*f*’ needs to be analyzed in the DOE. The four rounds of COV made it possible to determine which parameters variation implicates in the most changes in the final optimization. For the DOE, ‘load’, ‘nu’ and ‘*f*’ are going to be analyzed.

3.3 DOE analyzes

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After the conclusion of the COV experiments, it is possible to use the resulting parameters into a DOE experiment. The volume fraction is set to vary from 0.4 to 0.5, maintaining a 10% of variation in the total mass value. The '-' level for the Poisson coefficient is set by the ABNT norm for concrete and the '+' level is adopted as a possible variation demonstrated by Pinto *et al.* (2010). Table 5 summarizes the varying parameters used for the DOE experiment. Nelx is fixed in 60, Nely in 20, E is set to be 1 and the boundary condition used is also 1.

Table 5 – DOE reference parameters

Factor	Level -	Level +
Load	Applied in center	Applied 5% to right
Nu	0,2	0,3
F	0,4	0,5

A two level full factorial design is used. Figure 8(a) and Fig. 8(b) finalize the parameters analyzes.

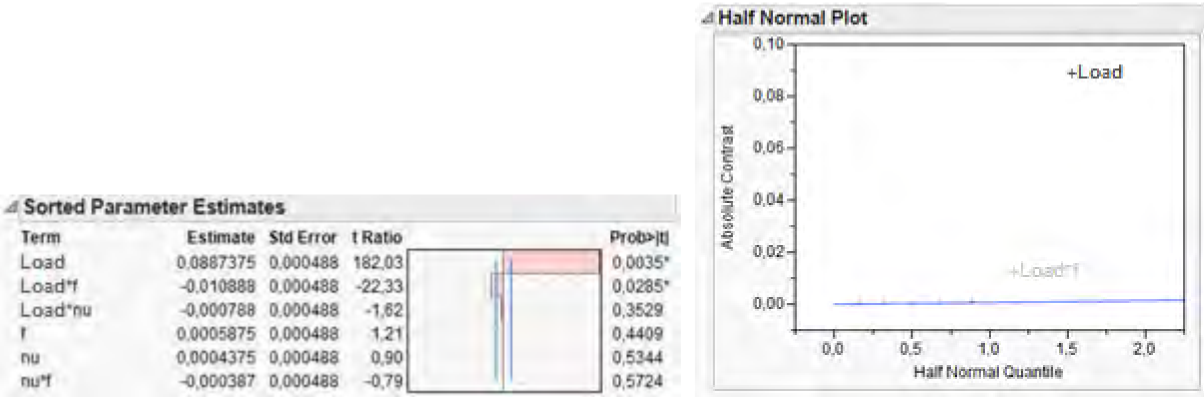

Figure 8 – (a) Parameters effects graphic, (b) Normal probability graphic

In Fig. 8(a) the parameters effects can be easily seen. It is noticeable that 'load' and 'load*f' are the parameters that most interfere in the end topology. Figure 8(b) assures the two parameters mentioned before as the main causes of variation in the optimal topology. Being this the case, these are the parameters to be studied for a robust topology.

3.4 Robust Topology

In order to present a possible robust design that incorporates the variation studied earlier in this paper, the superimposition theorem is used. The parameters variations are shown in Tab. 6.

Table 6 – Parameters adopted to robust design

Nelx	Nely	*F*	Nu	Boundary condition	Load
60	20	Variation within 0,4 and 0,43	0,2	1	10% variation, 5% at the left and 5% at the left of the domain center

Two different cases are assumed: in case 1 the parameters vary within a bell curve; in case 2 the parameters vary uniformly. The weights used to the bell curve distribution are shown in Fig. 9 and it is used to find the robust topology.

ISSN 2176-5480

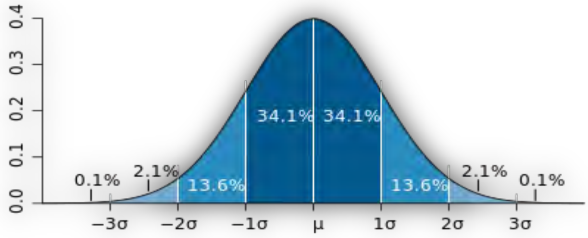

Figure 9 – Bell Curve and its probabilities Font: Wikimedia commons (2001)

The robust design using this parameters variation is hence normalized, in order to maintain the total mass in the system. The result can be seen in Fig. 10(b). A comparison can be performed with Fig. 10(a), which is the result of a deterministic simulation.

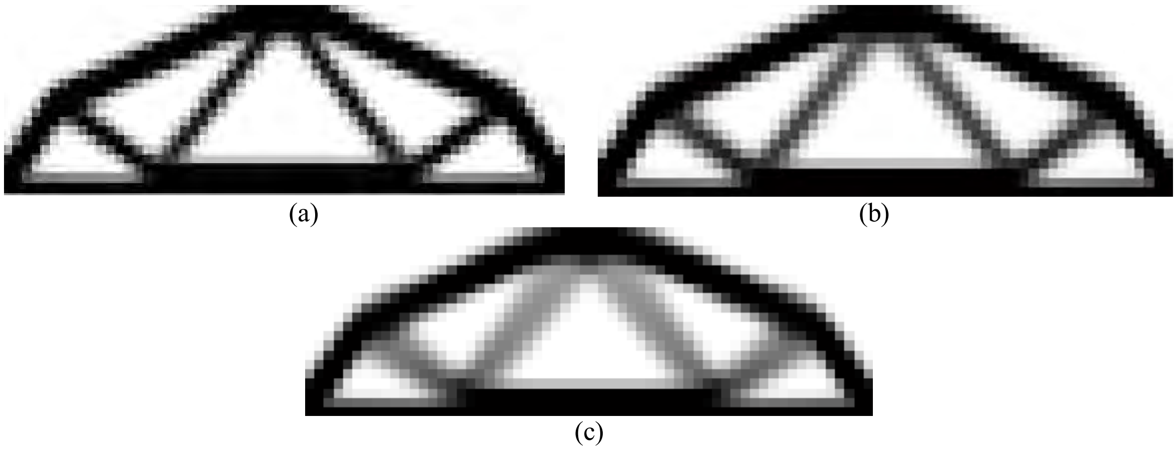

Figure 10 – (a) Deterministic topology, (b) Robust topology – Bell Curve, (c) Robust topology – uniform variations

A visual comparison between the two images can easily be performed and it is noted that the robust topology is more round. Also, the trusses are thicker, resulting of the sum of various different input parameters.

In order to provide a robust design using uniformed variation in the parameters, no weight needs to be used. The mass matrix should be normalized after the sum of all the result simulation. The result is seen in Fig. 10(c).

In this case the influence of parameter variations are more pronounced, which may be seen by the thicker trusses.

4. CONCLUSION

Statistical tools were used to study the variation in main parameters in a topology optimization study. Overall, the methods used to measure the variation were proven useful, easy and efficient.

The work demonstrates that using COV and DOE as a mean to verify the major causes of variation is not only possible but may also be highly recommended. These methods rapidly generate graphics that are easy to understand and manipulate, needing few computer time. The use of the MAC number, furthermore, is proven to be simple and correct. At last, the topology optimization used together with the friendly interface defines a rapid way to visualize the optimal topology, providing a matrix easy to manipulate.

The proposed robust topology, discussed at the last portion of the article, demonstrates visually the importance in considering variation in the projects. By knowing that the factor *f* and nu are the most important cause of variation, an optimal topology is successfully presented considering these variations.

For next studies, the authors propose a validation of the robust topology, using structural and probability considerations. Moreover, it is proposed a new study with larger fonts of variation, which may generate diverse topologies.

5. ACKNOWLEDGMENTS

Giuliana S. Venter and Maíra M. da Silva are thankful for their grants CNPq 130381/2013-6 and FAPESP 2012/22470-6 and CNPq 02588/2011-6, respectively.

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7. RESPONSIBILITY NOTICE

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