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THE INPUT-STATE LINEARIZATION OF A MAGLEV VEHICLE TYPE

Célia Aparecida dos Reis
Faculdade de Engenharia de Ilha Solteira - FEIS/UNESP, Ilha Solteira SP, Brasil
Célia@mat.feis.unesp.br

José Manoel Balthazar
UNESP, Departamento de Estatística, Matemática Aplicada e Computação, Rio Claro, SP, Brasil
jmbaltha@rc.unesp.br

Luciano Barbanti
Faculdade de Engenharia de Ilha Solteira - FEIS/UNESP, Ilha Solteira SP, Brasil
barbanti@mat.feis.unesp.br

Abstract: *This paper presents the input-state linearization of a vehicle type MAGLEV (magnetic levitation transport), by using standard Lie derivatives techniques, based in the simplified nonlinear model. Necessary and sufficient conditions for controllability are presented and furthermore, the analytic form of the controller ´u´ so that the nonlinear system dynamics is transformed into an equivalent linear time-invariant dynamics was exhibited. More specifically we find a diffeomorphism and a nonlinear feedback control law 'u' such that the new state variables and the new input satisfy a linear time-invariant dynamics.*

Keywords: *MAGLEV System, input-state linearization, Lie derivative, controllability.*

1. INTRODUCTION

The feedback exact linearization is a design methodology of nonlinear systems. This procedure allows changing the dynamics of a nonlinear system, in a linear dynamics one through a previously nonlinear state feedback or nonlinear output chosen.

With this in mind, it is almost always necessary to make a state variable change, and more, to be introducing a variable auxiliary input (Slotine, 1991; Isidori, 1995).

This methodology has been the subject of research for many researchers in recent years. This procedure has been used successfully in a wide range of applications, such as tracking problems in control robotic arms and manipulators, artillery, helicopters, airplanes and satellites, as well as being used in medical apparatus and in the chemistry and pharmaceutics (Alvarez-Gallegos, 1994; Barbanti, 2012; Chem, 1998, 2000, 1999; Isidori, 1995; Reis, 2012-a, 2012-b; Silva, 2003; Slotine, 1991; Ray, 2012; Yabuno 2004, 1991, 2003, 1989).

In the input-state feedback, we consider a nonlinear system of the type $\dot{x} = f(x) + g(x)u$, with *f (x)* and *g (x)* being smooth fields in $\Re^n$. The problem is to designing a control input *u* aiming to transform the nonlinear dynamics or part this, in a linear dynamics takes place in two steps: first we show the existence of a diffeomorphism ϕ *(x)* defined in a region Ω of $\Re^n$ and a nonlinear control law $\dot{z} = Az + b\upsilon$, in such a way that the new state variable $z = \phi(x)$ and the new entry υ satisfy both a linear time invariant relation, $\dot{z} = Az + b\upsilon$ with constant matrices *A* and *b*. After, we use the standard υ projection.

This procedure is justified since the Taylor series linearization has a local character, that is, it is true only for a region around a point, while the feedback linearization is global, i.e., applied to the whole state or output spaces, with the possible exception on isolated points. Moreover, while the linearization due to the analysis of the Jacobian is approximate by the feedback linearization it is exact (Isidori, 1995, Silva, 2003; Slotine, 1991).

In this work we present the input-state linearization of a vehicle type MAGLEV (magnetic levitation transport). The MAGLEV is a new technology for mass transport, which employs magnetic fields to levitate and propel direct high-speed trains, adding safety, low environmental impact and minimal maintenance costs. Hence the interest in to be considering the task in countries like Brazil, Germany, Japan, China, United States, Australia, Thailand, etc ...

Here we consider a simplified nonlinear model of such a system described in the state space obtained by Yabuno (2004), (1989), with scalar functions as output. Necessary and sufficient conditions for controllability will be presented, and in addition, the analytical form of the controller *u* and the ultimate form of the linearized dynamics are displayed too.

This paper is organized as follows. Section 2 presents a simplified mathematical model of the vehicle MAGLEV beyond the theoretical conditions for the realization of input-state linearization. Section 3 presents the input-state linearization, including the construction of the diffeomorphism, and the necessary and sufficient conditions for the

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application and the new nonlinear dynamics in the new state variables introduced. In section 4 we will do the conclusions of the work and in section 6 the references.

2. SIMPLIFIED MODEL FOR THE MAGLEV VEHICLE

Here we consider a simplified model of a vehicle MAGLEV type as in Fig. 1, obtained by Yabuno (2004). 0 is the origin of the Cartesian plane and we suppose that the levitated body moves freely only in the direction z. Furthermore, z_d measures the vertical displacement, m_1 is the mass of the main body, z_{st} is the distance between the magnets, Z_{b1} is the amplitude of excitation of the magnet base, ω is the excitation frequency of the magnet base, ω_z is the natural frequency of the body z_b is the vertical displacement of the basis of the magnet and $t^* = t\,\omega_z$ and $z^* = z_d / z_{st}$, respectively are dimensionless variables, $\nu = \omega / \omega_z$ and $\varepsilon = z_{b1}/z_{st}$ are parameters, as Yabuno (2004), (1989).

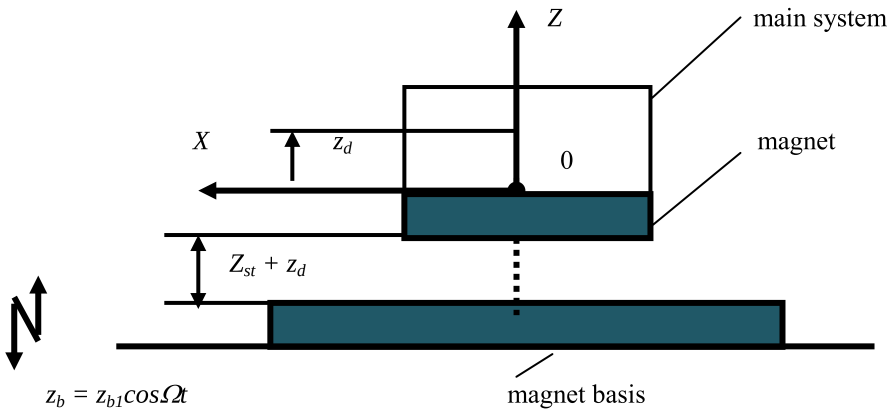

Figura 1: Modelo do corpo de levitação magnética (Yabuno, 2004).

Here we consider the repulsive magnetic force among the nonlinear magnets for small variations and finite distance z_{st} can be approximated by a polynomial with cubic and quadratic terms in which the basis is excited with vertical displacement $z_b = z_{b1}cost\Omega$ as Yabuno (1989). Considering the point as the derivative with respect to the time, the following nondimensional equation is found (Yabuno 2004):

$$\ddot{z}^* = -z^* - \mu_z \dot{z}^* + \varepsilon \cos \nu t^* + 2\alpha_{zz} \varepsilon z^* \cos \nu t^* - \alpha_{zz} z^{*2} - \alpha_{zzz} z^{*3} \tag{1}$$

where $\mu_z \dot{z}^*$ is viscosity linear force acting on the main system α_{zz} and α_{zzz} are the coefficients of z^2 and z^3 in the Taylor series of the magnetic force (Yabuno, 1989).

If we reorganize the Eq. (1) we get:

$$\ddot{z}^* = -z^* - \mu_z \dot{z}^* - \alpha_{zz} z^{*2} - \alpha_{zzz} z^{*3} + \varepsilon(1 + 2\alpha_{zz} z^*) \cos \nu t^* . \tag{2}$$

Defining $x = (x_1, x_2, x_3)$ as the state vector we have:

$$x_1 = z^* \text{ and } x_3 = t^* . \tag{3}$$

In this way the Eq. (2) can be transformed into the nonlinear system:

$$\dot{x} = f(x), \tag{4}$$

where $f(x)$ is a smooth field in $\Re^3$ done by

$$f(x) = \begin{pmatrix} x_2 \\ \eta(x) \\ \omega_z \end{pmatrix}, \tag{5}$$

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$$\eta(x) = -x_1 - \mu_z x_2 - \alpha_{zz} x_1^2 - \alpha_{zzz} x_1^3 + \varepsilon(1 + 2\alpha_{zz} x_1)\cos \nu x_3 . \qquad (5-a)$$

Aiming the input-state linearization for the MAGLEV vehicle, we make the following considerations:
(a) $y = h(x) = x_1$ is the output,
(b) $u(t)$ the system input, (6)
(c) $g(x)$ is the smooth field $g(x) = \begin{pmatrix} 0 \\ 0 \\ \gamma \end{pmatrix}$, with $\gamma \neq 0$ take as a real number.

Upon considerations (5) and (6), the Eq. (2) and (4) can be written as:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) = x_1 \end{aligned} \qquad (7)$$

where f and g are smooth vector fields in $\Re^3$ and $h(x)$ is a scalar function representing the output of the nonlinear system.

According Slotine (1991) and Isidori (1995), a dynamic represented in the form of state equations as in Eq. (7),where f and g are smooth vector fields on $\Re^3$, it is input-state linearizable if there exists a region Ω in $\Re^n$, and a diffeomorphism $\phi : \Omega \to \Re^n$ and a feedback control nonlinear law $u = \alpha(x) + \beta(x)\upsilon$ such that the new state $z = \phi(x)$ and the new entry satisfy a linear time-invariant having the form $\dot{z} = Az + b\upsilon$ where A and b are constant matrices expressed in the companion form.

The question that arises at this point is: all the nonlinear dynamics in the form (7) can be linearized by means of a nonlinear state feedback?

It is known in the literature (Slotine, 1991; Isidori, 1995), that the nonlinear dynamics given by Eq. (7) is input-state linearizable if and only if:

1. The fields $\{g, ad_f g, .. ad_f^{n-1} g\}$ are linear independent; (8)
2. The set $\{g, ad_f g, .. ad_f^{n-1} g\}$ is involutive in the region Ω of the $\Re^n$; (9)

where $ad_f^n g(x) = [f, ad_f^{n-1} g](x)$, $ad_f^0 g = g$ and $ad_f g = [f, g]$ is the Lie bracket respect to the fields f and g.

In this way, under the conditions (8) and (9), the following steps for the application of this technique can be adopted (Slotine, 1991; Isidori, 1995):

1. Give the fields $\{g, ad_f g, .. ad_f^{n-1} g\}$; (10)
2. Check where the conditions regarding the controllability and the involutiviness are being true; (11)
3. After the second step give the first state variable z_1 in the equations:

$$\begin{aligned} &\nabla z_1 . ad_f^i g = 0 \quad i = 0, 1, \ldots, n-2 \\ &\qquad ; \\ &\nabla z_1 . ad_f^{n-1} g \neq 0 \end{aligned} \qquad (12)$$

4. Define the diffeomorphism ϕ (x) and a nonlinear control feedback law $u = \alpha(x) + \beta(x)\upsilon$ with:

$$z(x) = \left[z_1 \quad L_f z_1 \quad \ldots \; L_f^{n-1} z_1 \right]^T, \quad \alpha(x) = -\frac{L_f^n z_1}{L_g L_f^{n-1} z_1} \text{ and } \beta(x) = \frac{1}{L_g L_f^{n-1} z_1}. \qquad (13)$$

In Eq. (13), $L_f^{n-1} z_1$ is the Lie derivative of the scalar function z_1 with respect to the vector field f. In the next section, this procedure will be used for non-linear dynamics given by Eq. (7) representing the MAGLEV vehicle.

3. THE INPUT-STATE LINEARIZATION FOR THE MAGLEV VEHICLE

According to Eq. (10), for the construction of fields $\{g, ad_f g, ad_f^2 g\}$, we have for $i = 2$ that:

$$adj_f^2 g(x) = [f, adj_f^1 g](x) = \nabla adj_f^1 g . f - \nabla f . adj_f^1 g$$

where ∇f is the gradient of the field *f*. Further, from Eq. (5) and (6) it follows that:

$$\nabla g = (0) \text{ and } \nabla f = \begin{pmatrix} 0 & 1 & 0 \\ -1 - 2\alpha_{zz} x_1 - 3\alpha_{zzz} x_1^2 + 2\varepsilon\alpha_{zz} \cos vx_3 & -\mu_z & -v\varepsilon(1 + 2\alpha_{zz} x_1) senvx_3 \\ 0 & 0 & 0 \end{pmatrix}.$$

In this way, we get

$$adj_f g = \begin{pmatrix} 0 \\ \gamma v\varepsilon(1 + 2\alpha_{zz} x_1) senvx_3 \\ 0 \end{pmatrix}. \tag{14}$$

But

$$\nabla ad_f g = \begin{pmatrix} 0 & 0 & 0 \\ 2\gamma\, v\varepsilon\alpha_{zz} x_2 senvx_3 & 0 & \gamma v^2 \varepsilon(1 + 2\alpha_{zz} x_1) \cos vx_3 \\ 0 & 0 & 0 \end{pmatrix},$$

and then:

$$adj_f^2 g = \begin{pmatrix} -\gamma v\varepsilon(1 + 2\alpha_{zz} x_1) senvx_3 \\ 2\gamma\, v\varepsilon\alpha_{zz} x_2 senvx_3 + \gamma v\varepsilon(1 + 2\alpha_{zz} x_1)(v \cos vx_3 + \mu_z sen\, vx_3) \\ 0 \end{pmatrix}. \tag{15}$$

In this way, from the Eq. (6), (14) and (15) the first step to be having the input-state linearization for the MAGLEV system is concluded.

To check the conditions for have controllability and involutiveness, we must initially search for conditions for have $\{g, ad_f g, \ldots, ad_f^2 g\}$ to be linearly independent. This means that the determinant of the controllability matrix has to be different from 0, that is:

$$\left| g, ad_f g, ad_f^2 g \right| \neq 0 . \tag{16}$$

But

$$\left| g, ad_f g, ad_f^2 g \right| = \begin{vmatrix} 0 & 0 & -\gamma v\varepsilon(1 + 2\alpha_{zz} x_1) senvx_3 \\ 0 & \gamma v\varepsilon(1 + 2\alpha_{zz} x_1) senvx_3 & 2\gamma\, v\varepsilon\alpha_{zz} x_2 senvx_3 + \gamma v\varepsilon(1 + 2\alpha_{zz} x_1)(v \cos vx_3 + \mu_z sen\, vx_3) \\ \gamma & 0 & 0 \end{vmatrix}$$

$$= \gamma^3 v^2 \varepsilon^2 (1 + 2\alpha_{zz} x_1)^2 sen^2(vx_3) . \tag{17}$$

Then

$$\left| g, ad_f g, ad_f^2 g \right| \neq 0 \Leftrightarrow (1 + 2\alpha_{zz} x_1)^2 \neq 0 \text{ or } sen^2(vx_3) \neq 0$$

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$$\Leftrightarrow \; x_1 \neq -\frac{1}{2\alpha_{zz}} \text{ or } x_3 \neq \frac{k}{\nu}\pi, \;\; k = 0, \pm 1, \pm 2, \ldots.$$

In this way we have

$$x_1 \neq -\frac{1}{2\alpha_{zz}} \qquad \text{or} \qquad x_3 \neq \frac{k}{\nu}\pi, \;\; k = 0, \pm 1, \pm 2, \ldots. \tag{18}$$

and the fields $\{g, ad_f g, ad_f^2 g\}$ are linearly independent. In such manner then along with the restriction (18) the controllability condition is true.

In order to check the involutivity, one has to prove that:

$$\left| g \;\; ad_f g \;\; [g,\, ad_f g] \right| = 0. \tag{19}$$

But

$$\left| 0 \;\; ad_f g \;\; [g,\, ad_f g] \right| = \begin{vmatrix} 0 & 0 & \\ 0 & \gamma\nu\varepsilon(1+2\alpha_{zz}x_1)\mathit{sen}\nu x_3 & -\gamma^2\nu^2\varepsilon(1+2\alpha_{zz}x_1)\cos\nu x_3 \\ \gamma & 0 & 0 \end{vmatrix} = 0. \tag{20}$$

In this way, $\{g, ad_f g\}$ are involutive.

Thus, from Eq. (20) it follows that the fields $\{g, ad_f g, .. ad_f^{n-1} g\}$ are involutive and from Eq. (18), the controllability conditions are true if and only if:

$$x_1 \neq -\frac{1}{2\alpha_{zz}} \qquad \text{or} \qquad x_3 \neq \frac{k}{\nu}\pi, \;\; k = 0, \pm 1, \pm 2, \ldots. \tag{21}$$

Because the steps 1. and 2. are satisfied, according the Frobenius theorem (Slotine, 1991), there is a scalar function $z_1(x)$ (the output function leading to the input-output linearization of degree 3) that can be derived from the equations:

$$\begin{aligned} &\nabla z_1 . ad_f^i g = 0 \qquad i = 0, 1, \ldots, n-2 \\ &\nabla z_1 . ad_f^{n-1} g \neq 0 \end{aligned} \; ; \tag{22}$$

Cause $n = 3$, we have that the Eq. (22) are in the form:

$$\begin{aligned} &\nabla z_1 . ad_f^o g = \nabla z_1 . g = 0; \\ &\nabla z_1 . ad_f^1 g = 0; \\ &\nabla z_1 . ad_f^2 g \neq 0; \end{aligned} \tag{23}$$

But from the controllability matrix, we get the following partial Eq. (23):

$$\begin{aligned} &\gamma \frac{\partial z_1}{\partial x_3} = 0 \\ &(\gamma\nu\varepsilon(1+2\alpha_{zz}x_1)\mathit{sen}\nu x_3)\frac{\partial z_1}{\partial x_2} = 0 \\ &(-\gamma\nu\varepsilon(1+2\alpha_{zz}x_1)\mathit{sen}\nu x_3)\frac{\partial z_1}{\partial x_1} + (2\gamma\nu\varepsilon\alpha_{zz}x_2\mathit{sen}\nu x_3 + \gamma\nu\varepsilon(1+2\alpha_{zz}x_1)(\nu\cos\nu x_3 + \mu_z \mathit{sen}\,\nu x_3))\frac{\partial z_1}{\partial x_2} \neq 0 \end{aligned} \; . \tag{24}$$

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From Eq. (6) and (21), we have that both, $\gamma \neq 0$ and $(\gamma v\varepsilon(1+2\alpha_{zz}x_1)senvx_3) \neq 0$, and therefore, from Eq. (24) we can conclude that:

$$\begin{aligned} &\frac{\partial z_1}{\partial x_3} = 0 \\ &\frac{\partial z_1}{\partial x_2} = 0 \; . \\ &\frac{\partial z_1}{\partial x_1} \neq 0 \end{aligned} \tag{25}$$

Hence, from Eq. (25) we have $z_1 = h(x) = x_1$ being a function that leads to the degree $r = 3$. Different states can be obtained z_1. In fact,

$$z_2 = L_f z_1 = (1\ 0\ 0)\begin{pmatrix} x_2 \\ \eta(x) \\ \omega_z \end{pmatrix} = x_2 \, ; \tag{26}$$

$$z_3 = L_f L_f z_1 = (0\ 1\ 0)\begin{pmatrix} x_2 \\ \eta(x) \\ \omega_z \end{pmatrix} = \eta(x).$$

From Eq. (26) we see that the state transform is of the form:

$$\phi(x) = z(x) = \begin{bmatrix} z_1 & L_f z_1 & L_f^2 z_1 \end{bmatrix}^T = [z_1 \ \ z_2 \ \ z_3]^T = [x_1 \ \ x_2 \ \ \eta(x)]^T \tag{27}$$

Note that from Eq. (5a) and (18) we have that $\phi(x) = z(x)$ is a diffeomorphism because

$$|\nabla\phi| = \begin{vmatrix} 1 & 0 & -1-\mu_z - 2\alpha_{zz}x_1 - 3\alpha_{zzz}x_1^2 + 2\varepsilon\alpha_{zz}\cos vx_3 \\ 0 & 1 & -\mu_z \\ 0 & 0 & -\varepsilon v(1+2\alpha_{zz}x_1)senvx_3 \end{vmatrix} = -\varepsilon v(1+2\alpha_{zz}x_1)senvx_3 \neq 0 \, .$$

As a consequence of the Eq. (27) we have that $\phi(x)$ is not a global diffeomorphism, because according to the Eq. (18), $x_1 \neq -\frac{1}{2\alpha_{zz}}$ or $x_3 \neq \frac{k}{v}\pi$, $k = 0, \pm 1, \pm 2, \ldots$. Now, from (27) we conclude :

$$\dot{z}_1 = x_2 = z_2 \tag{28}$$

$$\dot{z}_2 = \dot{x}_2 = \eta(x) = z_3 \tag{29}$$

$$\dot{z}_3 = \dot{\eta}(x) = -\dot{x}_1 - \mu_z\dot{x}_2 - 2\alpha_{zz}x_1\dot{x}_1 - 3\alpha_{zzz}x_1^2\dot{x}_1 + 2\varepsilon\alpha_{zz}\dot{x}_1\cos vx_3 - v\varepsilon(1+2\alpha_{zz}x_1)senvx_3\dot{x}_3$$

$$= -x_2 - \mu_z\eta(x) - 2\alpha_{zz}x_1x_2 - 3\alpha_{zzz}x_1^2x_2 + 2\varepsilon\alpha_{zz}x_2\cos vx_3 - v\varepsilon(1+2\alpha_{zz}x_1)senvx_3(w_z + \gamma u)$$

$$= -z_2 - \mu_z\eta(x) - 2\alpha_{zz}z_1z_2 - 3\alpha_{zzz}z_1^2z_2 + 2\varepsilon\alpha_{zz}z_2\cos vx_3 - v\varepsilon(1+2\alpha_{zz}z_1)senvx_3(w_z + \gamma u).$$

Hence

$$\dot{z}_3 = a(x) + b(x)u \, , \tag{30}$$

with

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$$a(x) = -x_2 - \mu_z \eta(x) - 2\alpha_{zz} x_1 x_2 - 3\alpha_{zzz} x_1^2 x_2 + 2\varepsilon\alpha_{zz} x_2 \cos \nu x_3 \\ - \nu \in w_z (1 + 2\alpha_{zz} x_1) sen \nu x_3, \tag{30-a}$$

$$b(x) = -\gamma\nu\varepsilon(1 + 2\alpha_{zz} z_1) sen \nu x_3. \tag{30-b}$$

From Eq. (28) - (30-b) we have the nonlinear dynamics can be transformed into the (non-linear dynamics)

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} z_2 \\ z_3 \\ a(x) + b(x)u \end{bmatrix} \tag{31}$$

with *a(x)* and *b(x)* the same as in Eq. (30-a) and (30-b). By considering the nonlinear dynamics given by Eq. (27) with the control law $u = \alpha(x) + \beta(x)\upsilon$, and if

$$\alpha(x) = -\frac{L_f^3 z_1}{L_g L_f^2 z_1} \quad \text{and} \quad \beta(x) = \frac{1}{L_g L_f^2 z_1}, \tag{32}$$

the linearized system is obtained:

$$\dot{z} = \begin{bmatrix} z_2 \\ z_3 \\ \upsilon \end{bmatrix}. \tag{33}$$

In fact, from Eq. (5-a) then:

$$\eta(x) = -x_1 - \mu_z x_2 - \alpha_{zz} x_1^2 - \alpha_{zzz} x_1^3 + \varepsilon(1 + 2\alpha_{zz} x_1) \cos \nu x_3.$$

Now:

$$\alpha(x) = -\frac{L_f^3 z_1}{L_g L_f^2 z_1} = -\frac{L_f\left(L_f^2 z_1\right)}{L_g L_f^2 z_1} = -\frac{L_f(\eta(x))}{L_g(\eta(x))}. \tag{34}$$

But:

$$-L_f(\eta(x)) = \left(-1 + 2\alpha_{zz} x_1 + 3\alpha_{zzz} x_1^2 - 2\varepsilon\alpha_{zz} \cos \nu x_3\right) x_2 + \mu_z \eta(x) + \\ w_z \nu\varepsilon(1 + 2\alpha_{zz} x_1) sen \nu x_3. \tag{35}$$

$$L_g(\eta(x)) = \left(-1 + 2\alpha_{zz} x_1 + 3\alpha_{zzz} x_1^2 - 2\varepsilon\alpha_{zz} \cos \nu x_3\right) x_2 + \mu_z \eta(x) + w_z \nu\varepsilon(1 + 2\alpha_{zz} x_1) sen \nu x_3. \tag{36}$$

So, from Eq. (35) and (36) we have:

$$\alpha(x) = \frac{\left(-1 + 2\alpha_{zz} x_1 + 3\alpha_{zzz} x_1^2 - 2\varepsilon\alpha_{zz} \cos \nu x_3\right) x_2 + \mu_z \eta(x) + w_z \nu\varepsilon(1 + 2\alpha_{zz} x_1) sen \nu x_3}{-\gamma\nu\varepsilon(1 + 2\alpha_{zz} x_1) sen \nu x_3}. \tag{37}$$

We have:

$$\beta(x) = \frac{1}{L_g L_f^2 z_1} = \frac{1}{L_g(\eta(x))}.$$

In this manner:

$$\beta(x) = \frac{1}{-\gamma v\varepsilon(1+2\alpha_{zz}x_1)senvx_3}. \tag{38}$$

Therefore, with the state transformation (27) and the input transformation $u = \alpha(x) + \beta(x)\upsilon$ given by Eq. (37) and (38), the problem of stabilizing the nonlinear dynamics (7) by using the original control input *u*, was transformed into the problem of stabilizing the new dynamics given by Eq. (31), with input υ.

It is noteworthy that the use of linear techniques could be used to design the input control υ. For example, the technique of imposing poles can be used. Thus, the stability of the closed-loop dynamics can be analyzed. For this purpose, the linear feedback control law:

$$\upsilon = \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 \tag{39}$$

can arbitrarily assign the poles of system given by Eq. (33) (Chen, 1998; Slotine, 1991; Isidore, 1995).

Substituting Eq. (37), (38) and (39) into equation $u = \alpha(x) + \beta(x)\upsilon$ gives the nonlinear feedback control law:

$$u = \left(\frac{-1}{\gamma v\varepsilon(1+2\alpha_{zz}x_1)senvx}\right)[(-1+\alpha_2+2\alpha_{zz}x_1+3\alpha_{zzz}x_1^2-2\varepsilon\alpha_{zz}\cos vx_3)x_2+(\mu_z+\alpha_3)\eta(x)+ \\ (\alpha_1+w_z v\varepsilon(1+2\alpha_{zz}x_1)senvx_3)x_1]. \tag{40}$$

How future goals we wish to introduce numerical examples control beyond the study of asymptotic stability of nonlinear dynamics.

Notice that despite the result to be true in a wide region of the state space, the result is not global since the control law is not defined when $x_1 \neq -\frac{1}{2\alpha_{zz}}$ or $x_3 \neq \frac{k}{v}\pi, \quad k = 0, \pm 1, \pm 2, \ldots$.

4. CONCLUSION

In this work we performed the input-state linearization of a MAGLEV vehicle, in which it was considered a simplified nonlinear model obtained by Yabuno (2004).

Necessary and sufficient conditions for controllability were obtained, in Eq. (21) and depending on such conditions, we determined the diffeomorphism and a new set of states generating a nonlinear dynamics given by Eq. (31). From this equation it was determined a control law of the form given by Eq. (37) and (38) which transform the nonlinear dynamics (31) in a dynamic linear, in the Eq. (33). Thus, with the diffeomorphism given by Eq. (27) and the input transformation $u = \alpha(x) + \beta(x)\upsilon$ given by Eq. (37) and (38), the problem of to be stabilizing the nonlinear dynamics (7) by using the original control input *u*, was transformed into problem in to be stabilizing the new dynamics given by Eq. (31), with input υ.

It has been proved that the result obtained is not global, even though being valid in a wide region of state space, since the control law is not well defined when $x_1 \neq -\frac{1}{2\alpha_{zz}}$ or $x_3 \neq \frac{k}{v}\pi, \quad k = 0, \pm 1, \pm 2, \ldots$.

We want to be mentioning that the use of linear techniques can be applied to design the input control. For example, the technique of imposing poles could be used. In this way, the stability of the closed-loop dynamics could be analyzed.

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