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ASYMPTOTIC STABILIZATION AND INTERNAL DYNAMICS OF A SIMPLIFIED MODEL OF A MAGLEV SYSTEM

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Abstract. *This paper presents the input-output linearization of a vehicle type MAGLEV (magnetic levitation transport), when the relative degree of the system is two, based in the simplified nonlinear model. By using standard Lie derivatives techniques it is possible to split the nonlinear dynamics in a linear external (input-output) part and in a nonlinear internal one not observable, allowing in this way considerations on the zero dynamics in the system. More specifically we find a class of diffeomorphisms in such a way the original system, can be placed in the normal form by using the outputs and their derivatives as part of a new set of states. This allows us to determine the internal and the zero dynamics for each diffeomorphism. Conditions for analysis of local asymptotic stability of the origin are presented and further it was exhibited one one-parameter family of implicit solutions for the dynamic zero.*

Keywords: *MAGLEV System, input-state linearization, Lie derivative, controllability.*

1. INTRODUCTION

Feedback linearization is an approach that allows transforming nonlinear system models into a (fully or partially) linear system through of a nonlinear feedback control law of the state or output previously chosen. This methodology has been used successfully to address some practical control problems such as tracking problems, control of helicopters, high performance aircraft, control robotic arms and manipulators, artillery and satellites, as well as being used in biomedical devices and chemical and pharmaceutical industry pharmaceutics (Alvarez-Gallegos, 1994; Barbanti, 2012; Chem, 1998, 2000, 1999; Isidori, 1995; Reis, 2012-a, 2012-b; Silva, 2003; Slotine, 1991; Ray, 2012; Yabuno 2004, 1991, 2003, 1989).

The main idea of feedback linearization is to find a coordinate transformation and a feedback control law such that the input-state of the input-output relationship of the closed-loop system, in the new coordinates, are linear. After the nonlinear system has been modified so that the system or part of it to behave as linear, it is possible to use linear control techniques.

This paper presents the input-output linearization of a vehicle type MAGLEV (magnetic levitation transport), when the relative degree of the system is two, based in the simplified nonlinear model (Yabuno, 2004, 1989).

The MAGLEV is a new technology for mass transportation, which employs the generation of magnetic fields to levitate, direct and propel high-speed trains, adding safety, low environmental impact and minimal maintenance costs. Hence the interests of study in countries like Brazil, Germany, Japan, China, United States, Australia, Thailand, etc... (www.rtri.or.jp/rd/maglev/html/english/maglev_frame_E.html - www.ferrovia.com.br/ligacao.htm).

Based in a simplified nonlinear model (Yabuno, 2004, 1989) described in state space, with the output vector field scale, the nonlinear dynamics in a linear external (input-output) part and in a nonlinear internal one not observable. More specifically we find a class of diffeomorphisms in such a way the original system, can be placed in the normal form by using the outputs and their derivatives as part of a new set of states. This allows us to determine the internal and the zero dynamics for each diffeomorphism. Conditions for analysis of local asymptotic stability of the origin are presented and further it was exhibited one one-parameter family of implicit solutions for the dynamic zero.

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The paper is organized as follows. The section 2 presents a simplified mathematical model of a MAGLEV system. Section 3 presents the input-output linearization of the MAGLEV and its internal dynamic. In the section 4 performs the analysis of local asymptotic stability and determines the zero dynamic and concluding remarks are given in Section 5.

2. THE SIMPLIFIED MODEL OF A MAGLEV SYSTEM

We consider a simplified mechanical model and the derivation of governing equations done by Yabuno (2004) for the MAGLEV, as shown in Fig. 1.

The origin *O* of an inertial Cartesian reference frame is set at the point of the pendulum on the levitated body in its equilibrium point state (the gap between the magnet on the base and the magnet on the body in this state is denoted by z_{st}). The levitated body, whose mass is m_1, is restrained to move freely only in the z-direction. The motion is expressed by the displacement of the pivot from O in the vertical direction and is denoted by z_d. The base is sinusoidally excited in the vertical direction with a prescribed displacement, $z_b = z_{b1} \cos \Omega t$, where z_{b1} and Ω are the amplitude and frequency of the base excitation, respectively. The natural frequency of the body is denoted by Ω_z. The dimensionless variables t^* and z^* are defined as $t^* = t\ \omega_z$ and $z^* = z_d / z_{st}$, respectively. $\nu = \omega / \omega_z$ and $\varepsilon = z_{b1}/z_{st}$ are dimensionless parameters (Yabuno, 1989; Yabuno, 1991).

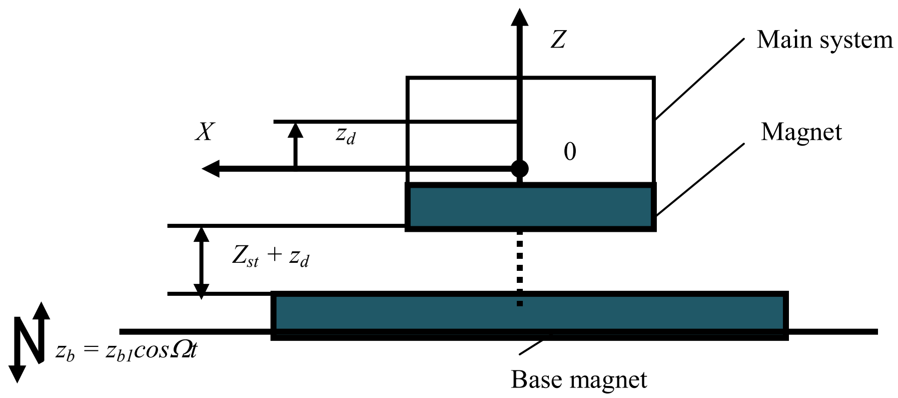

Figure 1: The simplified model of a maglev system (Yabuno, 2004).

The repulsive magnetic force between the magnet on the body and the magnet on the base, for finite but small variations of the gap between the magnets, can be well approximated by a polynomial function with quadratic and cubic terms (Yabuno, 1989; Yabuno, 1991). The (˙) represents differentiation with respect the dimensionless time.

The equation describing the system is the presented in Yabuno (2004) as follows:

$$\ddot{z}^* = -z^* - \mu_z \dot{z}^* + \varepsilon \cos \nu t^* + 2\alpha_{zz} \varepsilon z^* \cos \nu t^* - \alpha_{zz} z^{*2} - \alpha_{zzz} z^{*3} \tag{1}$$

where $\mu_z \dot{z}^*$ express the linear viscous-type acting in the main system, α_{zz} and α_{zzz} are the coefficients of z^2 and z^3 respectively, in the Taylor series expansion of the magnetic force (Yabuno, 1989, Yabuno, 2004).

By rearranging the Eq. (1), then:

$$\ddot{z}^* = -z^* - \mu_z \dot{z}^* - \alpha_{zz} z^{*2} - \alpha_{zzz} z^{*3} + \varepsilon (1 + 2\alpha_{zz} z^*) \cos \nu t^* . \tag{2}$$

Defining the state variables:

$$x_1 = z^* \text{ e } x_3 = t^* . \tag{3}$$

the Eq. (2) can be put in the following form:

$$\dot{x} = f(x) , \tag{4}$$

where *f(x)* a smooth vector fields in $\Re^3$ defined by:

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$$f(x) = \begin{pmatrix} x_2 \\ \eta(x) \\ \omega_z \end{pmatrix}, \tag{5}$$

$$\eta(x) = -x_1 - \mu_z x_2 - \alpha_{zz} x_1^2 - \alpha_{zzz} x_1^3 + \varepsilon(1 + 2\alpha_{zz} x_1)\cos vx_3. \tag{5-a}$$

For the input-output linearization of MAGLEV, let be in Eq. (4) - (5-a):

(a) $y = h(x) = x_1$ is the output;

(b) *u(t)* is the input of the system; (6)

(c) *g(x)* a smooth vector fields in $\Re^3$ defined by $g(x) = \begin{pmatrix} 0 \\ \beta \\ \gamma \end{pmatrix}$, where β and γ are nonzero real numbers.

By using Eq. (5) and (6), the Eq. (4) - (6) can be put in the following form:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) = x_1 \end{aligned} \tag{7}$$

where *f* and *g* are smoothing vector fields and *h* is a smoothing scalar function.

The system given by Eq. (7) has relative degree *2* (Slotine, 1991; Isidore, 1995). In fact, repeated differentiation of the output, then:

$$\ddot{y} = L_f^2 h(x) + L_g L_f h(x)u = \eta(x) + \beta u \tag{8}$$

with $L_f^o h(x) = h(x)$ and $L_f\left[L_f^{i-1} h(x)\right] = \nabla L_f^{i-1} h(x) f(x)$, *i = 1, 2, ...* are Lie derivative of the fields *h* and *f*. But β and γ are nonzero, following the results.

According to Slotine (1991) and Isidori (1995), the dynamics given by Eq. (7) is said to be *input-output linearizable* if there exists a region Ω in $\Re^3$, a diffeomorphism $\phi : \Omega \to \Re^3$ and a nonlinear feedback control law $u = u(x, \upsilon)$ such that the new state variables $z = \phi(x)$ and the new input υ satisfy a linear time-invariant relation.

By means of input-output linearization, the dynamics of a nonlinear system by Eq. (7) is decomposed into an external (input-output) part and an internal unobservable part, a so-called normal form. To show that the nonlinear system by Eq. (7) can indeed be transformed into the norm form, we have to show not only that a coordinate transformation exists, but also that it is a true state transformation. In other words, we need to show that we can construct a

Showing that the system given by Eq. (7) can be input-output linearized means to show that the nonlinear dynamics (7) can be decomposed into a linear outer part which relates input-output, and an internal unobservable one. This form is said normal form. For do this it is mandatory, then, to show not only that this form exists, but also that it is a true state transformation, that is, it is necessary to show that we could construct a local diffeomorphism ϕ (x) allows this normal form verified (Slotine, 1991; Isidori, 1995).

After to have decomposed the system in its normal way, the analysis of the internal dynamics of the original system can be made. From the knowledge of the internal dynamics and its equation, it is possible to analyze the dynamic zero and asymptotic stability issues. These are the goals of this work.

3. THE LINEARIZATION INPUT-OUTPUT MAGLEV VEHICLE AND ITS DYNAMIC INTERNAL

We wish to determine a diffeomorphism as in Slotine (1991) and Isidori (1995) such that the nonlinear system given by Eq. (7) can be placed in its normal form. From Eq. (5), (6) and (7), just define the set of states:

$$\mu_1 = y = x_1 \quad \text{and} \quad \mu_2 = L_f h(x) = \dot{y} = x_2. \tag{9}$$

For the determination of *ψ(x),* we must to be requiring (Slotine, 1991; Isidori, 1995):

$$\nabla \psi g = 0 \text{ or } L_g \psi = 0, \tag{10}$$

that is, $\psi(x)$ to be a solution of the differential equation:

$$\beta \frac{\partial \psi(x)}{\partial x_2} + \gamma \frac{\partial \psi(x)}{\partial x_3} = 0 \cdot \tag{11}$$

With the sake in to be determining the solution of Eq. (11) we are searching for solutions of the type:

$$\psi(x) = \gamma T_1(x_2) + \beta T_2(x_3) + \xi T_3(x_1) \tag{12}$$

In such a way $T_1(x_2)$, $T_2(x_3)$ e $T_3(x_1)$ are of the C^1 and invertible.

From Eq. (11) we get:

$$\beta\gamma \frac{dT_1}{dx_2}(x_2) + \gamma\beta \frac{dT_2(x_3)}{dx_3} = 0 . \tag{13}$$

Cause $T_1 = T_1(x_2)$ and $T_2 = T_2(x_3)$, from Eq. (13) we have:

$$\beta\gamma \frac{dT_1(x_2)}{dx_2}(x_2) = -\gamma\beta \frac{dT_2(x_3)}{dx_3} = C$$

and thus:

$$T_1(x_2) = \frac{C}{\beta\gamma} x_2 \text{ and } T_2(x_3) = -\frac{C}{\beta\gamma} x_3$$

where C is the constant of integration. Then a general class of the PDE given by Eq. (13) solutions are:

$$\psi(x) = \frac{C}{\beta} x_2 - \frac{C}{\gamma} x_3 + \xi T_3(x_1) . \tag{14}$$

Note that Eq. (14) represents a one-parameter family of solutions of the PDE given by Eq. (13), since the integration constant C ranges from $(-\infty, +\infty)$. Furthermore, $T_3 = T_3(x_1)$ can be varied as one wants. It may be linear or nonlinear. In Reis et al (2012) it was used the particular solution given by:

$$\psi(x) = \gamma x_2 - \beta x_3 .$$

So, depending on Eq. (9) and (14) the mapping $\phi(x)$ has the expression:

$$\phi(x) = \begin{pmatrix} x_1 & x_2 & \frac{C}{\beta} x_2 - \frac{C}{\gamma} x_3 + \xi T_3(x_1) \end{pmatrix} . \tag{15}$$

Note that the function $\phi(x)$ given by Eq. (15) is not singular for C and $\gamma \neq 0$ since $|\nabla\phi| = -\frac{C}{\gamma}$. Thus, $\phi(x)$ is a one-parameter family of diffeomorphisms (Slotine, 1991; Isidori 1995), whose inverses are given by:

$$(x_1, x_2, x_3) = \left(\mu_1, \mu_2, \frac{\gamma}{C}\left(-\psi(x) + \frac{C}{\beta}\mu_2 + \xi T_3(\mu_1) \right) \right) . \tag{16}$$

It is observed that when $C = 0$ we have $\psi(x) = \xi T_3(x_1)$. In this case, $|\nabla\phi| = 0$ and therefore ψ is not a diffeomorphism. It is not appropriated to the study of the proposed problem. Hence, from Eq. (16), the nonlinear dynamics given by Eq. (7) for each $C \neq 0$, possess the normal form:

$$\begin{aligned} \dot{\mu} &= \begin{bmatrix} \mu_2 \\ \eta(\mu,\psi) \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u \\ \dot{\psi} &= \frac{C}{\beta}\eta(\mu,\psi) - \frac{C}{\gamma} w_z + \xi \dot{T}_3(\mu_1) \\ y &= \mu_1 \end{aligned} \tag{17}$$

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where $\eta(\mu,\psi)$ is the one obtained from Eq.(5-a) and (15), which means:

$$\eta(\mu,\psi) = -\mu_1 - \mu_z\mu_2 - \alpha_{zz}\mu_1^2 - \alpha_{zzz}\mu_1^3 + \varepsilon(1+2\alpha_{zz}\mu_1)\cos v\left[\frac{\gamma}{C}\left(-\psi + \frac{C}{\beta}\mu_2 + \xi T_3(\mu_1)\right)\right]. \tag{17-a}$$

By facing Eq. (17) we can be doing the following observations:

1. The non-linear dynamics in the system by Eq. (7) was decomposed into an external portion (input-output u, μ_1 and μ_2) and into an inner unobservable ($\dot{\psi}$);
2. The inner part is independent from the input u, and it depends only up the state μ. This part is called dynamic internal system by Eq. (7).

The outer part given by Eq. (17) can be linearized. In fact, just take the control law (Slotine, 1991; Isidori, 1995):

$$u = \frac{1}{\beta}\left(-\eta(\mu,\psi) + \upsilon\right). \tag{18}$$

The replacement of Eq. (18) in Eq. (17) gives:

$$\begin{aligned} \dot{\mu} &= \begin{bmatrix} \mu_2 \\ \upsilon \end{bmatrix} \\ \dot{\psi} &= \frac{C}{\beta}\eta(\mu,\psi) - \frac{C}{\gamma}w_z + \xi\dot{T}_3(\mu_1) \\ y &= \mu_1 \end{aligned} \tag{19}$$

where $\eta(\mu,\psi)$ is the same as in Eq. (17-a). So, the system given by Eq. (7) is transformed into an exterior linear part over the input υ and an inner part that is non-linear on the states μ_1, μ_2 and ψ.

4. THE LOCAL ASYMPTOTIC STABILITY ANALYSIS AND THE DETERMINATION OF THE ZERO DYNAMICS

According Isidori (1995), the analysis of the local asymptotic stability of a nonlinear system can be done with the use of a linear approximation system, near a null critical point. This property can be observed in Proposition 2 in the Appendix. Another way for to do such analysis is by using the zero dynamics, which depends on the asymptotic stability.

Both techniques are not based on the knowledge of the critical points in the nonlinear dynamics. In this sense, from Eq. (17) it follows that:

$$\begin{aligned} \mu_2 &= 0 \\ u &= -\frac{1}{\beta}\eta(\mu,\psi) \\ \eta(\mu,\psi) &= \frac{\beta}{\gamma}w_z - \frac{\beta\xi}{C}\dot{T}_3(\mu_1). \end{aligned} \tag{20}$$

In this way, from Eq. (20) we obtain:

$$\cos v\left[\frac{\gamma}{C}\left(-\psi + \xi T_3(\mu_1)\right)\right] = \frac{1}{\varepsilon(1+2\alpha_{zz}\mu_1)}\left[\mu_1 + \alpha_{zz}\mu_1^2 + \alpha_{zzz}\mu_1^3 + \frac{\beta}{\gamma}w_z - \frac{\beta\xi}{C}\dot{T}_3(\mu_1)\right]. \tag{21}$$

From Eq. (21) we get:

$$\psi = \xi T_3(\mu_1) - \frac{C}{v\gamma}ar\cos\left[\frac{1}{\varepsilon(1+2\alpha_{zz}\mu_1)}\left(\mu_1 + \alpha_{zz}\mu_1^2 + \alpha_{zzz}\mu_1^3 + \frac{\beta}{\gamma}w_z - \frac{\beta\xi}{C}\dot{T}_3(\mu_1)\right)\right]. \tag{22}$$

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From Eq. (20) and (22) we have that the critical point *P*, in the nonlinear dynamics given by Eq. (17) has as coordinates:

$$P = (\mu_1,\ 0,\ \psi), \tag{23}$$

with ψ being given as in Eq. (22).

As the object of our interest is the point $P = (0,\ 0,\ 0)$ by making $\mu_1 = 0$ the critical point in Eq. (23) has the coordinates:

$$P = \left(0,\ 0,\ \xi T_3(0) - \frac{C}{v\gamma} ar\cos\theta_o\right) \tag{24}$$

where

$$\theta_o = \frac{\beta}{\varepsilon\gamma} w_z - \frac{\beta\xi}{\varepsilon C}\dot{T}_3(0) \text{ and } u = -\frac{1}{\beta}\eta(\mu_1, 0, \psi). \tag{24 - a}$$

Making the translation for have *P* coincident with the origin in the new coordinate system, and because $\mu_1 = \mu_2 = 0$, if we define:

$$\psi^* = \psi - \xi T_3(0) + \frac{C}{v\gamma} ar\cos\theta_o, \tag{25}$$

for $\theta_o = \frac{\beta}{\varepsilon\gamma} w_z - \frac{\beta\xi}{\varepsilon C}\dot{T}_3(0)$, we obtain the new nonlinear dynamics whose critical point is the origin given by:

$$\begin{aligned}
\dot{\mu} &= \begin{bmatrix} \mu_2 \\ \eta(\mu,\psi^*) \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u \\
\dot{\psi}^* &= \frac{C}{\beta}\eta(\mu,\psi^*) - \frac{C}{\gamma} w_z + \xi \dot{T}_3(\mu_1) \\
y &= \mu_1
\end{aligned} \tag{26}$$

with

$$\begin{aligned}
\eta(\mu,\psi^*) &= -\mu_1 - \mu_z\mu_2 - \alpha_{zz}\mu_1^2 - \alpha_{zzz}\mu_1^3 \\
&+ \varepsilon(1 + 2\alpha_{zz}\mu_1)\cos v\left[\frac{\gamma}{C}\left(-\psi^* - \xi T_3(0) + \frac{C}{v\gamma} ar\cos\theta_o + \frac{C}{\beta}\mu_2 + \xi T_3(\mu_1)\right)\right].
\end{aligned} \tag{26-a}$$

To simplify the problem, it is assumed from now on that $T_3(\mu_1) \equiv 0$. This assumption, although reducing the classes of nonlinear dynamics treated, yet allows an analysis of the dynamics of large amplitude, which are given for each real *C* by:

$$\begin{aligned}
\dot{\mu} &= \begin{bmatrix} \mu_2 \\ \eta(\mu,\psi^*) \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u \\
\dot{\psi}^* &= \frac{C}{\beta}\eta(\mu,\psi^*) - \frac{C}{\gamma} w_z \\
y &= \mu_1
\end{aligned} \tag{27}$$

$$\begin{aligned}
\eta(\mu,\psi^*) &= -\mu_1 - \mu_z\mu_2 - \alpha_{zz}\mu_1^2 - \alpha_{zzz}\mu_1^3 \\
&+ \varepsilon(1 + 2\alpha_{zz}\mu_1)\cos v\left[\frac{\gamma}{C}\left(-\psi^* + \frac{C}{v\gamma} ar\cos\theta_o + \frac{C}{\beta}\mu_2\right)\right].
\end{aligned} \tag{27-a}$$

If we make in Eq. (27-a)

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$$\Lambda = \frac{\gamma}{C}\left(-\psi^* + \frac{C}{v\gamma} ar\cos\theta_o + \frac{C}{\beta}\mu_2 \right)$$

for $\theta_o = \frac{\beta}{\varepsilon\gamma} w_z$ and taking into account the Taylor series expansion around the origin of the function $cos(v\Lambda) = cos(\psi^*, \mu_2)$ and making $\Gamma = sen(ar\cos\theta_o)$, then the nonlinear dynamics given by Eq. (27) can be rewritten as:

$$\begin{bmatrix} \dot{\mu}_1 \\ \dot{\mu}_2 \\ \dot{\psi}^* \end{bmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ \left(-1 + \frac{2\alpha_{zz}\beta w_z}{\gamma} \right) & -\left(\mu_z + \frac{\varepsilon\gamma v}{\beta}\Gamma \right) & \frac{\varepsilon\gamma v}{C}\Gamma \\ \frac{C}{\beta}\left(-1 + \frac{2\alpha_{zz}\beta w_z}{\gamma} \right) & -\frac{C}{\beta}\left(\mu_z + \frac{\varepsilon\gamma v}{\beta}\Gamma \right) & \frac{\varepsilon\gamma v}{\beta}\Gamma \end{vmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \psi^* \end{bmatrix} +$$

$$+ \begin{vmatrix} 0 \\ \frac{\beta}{\gamma} w_z - \alpha_{zz}\mu_1^2 - \alpha_{zzz}\mu_1^3 + \frac{2\varepsilon\alpha_{zz}\gamma v\Gamma}{C}\mu_1\psi^* - \frac{2\varepsilon\alpha_{zz}\gamma v\Gamma}{\beta}\mu_1\mu_2 + \ldots\ldots \\ \frac{C}{\beta}\left\{ -\alpha_{zz}\mu_1^2 - \alpha_{zzz}\mu_1^3 \right\} + \frac{2\varepsilon\alpha_{zz}\gamma v\Gamma}{\beta}\mu_1\psi^* - \frac{2\varepsilon\alpha_{zz}\gamma vC\Gamma}{\beta^2}\mu_1\mu_2 + \ldots.. \end{vmatrix} + \begin{bmatrix} 0 \\ \beta \\ 0 \end{bmatrix} u \tag{28}$$

and then the matrix A due to the linear approximation it is done as:

$$A = \begin{vmatrix} 0 & 1 & 0 \\ \left(-1 + \frac{2\alpha_{zz}\beta w_z}{\gamma} \right) & -\left(\mu_z + \frac{\varepsilon\gamma v}{\beta}\Gamma \right) & \frac{\varepsilon\gamma v}{C}\Gamma \\ \frac{C}{\beta}\left(-1 + \frac{2\alpha_{zz}\beta w_z}{\gamma} \right) & -\frac{C}{\beta}\left(\mu_z + \frac{\varepsilon\gamma v}{\beta}\Gamma \right) & \frac{\varepsilon\gamma v}{\beta}\Gamma \end{vmatrix}. \tag{29}$$

The eigenvalues of A are:

$$\lambda = 0 \quad \text{and} \quad \lambda = \frac{-\mu_z \pm \sqrt{\mu_z^{\,2} + 4\left(-1 + \frac{2\alpha_{zz}\beta w_z}{\gamma} \right)}}{2}. \tag{30}$$

Taking the matrix $B = \begin{bmatrix} 0 \\ \beta \\ 0 \end{bmatrix}$ it is possible to show that the linear approximation is asymptotically stable, namely, the pair *(A, B)* is controllable if $\frac{\gamma}{\beta} \neq 2\alpha_{zz} w_z$ (see Proposition 1 in Appendix). In this case, we have rank *(A, B)* < *3*, and in addition, $\lambda = -\mu_z < 0.$

According the Proposition 2, every linear feedback which stabilizes asymptotically the linear approximation also stabilizes the original nonlinear system, at least locally.

A question for future works will be to find the dynamic that stabilizes the linear approximation near the origin and in this way making stable the nonlinear dynamics.

The zero dynamics of this non-linear system is achieved when y (t) = 0 for all real t> 0, which means that $\mu_1(t) = \mu_2(t) = 0 \ \forall t.$ Under these conditions the zero dynamics is given by:

$$\dot{\psi} = \frac{C}{\beta}\eta(0,\psi) - \frac{C}{\gamma} w_z \qquad y = 0 \tag{31}$$

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where:

$$\eta(0,\psi) = \varepsilon \cos v\left\lfloor -\frac{\gamma}{C}\psi \right\rfloor . \tag{31-a}$$

But since $y = x_1(t) = 0$, the manifold Mo is described as the locus of the points:

$$M_o = \{x \in \Re^3 : x_1 = 0\}$$

which is a plane passing through the origin. In this manifold, the system dynamics given by Eq. (27) and (27-a) is governed by the dynamic zero of the Eq. (31). The critical point of the nonlinear dynamics done by Eq. (31) is different from zero and is given by:

$$P = -\frac{C}{v\gamma} arccos\left(\frac{\beta w_z}{\varepsilon\gamma}\right). \tag{32}$$

By means of a translation of coordinates, the critical point in Eq. (32) will be taken at the origin of the new coordinates systems. In this case is denoted by ψ^*, as follows:

$$\begin{aligned}(\psi^*)' &= -\frac{C}{\gamma}w_z + \frac{C\varepsilon}{\beta}\cos v\left(\frac{\gamma}{C}\left(-\psi^* + \frac{C}{v\gamma}ar\cos(\theta_o)\right)\right)\\ &= -\frac{C}{\gamma}w_z + \frac{C\varepsilon}{\beta}\cos\left(-\frac{v\gamma}{C}\psi^* + ar\cos\theta_o\right)\end{aligned} \tag{33}$$

where $\theta_o = \frac{\beta w_z}{\varepsilon\gamma}$.

The zero dynamics given by Eq. (33) has a critical point at the origin. Note that this equation is not linear and for the determination of its general solution, just do:

$$\begin{aligned}&\text{i. } A = -\frac{C}{\gamma}w_z ;\\ &\text{ii. } K = \frac{C\varepsilon}{\beta} ;\\ &\text{iii. } R = -\frac{v\gamma}{C} ;\\ &\text{iv. } S = ar\cos\theta_o ;\\ &\text{v. } y = R\psi^* + S .\end{aligned} \tag{34}$$

In this way by using Eq. (34) the Eq. (33) is done by

$$\frac{d\psi^*}{dt^*} = A + K\cos y. \tag{35}$$

By using the Taylor approximation of order *2* about the origin in the cosine function the Eq. (35) can be written as:

$$\frac{d\psi^*}{dt^*} = A + K - \frac{K}{2}y^2 \tag{36}$$

where *A, K* and *y* are the same that the ones in Eq. (34). From Eq. (34) - v, it follows that $d\psi^* = \frac{1}{R}dy$. Therefore, the Eq. (36) can be written as:

$$\frac{1}{R}\frac{dy}{dt^*} = A + K - \frac{K}{2}y^2, \tag{37}$$

where A, K and R are the same that the ones in Eq. (34). The general solution of the Eq. (37) or Eq. (36) is:

$$\frac{\sqrt{-\frac{K}{2}(A+K)}}{-\frac{K}{2}(A+K)} actg\left[\frac{\sqrt{-\frac{K}{2}(A+K)}}{A+K}\left(R\psi^* + S\right)\right] - Rt^* = C_1 \tag{38}$$

In an explicit way we can do

$$\psi^* = \frac{-2\sqrt{-\frac{K}{2}(A+K)}}{KR} tg\left[\sqrt{-\frac{K}{2}(A+K)}\left(Rt^* + C_1\right)\right] - \frac{S}{R} \tag{39}$$

and C_1 is is the constant of integration. Therefore, Eq. (39) represents an one-parameter family of implicit solutions of Eq. (37).

5. CONCLUSIONS

This work concerns the dynamics of a MAGLEV (Magnetic Levitation transport) device, whose simplified model is obtained Yabuno (2004), (1989), where such nonlinear dynamics has relative degree 2. It is shown that the nonlinear dynamics can be decomposed, in a linear external part and in a nonlinear internal one. To this end, it presents a class of diffeomorphisms $\phi_i(x)$ (equation (15)) such that the original system, for each i, may be placed in the normal manner from the use of departure and its derivatives as part of a new set of states, allowing the determination of internal dynamics and the dynamics of zero, for each diffeomorphism $\phi_i(x)$.

It was presented conditions for asymptotic stabilization of the linear approximation and proved that any linear feedback which asymptotically stabilizes the linear approximation is able to asymptotically stabilize the original nonlinear system, at least locally.

A family of dynamic systems representing the zero dynamics was obtained and the one-parameter family of solutions of this dynamic was determined, for each i.

For future work we also will be determine a feedback control that stabilizes the linear approximation, which also stabilize, at least locally, the original nonlinear dynamics. In addition, we intend to make the analysis of local asymptotic stability via the dynamic zero. If the zero dynamics of the system are asymptotically at 0, it is intended to determine a feedback control that stabilizes asymptotically and locally closed loop.

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7. REFERENCES

Alvarez-Gallegos, J. (1994). Nonlinear Regulation of a Lorenz System by Feedback Linearization Techniques, *Dynamics and Control*, 4, pp. 277-298.

Barbanti, L. (2012). Hartman-Grobman Decomposition When in Presence of Jumps in the Relative Degree of the Dynamics in an Energy Harvesting Device In: 9th International conference Problems in Engineering, Aerospace and Sciences, 2012. American Institute of Phisics p.84 - 87.

Chem Liqun, Yanzhu, L. (1998). Control of the Lorenz Chaos by the Exact Linearization, *Applied Mathematics and Mechanics*, 19, pp. 67-73.

Chem Liqun (2000). Controlling Chaotic Oscillations With Input-Output Linearization, *Journal of Shanghai University*, 4. pp. 175-178.

Chem Liqun, Yanzhu, L. (1999). A Modified Exact Linearization Control for Chaotic Oscillators, *Nonlinear Dynamics, 20. pp. 309-317.*

Isidore, A. *Nonlinear Control Systems*, 3ed., Springer-Verlag, Roma, 1995.

Silva, G. V. M. *Controlo Não Linear*, Escola Superior de Tecnologia Setúbal, Lisboa, 2003.

Slotine, J.; LI, W.. *Applied Nonlinear Control*. New Jersey: Prentice Hall, 1991.

Ray, A., Chowdhury, A. R. (2012). Nonlinear Control of Hiperchaotic System, Lie Derivative and State Space Linearization, *J. Comput. Nonlinear Dynamics*, 7.
Reis, C. A *et al.*, (2012-a) A Análise da Dinâmica Interna de Um Trem Maglev In: VIII Congresso Nacional de Engenharia Mecânica, 2012, São Luiz. Anais do VIII Congresso Nacional de Engenharia Mecânica.
Reis, C. A. *et al.,* (2012-b) A Linearização Entrada-Saída de um Veículo MAGLEV. In: CNMAC 2012: 34º Congresso Nacional de Matemática Aplicada e Computacional, Águas de Lindoía, S. P., 2012, Anais do 34^O Congresso Nacional de Matemática aplicada e Computacional, SBMAC, São Carlos.
Yabuno, H., Kanda, R., Lacarbonara, W., Aoshima, N., (2004). nonlinear Active Cancellation of the Parametric Resonance in a Magnetically Levitated Body, *Jounal of Dynamics system, Measurement and Control*, 126, pp. 433-442.
Yabuno, H., Fujimoto, N., Yoshizawa, M., and Tsujioka, Y. (1991). Bouncing and Pitching Oscillations of Magnetically Levitated Body due to the Guideway Roughness, *JSME International Journal*, 34(2), pp. 192–199.
Yabuno, H., Murakami, T., Kawazoe, J., and Aoshima, N. (2003). Suppression of Parametric Resonance in Cantilever Beam with a Pendulum Effect of Static Friction at the Supporting Point of the Pendulum, *Trans. ASME Journal of Vibration and Acoustics*, 126, pp. 149-162.
Yabuno, H., Seino, T., Yoshizawa, M., and Tsujioka, Y. (1989). Dynamical Behavior of a Levitated Body with Magnetic Guides (Parametric Excitation of the Subharmonic Type Due to the Vertical Motion of Levitated Body), JSME *International Journal*, 32**(**3), pp. 428–435.
http://www.maglevpa.com/index.html, www.maglevpa.com/propul.html, www.fra.dot.gov/o/hsgt/index.htm, www.llnl.gov/str/Post.html, www.transrapid-international.de/english/welt.html, www.skytran.net/press/sciam01.htm, www.rtri.or.jp/rd/maglev/html/english/maglev_frame_E.html, www.ferrovia.com.br/ligacao.htm.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.

APPENDIX:

Proposition 1: The linear approximation of the nonlinear dynamics given by Eq. (28) is asymptotically stable if $\frac{\gamma}{\beta} \neq 2\alpha_{zz} w_z$.

Proof: To show that the linear approximation is asymptotically stable is equivalent to prove that the pair *(A, B)* is controllable. From Eq. (29) and (30), if $\lambda = 0$, then we have that:

$$\left(A,\ B\right)=\left|\begin{array}{cccc} 0 & 1 & 0 & 0 \\ \left(-1+\frac{2\alpha_{zz}\beta w_z}{\gamma}\right) & -\left(\mu_z+\frac{\varepsilon\gamma\nu}{\beta}\Gamma\right) & \frac{\varepsilon\gamma\nu}{C}\Gamma & \beta \\ \frac{C}{\beta}\left(-1+\frac{2\alpha_{zz}\beta w_z}{\gamma}\right) & -\frac{C}{\beta}\left(\mu_z+\frac{\varepsilon\gamma\nu}{\beta}\Gamma\right) & \frac{\varepsilon\gamma\nu}{\beta}\Gamma & 0 \end{array}\right]$$

The rank of *(A, B)* is equal to *3*. Se $\lambda \neq 0$, we get

$$\left(A,\ B\right)=\left|\begin{array}{cccc} \lambda & -1 & 0 & 0 \\ -\left(-1+\frac{2\alpha_{zz}\beta w_z}{\gamma}\right) & \lambda+\left(\mu_z+\frac{\varepsilon\gamma\nu}{\beta}\Gamma\right) & -\frac{\varepsilon\gamma\nu}{C}\Gamma & \beta \\ -\frac{C}{\beta}\left(-1+\frac{2\alpha_{zz}\beta w_z}{\gamma}\right) & \frac{C}{\beta}\left(\mu_z+\frac{\varepsilon\gamma\nu}{\beta}\Gamma\right) & \lambda-\frac{\varepsilon\gamma\nu}{\beta}\Gamma & 0 \end{array}\right]. \tag{1}$$

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In the Eq. (1), by making $a = -\left(-1 + \frac{2\alpha_{zz}\beta w_z}{\gamma}\right)$, $b = \left(\mu_z + \frac{\varepsilon\gamma\nu}{\beta}\Gamma\right)$ and $d = -\frac{\varepsilon\gamma\nu}{C}\Gamma$, we obtain:

$$(A,\ B) = \begin{vmatrix} \lambda & -1 & 0 & 0 \\ a & \lambda + b & d & \beta \\ \frac{C}{\beta}a & \frac{C}{\beta}b & \lambda - \frac{c}{\beta}d & 0 \end{vmatrix}. \tag{2}$$

So

$$(A,B) \sim \begin{vmatrix} a & \lambda + b & d & \beta \\ 0 & \lambda(\lambda+b)+a & \lambda d & \lambda\beta \\ 0 & 0 & \left(\lambda - \frac{2c}{\beta}d\right)(\lambda(\lambda+b)+a) - \frac{cd}{\beta}\lambda^2 & -C\left[\lambda^2 + (\lambda(\lambda+b)+a)\right] \end{vmatrix} \text{ or}$$

$$(A,\ B) \sim \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & \lambda(\lambda+b)+a & d\lambda & \beta\lambda \\ 0 & \frac{C}{\beta}(b\lambda + a) & \lambda\left(\lambda - \frac{c}{\beta}d\right) & 0 \end{vmatrix}.$$

By making $A_1 = (\lambda + b)$, $A_2 = (b\lambda + a)$ and $A_3 = \left(\lambda - \frac{c}{\beta}d\right)$, we have:

$$(A,\ B) \sim \begin{vmatrix} \lambda & -1 & 0 & 0 \\ 0 & A_1\lambda + a & d\lambda & \beta\lambda \\ 0 & 0 & \lambda\left(A_1A_3\lambda + aA_3 - A_2\frac{Cd}{\beta}\right) & -A_2C\lambda \end{vmatrix}. \tag{3}$$

Then,from the Eq.(3) we have the rank of $(A, B) < 3$ if and only if:

1. $\lambda\left(A_1A_3\lambda + aA_3 - A_2\frac{Cd}{\beta}\right) = 0$ or
2. $-A_2C\lambda = 0$.

From observing the condition 2, and because C ≠ 0, we have $A_2\lambda = 0$. Now,

$$A_2\lambda = 0 \Leftrightarrow A_2 = (b\lambda + a) = 0 \text{ or } \lambda = 0 \Leftrightarrow \lambda = -\frac{a}{b} \text{ or } \frac{\gamma}{\beta} = 2\alpha_{zz}w_z. \tag{4}$$

From Eq. (4) and $\lambda = -\frac{a}{b}$ we have

$$\lambda = \frac{-\mu_z \pm \sqrt{\mu_z^2 - 4a}}{2} = -\frac{a}{b} \Leftrightarrow -\frac{2a}{b} + \mu_z = \pm\sqrt{\mu_z^2 - 4a} \Leftrightarrow 4\frac{a}{b}\left[\frac{a}{b} + (b - \mu_z)\right] = 0 \Leftrightarrow \frac{\gamma}{\beta} = 2\alpha_{zz}w_z$$

or $\frac{a}{b} = \mu_z - b$. From the condition 1, we have

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$$\lambda = 0 \text{ or } A_1 A_3 \lambda + aA_3 - A_2 \frac{Cd}{\beta} = 0 \Leftrightarrow A_1 A_3 \lambda = -aA_3 + A_2 \frac{Cd}{\beta} \text{ or } \frac{\gamma}{\beta} = 2\alpha_{zz} w_z .$$

Proposition 2: (Slotine, 1991) Suppose that the linear approximation is asymptotically stable, namely, the pair (A, B) is controllable, or if the pair (A, B) is not controllable then the modes of non-controllability corresponds to eigenvalues with the real part non negative. Then, any linear feedback which stabilizes asymptotically the linear approximation also stabilizes the original nonlinear system, at least locally. If the pair (A, B) is not controllable and there is no controllability modes associated with eigenvalues with positive real part, the original non-linear system cannot be stabilized.