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APO - PRESSURIZATION EFFECTS IN TUBES: FEM SIMULATION OF EFFECTIVE AXIAL FORCE

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Abstract. *This paper will cover the effective axial force approach to analyze vibrational aspects of pressurized straight each tube's flexural modes. According to Fyrileiv and Collberg, the effective axial force is a concept that translates the pressure acting over the tube walls into axial forces acting on tube ends so the equations describing vibrational behavior may be simplified. The main purpose is to verify this simplification by simulating a straight tube using finite element method where both hydrostatic pressure and effective axial force will be applied while performing a modal analysis, where their flexural modes' natural frequencies will be compared. Results show that the effective axial force representation of hydrostatic pressure in straight tubes is very consistent and results only differs by, in maximum, 4% each other, inside the range where simulations were carried.*

Keywords: *Axial Force, Pressure, Flexural Vibration, Natural Frequency, Straight Tube*

1. INTRODUCTION

Few studies of the pressurization effects in tubes evaluating its' interference in vibratory aspect are found in literature. One may observe that these effects are not perfectly clear. Therefore, this paper covers exclusively the pressurization in tube vibration.

This analysis concerns to flexural modes of pressurized pipes only, resembling beam's modal forms, which natural frequencies can be calculated by Timoshenko's beam theory. The studies were based on works of Fyrileiv & Collberg (2005), Leissa (1993), and on the DNV-RP-F105 standard (2006). Leissa (1993) describes, in detailed form, various aspects relative to the effects of pressurization in cylindrical circular shells' vibration and vibration modes, and their respective analytic formulations. Fyrileiv & Collberg (2005) and the DNV standard (2006) discusses the pressure effect in a pipe as an equivalent axial force component, as covered in this article.

2. PRESENTATION OF ANALYTIC EQUATIONS

This work will approach the effective axial force by confronting several comparisons between analytic and numeric models to emphasize its differences. Therefore, it is mandatory, at least, one introduction for the used formulations.

The Timoshenko's (1974) beam theory gives the natural frequencies of flexural modes and may be written as

$$\omega_n = \omega_{n0}\left\{1 - \frac{1}{2}\frac{r_g^2\omega_{n0}}{a}\left(1 + \frac{E}{\kappa' G}\right)\right\}, \tag{1}$$

where r_g is the gyration radius, E the elasticity modulus, G the shear modulus and κ' the effective shear area constant given by Timoshenko (1974). The ω_{n0} is the natural frequencies given by Euler-Bernoulli's theory, in the form

$$\omega_{n0} = \frac{\pi^2 n^2}{L^2} a, \tag{2}$$

for a bi-supported beam/tube. The constant a is given by

$$a = \sqrt{\frac{EI}{\rho S}}, \tag{3}$$

where I represents the transversal section's inertia momentum, ρ the density and S, the transversal section area. Having this formulation in mind, one may use it with axial force correction, given by Euler-Bernoulli (Heckl *et al*, 1988; Timoshenko *et al*, 1974) in the form

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$$\omega_{na} = \omega_n \sqrt{1 + \frac{F_a a}{EI\omega_n}}, \tag{4}$$

where F_a is the axial force applied in one of the beam/tube ends. With equations (1.1) and (1.4), the natural frequencies can be calculated with respect to the effective axial force.

3. EFFECTIVE AXIAL FORCE CONCEPT

According to the DNV standard (2006) and analyzed by Fyrileiv & Collberg (2005), the effective axial force is a way to simplify the vibration's calculations in tubular structures subjected to internal or external pressure.

The concept is based in the decomposition of pressure acting over an open tubular section in two components. One of them refers to equivalent pressure acting over a similar closed tube section, and represents buoyancy or fluid weight of the section. The other, considers a resultant axial force, opposite to the "pressure-type" force, which intensity is given by pS_S on each side, being p the pressure, S_S the transversal section that suffers the pressure effect. This axial force pS_S from the second component, arithmetically cancels the "pressure force" from the first component, resulting in the original problem's mathematical representation. This "pressure axial representation" is called "effective axial force".

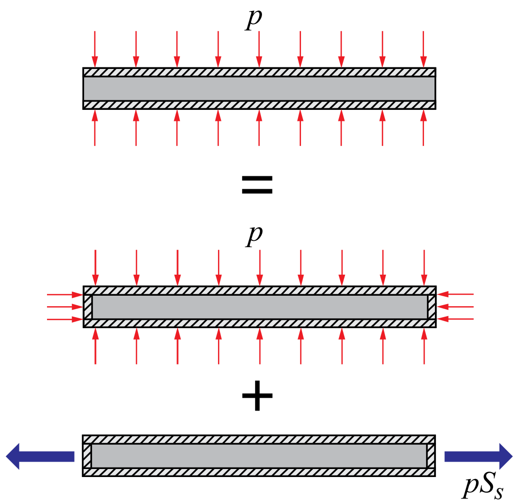

Figure 1 - Effective axial force concept's representation, that assumes the hydrostatic pressure subdivision in two components.

4. COMPARISON BETWEEN ANALYTIC AND NUMERIC MODELS

One wishes to analyze pressure effects in flexural modes' natural frequencies through three different models: the Timoshenko's analytic formulation of beams, numeric simulation using SHELL elements considering hydrostatic pressure (which are applied radially on tube walls) and a numeric simulation considering pressure equivalent axial forces from effective axial force concept.

In this analysis will be considered a simple supported tube, with 300 mm length, 6 mm external diameter, 3 mm internal diameter, using mechanical properties from a generic material with elastic modulus E = 0.63 GPa, density ρ = 2175 kg/m³ and Poisson's coefficient ν = 0.45.

The mesh used in numeric simulations had 24 elements in circumference and 247 elements across the length, totalizing 5928 elements.

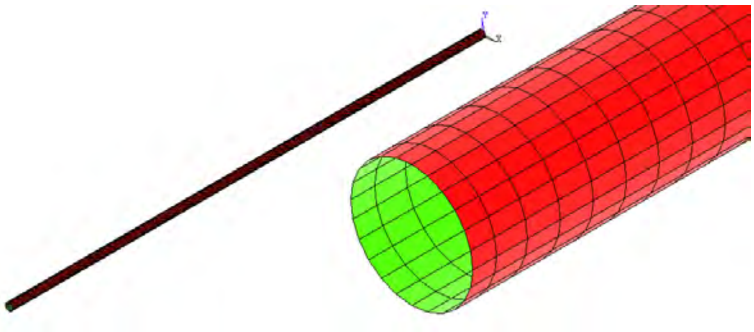

Figure 2 - Mesh used in numeric simulations to analyze pressure effect in flexural modes.

Initially, the results from Timoshenko's analytic model, considering rotatory inertia and shear deformation effects, were compared to a numeric FEM model, the later with shell elements. The 20 first flexural modes, and their respective natural frequencies, were analyzed. Some of the natural frequencies are present on Table 1 for comparison.

Table 1 - Flexural modes' natural frequencies [Hz] without pressure effect, obtained by Timoshenko's analytic model and numeric FEM model using SHELL elements.

Mode	**Analytic**	**Numeric**
1	15.7	15.0
2	62.8	60.0
3	140.7	134.4
5	385.9	368.7
10	1448.3	1397.1
20	4268.7	4738.9

Figure 3 shows the frequencies from respective flexural modes calculated by the two models, to give the results a clearer demonstration. One can observe the good concordance until 15th mode, with differences below 5%. Higher order modes, however, tend to bigger discrepancies between the results. This is due to limitations in Timoshenko's model, when the tube no longer behave as a beam, and results begin to loose physical meaning. Figure 4 shows these differences between calculated frequencies in percentage.

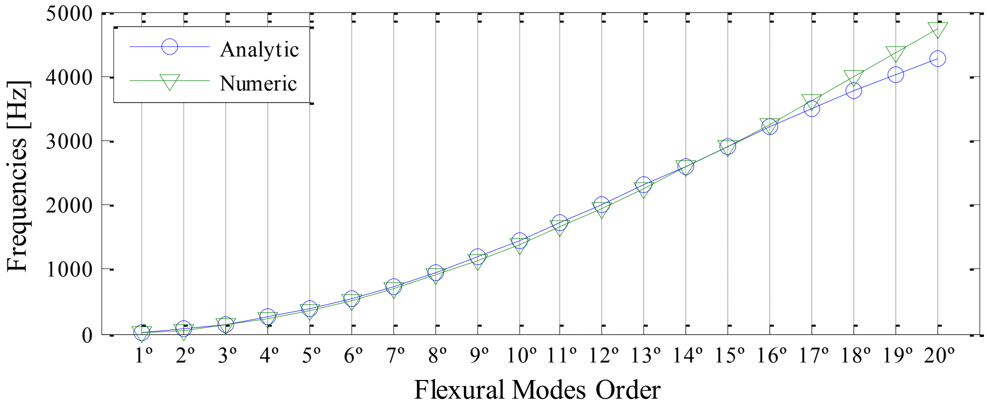

Figure 3 - Natural frequencies from respective flexural modes of unpressurized tubes, for Timoshenko's beam model and numeric model.

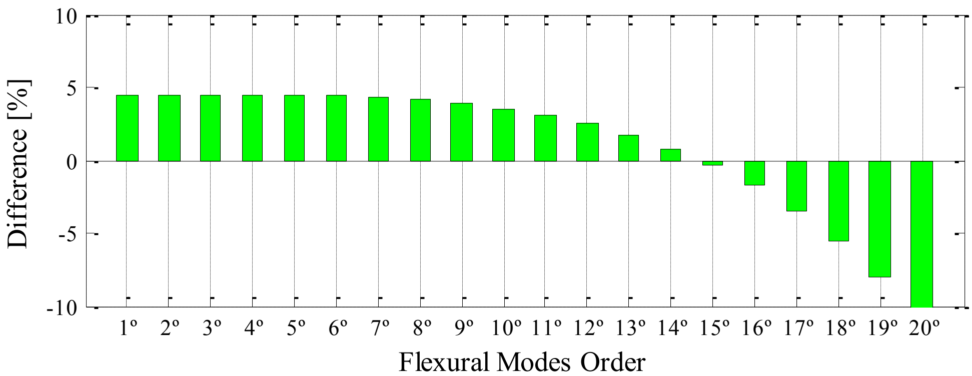

Figure 4 - Difference between natural frequencies calculated by Timoshenko's beam model and numeric model, without pressure.

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4.1 Internal Pressure Effect

Knowing the good concordance between the analytic and numeric models, the internal pressure effect on the tube was analyzed. Hence was assumed an internal pressure of 15 bar, maintaining all other analysis properties and configurations.

The analytical frequencies were calculated considering internal pressure represented by a compression effective axial force. One of the numeric analysis considered a hydrostatic pressure applied on pipe walls in radial direction and the other considered a compressive axial force equivalent to the applied pressure. Results are shown on Table 2 and Figure 5, and differences are on Figure 6.

Table 2 - Natural frequencies [Hz] of flexural modes considering internal pressure of 15 bar, obtained by Timoshenko's analytic model considering effective axial force, FEM numeric using shell element considering hydrostatic pressure (radial) and FEM numeric considering effective axial force.

Mode	**Analytic**	**$\text{Numeric}_{\text{pressure}}$**	**$\text{Numeric}_{\text{axial}}$**
3	92.1	72.9	73.8
5	343.9	317.5	319.2
10	1410.5	1347.6	1354.7
20	4241.1	4687.3	4721.7

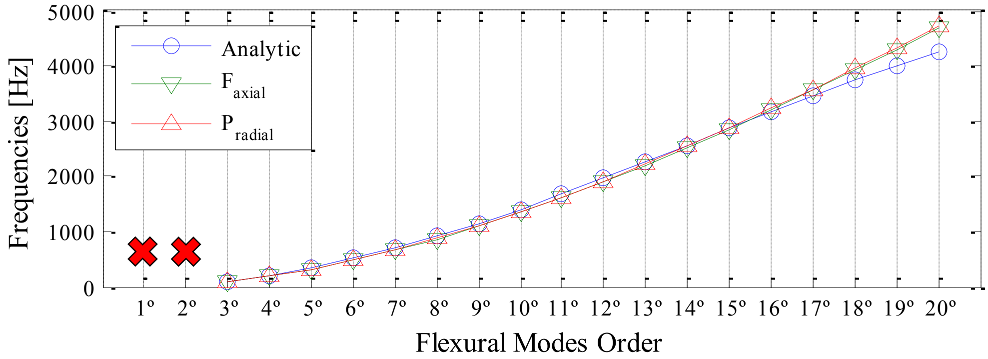

Figure 5 - Natural frequencies obtained mode-to-mode from the three models (Timoshenko's with axial force, numeric with internal pressure and numeric with radial force) with internal pressure of 15 bar.

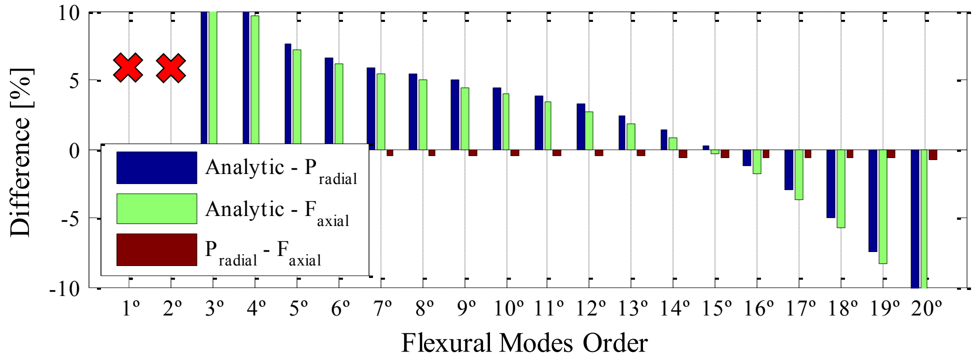

Figure 6 - Differences between models of Timoshenko's beam with axial force, numeric with hydrostatic pressure and numeric with axial force, considering 15 bar of internal pressure.

These results show that the effective axial force approach to simplify the pressurization effects in tubes is very consistent.

The difference between two numeric models remains practically constant over modal order, with less than 1%. However, one remembers that the analytic formulation have its correction factor based on Euler-Bernoulli's beam theory, which do not considers rotatory inertia nor shear effects. Has been said that, despite of Timoshenko's natural frequencies adaptation approximate the results to numeric analysis, its difference still noteworthy.

One concludes that the simulation of internal pressure effect represented by an equivalent compression axial force provides a mean difference of just 0.5% from flexural natural frequencies, addressing again to Figure 6. The difference

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related to analytic frequencies slightly grows on first modes (Fyrileiv & Collberg, 2005). For example, the difference for the third mode between Timoshenko's analytic result and numeric result is significant. One believes this occurrence is due to the value of resultant axial force be near to the critical buckling load relative to this mode.

Was observed that the first resonance, when tube submitted to 15 bar of internal pressure, occurs in 73 Hz, approximately, with modal form correspondent to the third flexural mode of vibration. This happened because, with internal pressure of 15 bar and current mechanical properties, the instability limit by buckling was reached. The natural frequencies in these modes do not present physical meaning, which occurs when the square root's argument from equation (1.4) is a negative number. For an internal pressure of 30 bar, the same behavior is observed, but now for the tree first flexural modes instead of two.

Table 3 - Natural frequencies [Hz] of flexural modes considering internal pressure of 30, bar, obtained by Timoshenko's analytic model considering effective axial force, FEM numeric using shell element considering hydrostatic pressure (radial) and FEM numeric considering effective axial force.

Mode	Analytic	$\text{Numeric}_{\text{pressure}}$	$\text{Numeric}_{\text{axial}}$
4	148.6	106.3	108.2
5	296.0	256.2	258.4
10	1371.7	1296.2	1303.8
20	4213.4	4635.1	4669.5

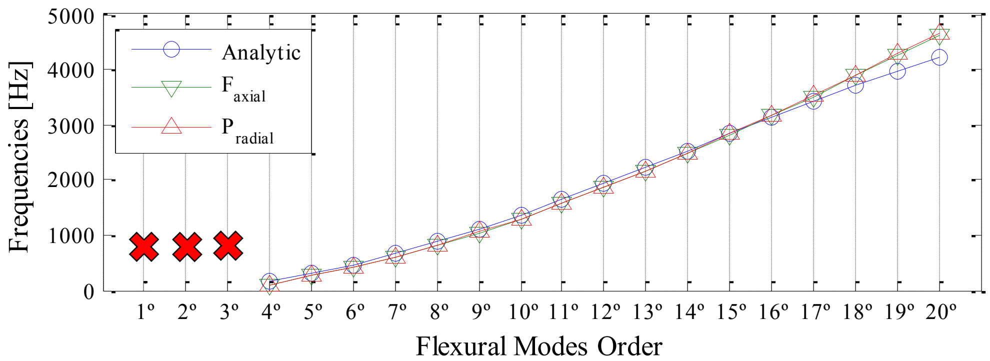

Figure 7 - Natural frequencies obtained mode-to-mode from the three models (Timoshenko's with axial force, numeric with internal pressure and numeric with radial force) with internal pressure of 30 bar.

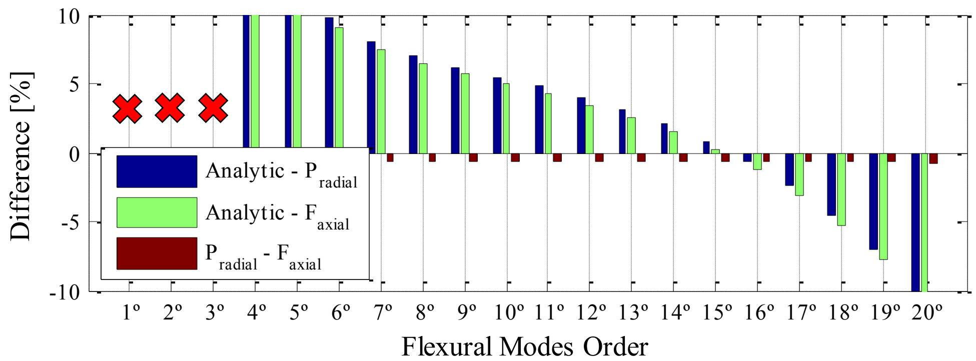

Figure 8 - Differences between models of Timoshenko's beam with axial force, numeric with hydrostatic pressure and numeric with axial force, considering 30 bar of internal pressure.

The difference between the two numeric approaches also slightly increases, to the order of 0.65%, but still a very small value and can be neglected.

One may conclude that the results shows good concordance between the hydrostatic pressure representation and with it translated in a compression effective axial force, and that the internal pressure make flexural mode's natural frequencies decrease as the pressure grows.

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4.2 External Pressure Effect

Continuing the validation procedures of effective axial force concept, simulations considering external pressure were also made. Resembling precedent tests, now assumes initially 15 bar of external pressure and 30 bar after. Obtained natural frequencies will be compared in graphical form varying the order of flexural modes and showing the differences between them in percent.

Table 4 - Natural frequencies [Hz] of flexural modes considering external pressure of 15, bar, obtained by Timoshenko's analytic model considering effective axial force, FEM numeric using shell element considering hydrostatic pressure (radial) and FEM numeric considering effective axial force.

Modes	**Analytic**	**Numeric$_{pressure}$**	**Numeric$_{axial}$**
1	39.0	40.6	40.6
2	95.0	96.3	96.5
3	176.5	175.5	175.9
5	423.7	413.5	414.8
10	1485.1	1444.8	1451.1
20	4296.0	4790.0	4824.5

Changes in the difference behavior can be observed between the models: the analytic natural frequencies from lesser order's modes show themselves smaller than the numeric ones. It is important to analyze this, since the imposed axial forces now have opposite direction from previous simulations.

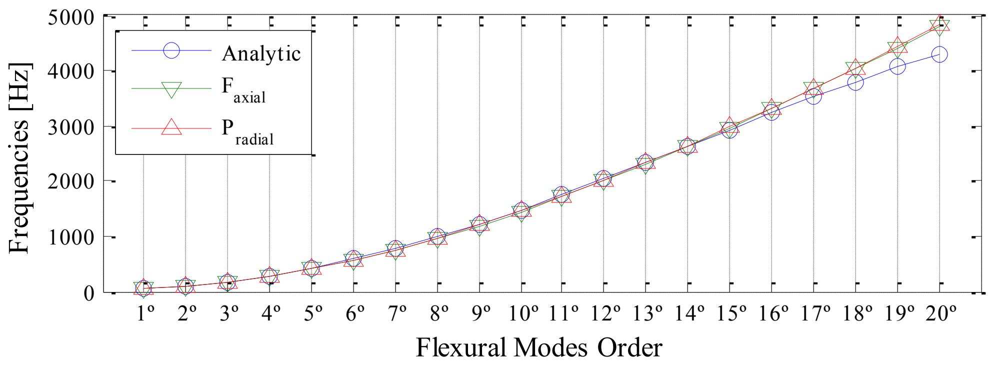

Figure 9 - Natural frequencies obtained mode-to-mode from the three models (Timoshenko's with axial force, numeric with external pressure and numeric with radial force) with external pressure of 15 bar.

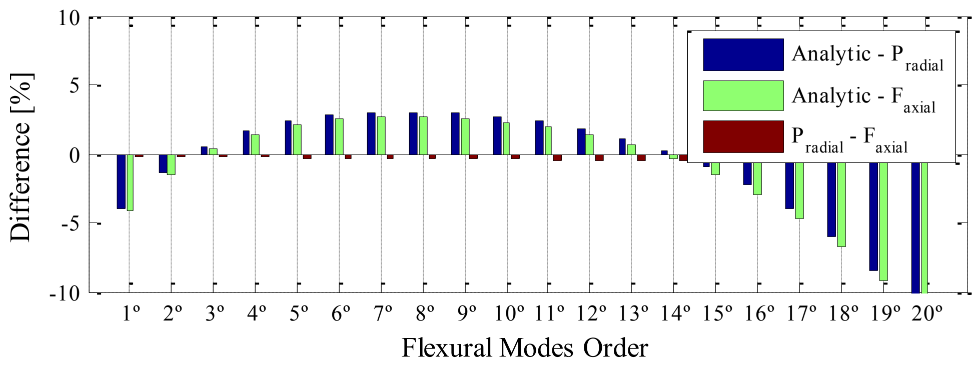

Figure 10 - Differences between models of Timoshenko's beam with axial force, numeric with hydrostatic pressure and numeric with axial force, considering 15 bar of external pressure.

Can be observed that the difference between the two numeric approaches shows a minor increase with mode's order, but remains less than 0,8% which still is negligible. It's good to remember that external pressure now have an equivalent tension-type axial force, so the natural frequencies have higher values than the previous cases.

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Increasing the external pressure up to 30 bar, the results are shown in Table 5 and Figure 11, and respective differences between model's results are in Figure 12.

Table 5 - Natural frequencies [Hz] of flexural modes considering external pressure of 30, bar, obtained by Timoshenko's analytic model considering effective axial force, FEM numeric using shell element considering hydrostatic pressure (radial) and FEM numeric considering effective axial force.

Mode	**Analytic**	**$Numeric_{pressure}$**	**$Numeric_{axial}$**
1	52.9	55.4	55.5
2	118.7	122.2	122.4
3	206.1	208.6	209.1
5	458.5	454.0	455.2
10	1521.0	1491.0	1496.9
20	4323.2	4840.4	4875.1

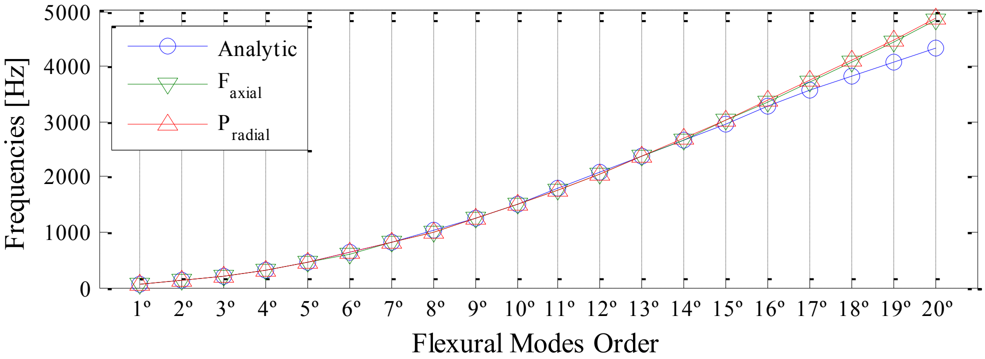

Figure 11 - Natural frequencies obtained mode-to-mode from the three models (Timoshenko's with axial force, numeric with external pressure and numeric with radial force) with external pressure of 30 bar.

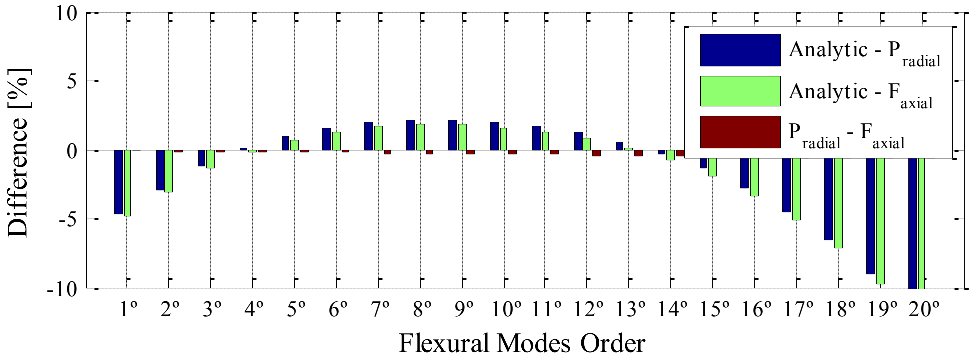

Figure 12 - Differences between models of Timoshenko's beam with axial force, numeric with hydrostatic pressure and numeric with axial force, considering 30 bar of external pressure.

With the differences between numeric models below 1% and between analytic-numeric below 5% before mathematical representation lost physical meaning, one may conclude that the representation of pressure as an equivalent axial force is robust and valid. Is good to emphasize that internal pressure decreases natural frequencies until instability by buckling effect, and external pressure increases them.

5. ACKNOWLEDGEMENTS

Numeric and analytic results shows that tubes submitted to internal pressure have its natural frequencies of flexural vibration modes decreased until the buckling limit is achieved (Fyrileiv & Collberg, 2005; Leissa, 1993; Timoshenko et al, 1974). To smaller frequencies than those correspondents to buckling's limit provided by equivalent axial force, the tube does not present resonances. Those cases are represented as unstable. On the other hand, when submitting the tube to an external pressure, the frequencies would increase indefinitely if, in the practice, the material didn't fail.

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Table 6 - Natural frequencies [Hz] given by Timoshenko's beam model, under effective axial force.

modes	$30bar_{external}$	$15bar_{external}$	Unpressurized	$15bar_{internal}$	$30bar_{internal}$
1	52.9	39.0	15.7	unstable	unstable
2	118.7	95.0	62.8	unstable	unstable
3	206.1	176.5	140.7	92.1	unstable
5	458.5	423.7	385.9	343.9	296.0
10	1521.0	1485.1	1448.3	1410.5	1371.7
20	4323.2	4296.0	4268.7	4241.1	4213.4

Leissa (1993) also affirms that, to vibration modes similar to beam-like ones (flexural), the hydrostatic internal pressure decreases its natural frequencies. This effect becomes very meaningful to small ratios of tube's mass and length (mR/l). The critical internal pressure, which causes buckling, is calculated by the expression $E\pi^2 Rh/l^2$, where E is the elasticity modulus, R the tube's external radius, h its thickness and l its length. One similar expression is also presented by Fyrileiv & Collberg (2005), with the term Rh substituted by the transversal section's inertia momentum I. However, Leissa warns that the buckling phenomenon happens only with flexural modes. Such behavior (decrease of natural frequencies with internal pressure's increase) does not apply to axial, radial and circumferential vibration modes.

Table 7 –Natural frequencies [Hz] given by FEM numeric model using shell elements, submitted to hydrostatic pressures on radial direction.

modes	$30bar_{external}$	$15bar_{external}$	Unpressurized	$15bar_{internal}$	$30bar_{internal}$
1	55.4	40.6	15.0	unstable	unstable
2	122.2	96.3	60.0	unstable	unstable
3	208.6	175.5	134.4	72.9	unstable
5	454.0	413.5	368.7	317.5	256.2
10	1491.0	1444.8	1397.1	1347.6	1296.2
20	4840.4	4790.0	4738.9	4687.3	4635.1

The tendency of natural frequencies' decrease with increase of internal pressure is clear, corroborating Fyrileiv & Collberg's studies (2005). It is noticed also a decrease of pressure's effect on higher frequencies by observing smaller differences between natural frequencies of same flexural modes' orders.

Table 8 –Natural frequencies [Hz] given by FEM numeric model, under effective axial force simulating pressure.

modes	$30bar_{external}$	$15bar_{external}$	Unpressurized	$15bar_{internal}$	$30bar_{internal}$
1	55.5	40.6	15.0	unstable	unstable
2	122.4	96.5	60.0	unstable	unstable
3	209.1	175.9	134.4	73.8	unstable
5	455.2	414.8	368.7	319.2	258.4
10	1496.9	1451.1	1397.1	1354.7	1303.8
20	4875.1	4824.5	4738.9	4721.7	4669.5

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