



## FLOW PATTERN IDENTIFICATION IN BISTABLE FLOWS BY MEANS OF SYMBOLIC DYNAMICS

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**Abstract.** *The bistable phenomenon consists on the deviation of the flow of two cylinders placed side-by-side, where a stable characteristic is acquired for a certain period of time. In general, bistability occurs in flows over sets of bluff bodies forming a flip-flopping wake characterized by a biased flow switching at irregular intervals. Therefore, bistability can represent an additional source of dynamic instabilities. By means of hot wire anemometry technique the presence of the phenomenon of the bistability in the flow on two cylinders side-by-side is investigated in an aerodynamic channel. The results from hot wires were filtered by means of wavelet transform, so that the turbulence is isolated from the bistable phenomenon. The resulting time series is analyzed by means of symbolic dynamics, thus being considered as a chaotic signal. The application of symbolic dynamics allows the identification of the flow patterns where the time series are partitioned according to an alphabet. The results show that the concentration of the flow modes is markedly in well-defined values.*

**Keywords:** *turbulent flow, hot-wire anemometry, bistable flow, wavelets, symbolic dynamics.*

### 1. INTRODUCTION

Circular cylinders nearly disposed are found in several engineering applications like heat exchangers, pipelines and transmission lines. Tube banks are the most used configuration for the analysis of the phenomena that occur in various arrangement types.

A very interesting phenomenon occurs when two circular cylinders placed side-by-size are submitted to a turbulent cross-flow. The flow that emanates through the gap between the cylinders is biased towards the rear surface of one of the cylinders, and has a narrow wake. This can be called as a flow mode. In the literature, bistability is the phenomenon where a floppy and random behavior of the gap flow changes intermittently the flow mode, from one cylinder to other at irregular time intervals. This process generates two dominant Strouhal numbers, each one associated with one of the two wakes formed: the wide wake is associated with a lower Strouhal number and the narrow wake with a higher one.

The study of the behavior of the bistability phenomenon in simplified geometries, as in the case of two tubes placed side-by-side, helps in understanding the parameters and variables that influence more complex geometries, as in the case of banks of tubes or rods, which are found in the nuclear and process industries, being the most common geometry used in heat exchangers. As flow induced vibration and structure-fluid interaction are very dependent of the arrangement or configuration of the cylinders and since bistability can be an additional excitation mechanism on the tubes, new studies are justified with new techniques to better understand and classify this phenomenon.

According to Zdravkovich and Stonebanks (1990), the leading feature of flow-induced vibration in tube banks is the randomness of dynamic responses of tubes, and even if the tubes are all of equal size, have the same dynamic characteristics, are arranged in regular equidistant rows and are subjected to an uniform steady flow, the dynamic response of tubes is non-uniform and random.

The symbolic dynamics technique can be applied to identify patterns in the flow through the analysis of a particular partition with an alphabet, and further study of histograms.

Ferrara and Prado (1999), comment that it is possible to develop a study of topological attractors using a completely different approach, based on symbolic dynamics.

Lehrman and Rechester (2001) applied a symbolic dynamics technique to extract from chaotic signals of turbulent fluctuations data, information related to the structure of unstable periodic orbits. The authors concluded that their methodology could be applied in the analysis of fluctuations in turbulent data measured experimentally.

Daw *et al.* (2003), present a simplified method of symbolization of experimental time series, as well as its use in describing a two-dimensional Poincaré section discretized.

Also, Brida and Punzo (2003), present a comprehensive review of a symbolic dynamics technique to study the dynamics of economic systems.

Rajagopalan *et al.* (2007), present a symbolic analysis of time series data from multidimensional measuring to identify patterns in dynamic systems.

Thus, the study of attractors based on symbolic dynamics involves the analysis and characterization of the time series using statistical density histograms representative of symbolic sequences. An analysis of the bistable phenomenon based on symbolic dynamics can be used not only in experimental time series, but also on the reconstructed attractor in the state space, aiming at identifying patterns contained within the data.

Wavelet analysis is a useful tool for the study of transient turbulent signals (Indrusiak *et al.*, 2005; Olinto *et al.*, 2009), in special the switching phenomenon in two side-by-side circular cylinders (Alam *et al.*, 2003), and its use helps in the comprehension of the phenomena studied in laboratory conditions. The wavelet analysis can be applied to time varying signals, where the stationary hypothesis cannot be maintained, to allow the detection of non permanent flow structures. A discrete wavelet transform (DWT) is used to make a multilevel decomposition of a time signal in several bandwidth values, accordingly with the selected decomposition level.

## 2. THE BISTABLE EFFECT

According to Sumner *et al.* (1999), the cross steady flow through same diameter circular cylinder ( $d$ ) placed side-by-side can present a wake with different modes, depending on distances between its centers, called pitch ( $p$ ). Different flow behaviors can be found for different pitch-to-diameter ratios  $p/d$ . When cylinders are in contact ( $p/d=1$ ), they behave as a bluff body, and due the increasing of the distance between the free shear layer from both sides, the vortex shedding is lower than of a single cylinder. At small pitch ratios ( $1.0 < p/d < 1.2$ ) two cylinders placed side-by-side still behave as a bluff body, but the high-momentum fluid that enters trough the gap between tubes increases the base pressure and reduces the drag forces of both cylinders, with a vortex-shedding frequency close to that observed for  $p/d=1$ . In addition, a single vortex street is observed in the combined wake of the two cylinders, and vortex shedding occurs only from the outer shear layer. Three types of behavior were observed. The most commonly observed has an asymmetrical near-wake region with a deflected or biased gap-flow, with a possible single vortex street forming downstream. Another, with a symmetrical near-wake formation of a single vortex street, and a gap-flow oriented parallel to the flow axis. And a third flow pattern showing no significant gap flow. At large pitch ratios ( $p/d > 2.2$ ) the biased flow disappears, and side-by-side circular cylinders behave more independent, as isolated bluff bodies. Nevertheless, there is still some interaction or synchronization occurring between them, predominantly as anti-phase vortex formation. At intermediate pitch ratios ( $1.2 < p/d < 2.0$ ) the flow is characterized by a wide near-wake behind a cylinder and a narrow near-wake behind the other, as shown in Fig. 1a and Fig. 1b. This phenomenon generates two dominants vortex-shedding frequencies, each one associated with a wake: the narrow wake is associated with a higher frequency and the wide wake with a lower one. Through the gap, flow is biased towards the cylinder, and has a narrow wake. Bistable flow is characterized by switch of this gap flow, from one side to other at irregular time intervals. Thereby, if the flow velocity is measured downstream the cylinders, by example along the tangent to their external generatrices, a switch mode can occur as shows the scheme in Fig. 1b. According to previous studies, this pattern is independent of Reynolds number, and it is not associated to cylinders misalignment or external influences, what suggest an intrinsically flow feature.

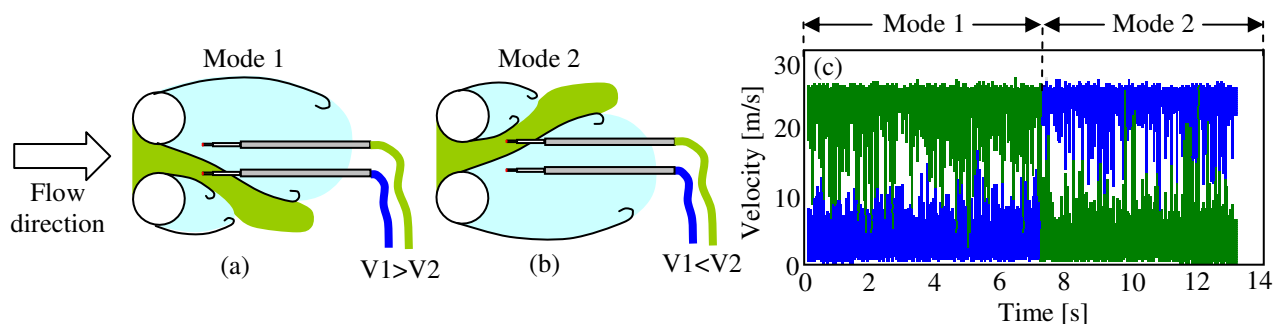


Figure 1. Bistability scheme for (a) mode 1 and (b) mode 2, and the respective characteristic signals (c).

Kim and Durbim (1988), present a study where the transition between the asymmetric states is completely random and it is not associated with a natural frequency. Through a dimensionless study, the authors concluded that the mean time between the transitions is on order  $10^3$  times longer than vortex shedding period, and the mean time intervals between the switches decreases with the increasing of Reynolds number. As Strouhal numbers are relatively independent from the Reynolds numbers (Žukauskas, 1972), they conclude that there is no correlation between the bistable feature and the vortex shedding.

De Paula (2013), studying bistable flows on the simplified geometry of two tubes placed side-by-side in an aerodynamic channel, performed a joint analysis of axial and transverse velocity fluctuation components of the flow,

and its probability density functions indicate the regions in the measurement plan where the phenomenon is manifested. Through the reconstructions of the trajectory of the filtered time series for certain frequency bands, was found that the experimental turbulent time series had chaotic-deterministic characteristics. The largest Lyapunov exponent of experimental series was positive, which is an indication of chaotic behavior.

### 3. OBJECTIVES

The purpose of this paper is to study the bistable flow after the simplified geometry of two tubes placed side-by-side in an aerodynamic channel to better comprehend the switching of the gap flow. Through the technique of symbolic dynamics applied to the experimental time series, a decimal representation is performed through histograms according to an alphabet.

### 4. METHODOLOGY

Time series of axial and transversal velocity obtained with the constant temperature hot wire anemometry technique in an aerodynamic channel are used as input data in a dynamic symbolic approach, enabling the analysis of the data. The average speed and speed fluctuations of the air are measured downstream the tubes.

Wavelet transforms were also applied to perform a joint time-frequency domain analysis of the experimental signals, in order to make a multilevel decomposition of the signals in several bandwidth values, accordingly with the selected decomposition level.

#### 4.1 Experimental technique

The aerodynamic channel used in the experiments is made of acrylic, with a rectangular test section of 0.146 m height, width of 0.193 m and 1.02 m of length (Fig. 2a). The air is impelled by a centrifugal blower of 0.64 kW, and passes through two honeycombs and two screens, which reduce the turbulence intensity to about 1% in the test section. Upstream the test section, placed in one of the side walls, a Pitot tube measures the reference velocity of the non-perturbed flow.

The velocity of the flow and its fluctuations are measured by means of a DANTEC *StreamLine* constant hot-wire anemometry system. One double straight/slant hot wire probe (type DANTEC 55P71 Special) was used in the experiments, where their position is shown in Fig. 1c. The double probe has a straight wire perpendicular to the main flow, and a slant wire 45° with the probe axis. Data acquisition is performed by a 16-bit A/D-board (NATIONAL INSTRUMENTS 9215-A) with USB interface. The acquisition frequency of time series was of 1 kHz, and a low-pass filter of 300 Hz was used to avoid aliasing.

The circular cylinders, with external diameter of 25.1 mm, are made of Polyvinyl chloride (PVC), are rigidly attached to the top wall of test section and their extremities are closed. The probe support is positioned with 3D transverse system placed 200 mm downstream the outlets (Fig. 2b).

The measurements were performed aligning the probes along the tangent to the external generatrices of the tubes (Fig. 2c). The mean error of the flow velocity determination with a hot wire was about +/- 3%. The Reynolds number of the experiment is 21,000, computed with the tube diameter and the reference velocity of 12.9 m/s.

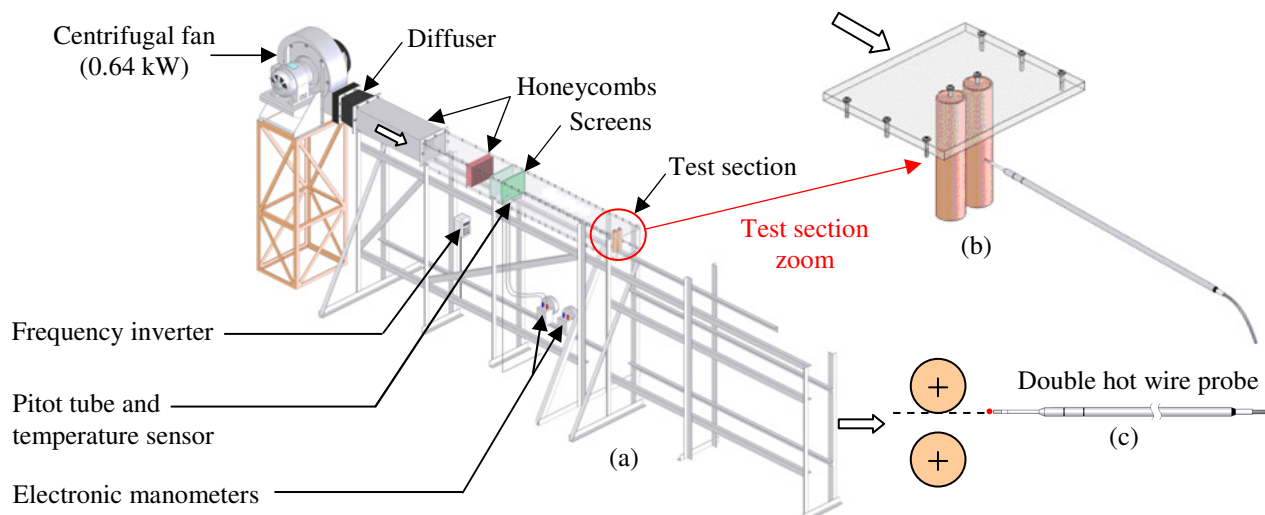


Figure 2. Schematic view of (a) the aerodynamic channel, (b) test section and (c) probe position.

## 4.2 Wavelet transforms

The wavelet analysis can be applied to time varying signals, where the stationary hypothesis cannot be maintained, to allow the detection of non permanent flow structures. The Fourier transform of a discrete time series gives the energy distribution of the signal in the frequency domain evaluated over the entire time interval. While the Fourier transform uses trigonometric functions as basis, the lowers of wavelet transforms are functions named wavelets, with finite energy and zero average that generates a set of wavelet basis.

The continuous wavelet transform of a function  $x(t)$  is given by

$$\tilde{X}(a, b) = \int_{-\infty}^{\infty} x(t) \psi_{a,b}(t) dt, \quad (1)$$

where  $\psi$  is the wavelet function and the parameters  $a$  and  $b$  are respectively scale and position coefficients ( $a, b \in \mathfrak{R}$ ) and  $a > 0$ .

The respective wavelet spectrum is defined as

$$P_{xx}(a, b) = |\tilde{X}(a, b)|^2. \quad (2)$$

In the wavelet spectrum, Eq. (2), the energy is related to each time and scale (or frequency) (Daubechies, 1992).

This characteristic allows the representation of the distribution of the energy of the signal over time and frequency domains, called spectrogram.

The discrete wavelet transform (DWT) is a judicious sub sampling of the continuous wavelet transform (CWT), dealing with dyadic scales, and given by (Percival and Walden, 2000)

$$d(j, k) = \sum_t x(t) \psi_{j,k}(t), \quad (3)$$

where the scale and position coefficients ( $j, k \in \mathbb{I}$ ) are dyadic sub samples of ( $a, b$ ).

Any discrete time series with sampling frequency  $F_s$  can be represented by

$$x(t) = \sum_k c(J, k) f_{J,k}(t) + \sum_{j \leq J} \sum_k d(j, k) y_{j,k}(t), \quad (4)$$

where the first term is the approximation of the signal at the scale  $J$ , which corresponds to the frequency interval  $[0, F_s/2^{J+1}]$  and the inner summation of the second term are details of the signal at the scales  $j$  ( $1 \leq j \leq J$ ), which corresponds to frequency intervals  $[F_s/2^{j+1}, F_s/2^j]$ .

The velocity signals were analyzed using discrete wavelet transforms to decompose the measured signal in wavelet approximations divided in frequency bands (Indrusiak *et al.*, 2005).

In this work, Daubechies Db20 functions were used as lowers of discrete wavelet transforms. Indrusiak *et al.* (2005) presents a study of wavelet transforms, applied to accelerating and decelerating turbulent flows in tube banks. Mathematical tools were developed using Matlab<sup>®</sup> software and its specific toolboxes for statistical, spectral and wavelet analysis.

## 4.3 Symbolic dynamics

The technique of symbolic dynamics applied to the experimental series in this work is the same as proposed by Daw *et al.* (2003). The steps used in this procedure are as follows:

- (a) the time series data is divided (or partitioned) into a convenient position;
- (b) the values of the time series are converted according the alphabet chosen, containing  $n$  symbols, based on the partitioning performed, generating a symbolic series;
- (c) a length  $k$  is chosen to represent words, being generated  $m$  words, where  $m = n^k$ ;
- (d) a transformation is performed in decimal representation of words generated;
- (e) the result is displayed using histograms according to the decimal representation. This result represents the statistical density of symbolic sequences, where the horizontal axis represents the symbolic sequences and the vertical axis the absolute or relative frequency of the sequences.

The necessary steps described above for the symbolization of time series to construct the histogram of symbolic sequences are shown schematically in Fig. 2.

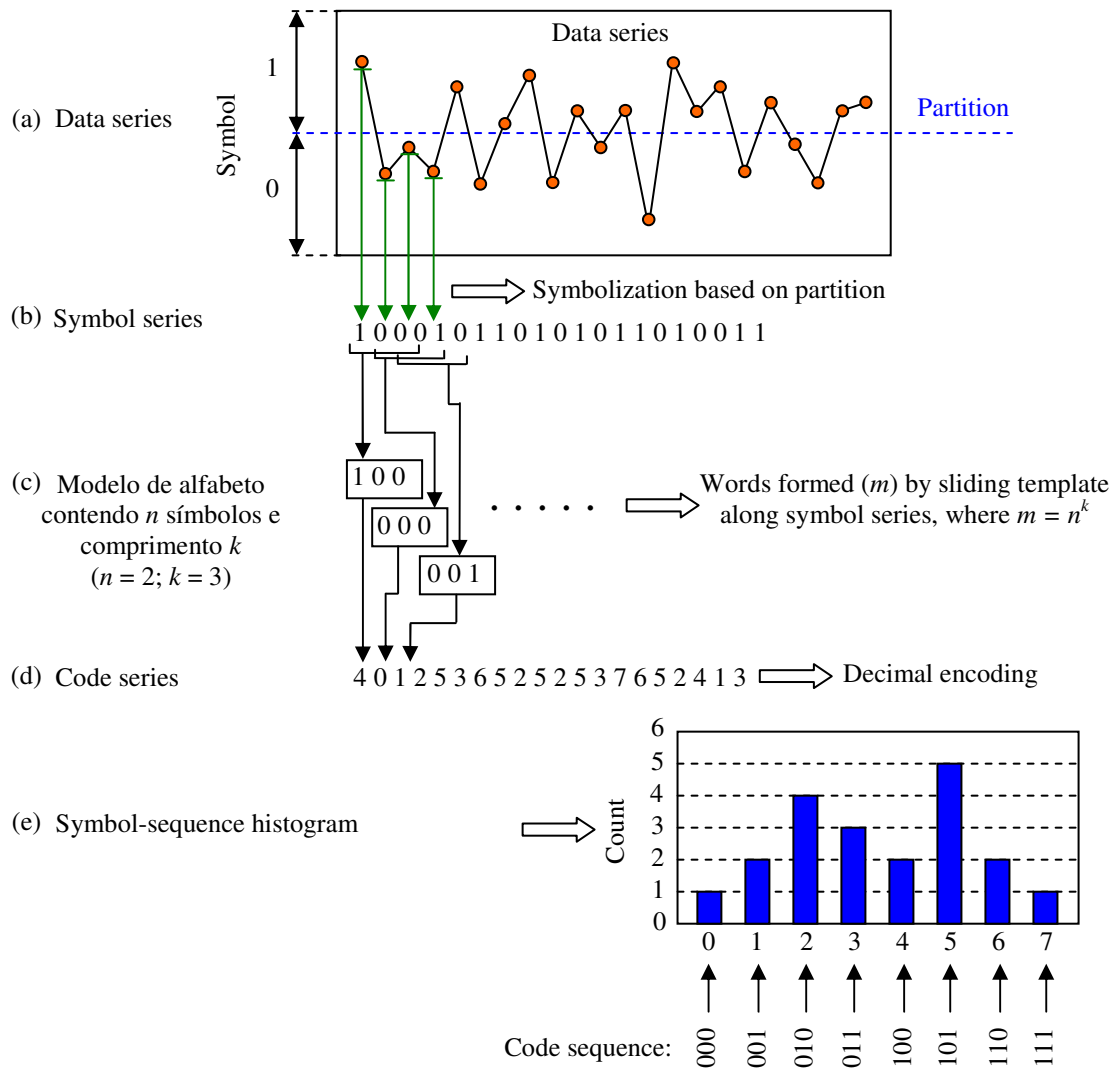


Figure 2. Process symbolizing a series of data to construct the histogram of the symbolic sequence (adapted from Daw *et al.*, 2003).

For the binary alphabet, for example, there are two letters (0 or 1), or  $n = 2$ . If the length for the representation of words was chosen equal 3 ( $k = 3$ ), there will be  $m = n^k = 2^3 = 8$  words (000, 001, 010, 011, 100, 101, 110 and 111). The probability of a binary episode  $i$ , denoted by  $p_i$ , will be between  $0 \leq p \leq 1$ . A partial representation of a symbolic binary tree for lengths of symbolic sequence  $k$  from 1 to 3 is illustrated in Fig. 3.

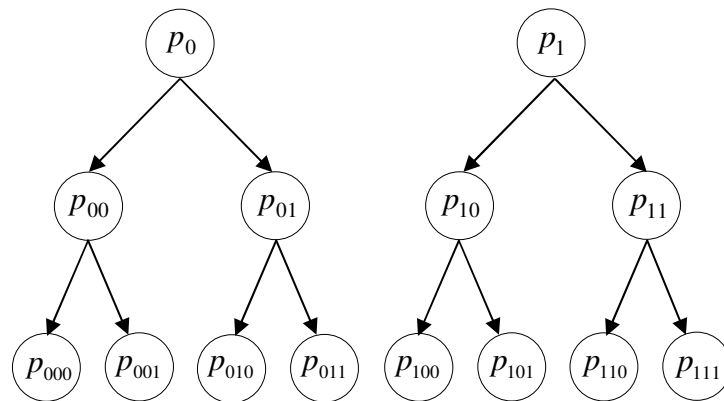


Figure 3. Partial representation of a symbolic binary tree for lengths of symbolic sequence  $k$  from 1 to 3 (adapted from Daw *et al.*, 2003).

For purely random data sequences, all symbolic sequences of length  $m$  are equiprobable, that is, the result should have uniform density. Thus, the detachment of such behavior indicates the structure of the deterministic data, that can be chaotic or not.

## 5. RESULTS

Figure 4 show the time series of axial and transversal velocity. Figure 4a shows several changes between two distinct velocity levels from the axial component, concerning to 3.0 m/s (wide near-wake - mode 1) and 18.6 m/s (narrow near-wake - mode 2). These changes are accompanied by the transversal component (Fig. 4b), as the flow changes direction, from the wide near-wake to the narrow near-wake mode.

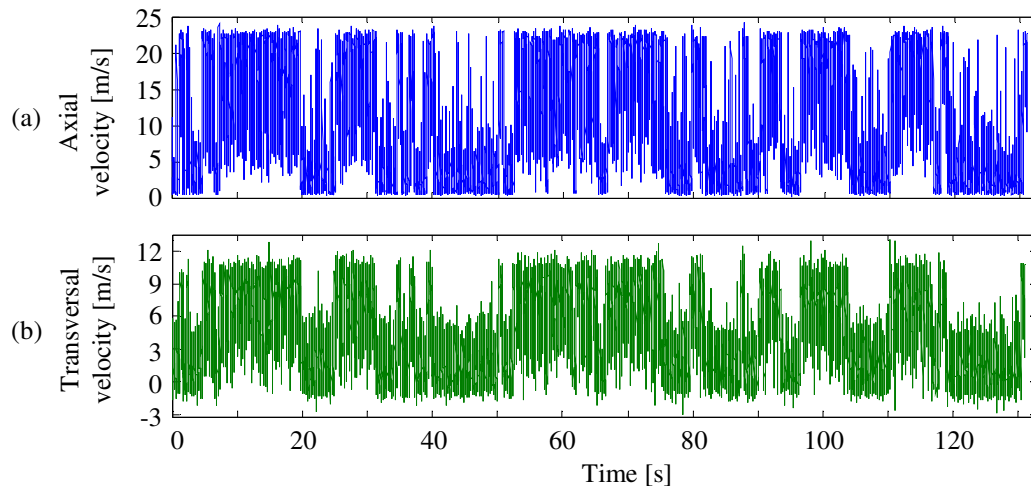


Figure 4. Velocity signals of the flow: (a) axial velocity and (b) transversal velocity.

The result of the transformation of the axial velocity signal (Fig. 4a) in binary alphabet has several changes between the states 0 and 1, for the two modes of flow (Fig. 5a). For this analysis, only the fluctuation signal is used, that is, the partition is done around the zero value. The graph appears similar to a barcode, which filled intervals represent the various exchanges in small time intervals between the states 0 and 1, and the blank intervals which are not actually occurring exchanges direction of flow. Similar behavior is observed in the transverse component of velocity (Fig. 5b).

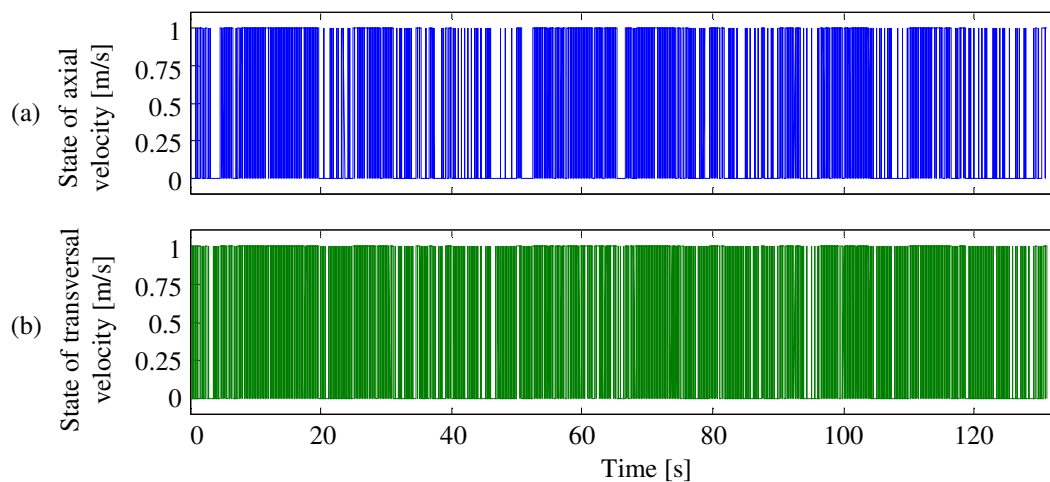


Figure 5. Transformation of the time series in binary alphabet: (a) axial velocity and (b) transversal velocity.

Figure 6 shows a magnification of the first 2 seconds of the Fig. 5, where the variations between the states 0 and 1 are observed with clarity.

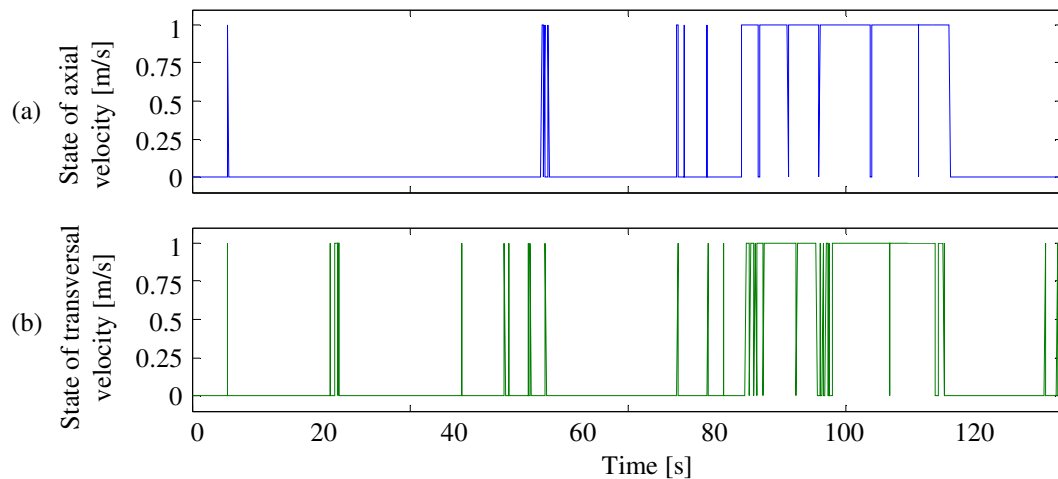


Figure 6. Magnification of the first 2 seconds of the Fig. 5: (a) axial velocity and (b) transversal velocity.

To access the information about the flow modes via symbolic dynamics, symbolic sequences are chosen with length ranging from 1 to 6, and the results of the relative frequencies ( $F_r$ ) of the time series are shown in Fig. 7.

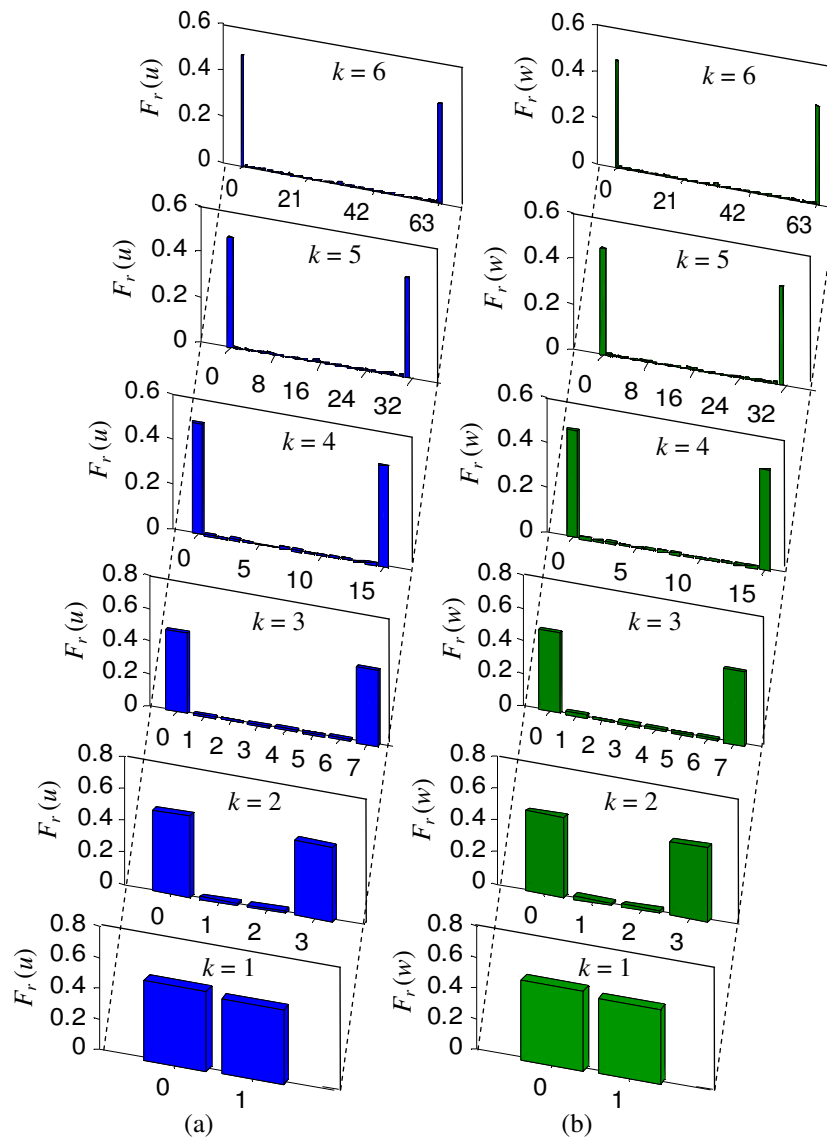


Figure 7. Relative frequencies ( $F_r$ ) of the time series, extracting the average values for various lengths of symbolic sequence  $k$ : (a) axial velocity and (b) transversal velocity of the flow.

From the partitions chosen in Fig. 7 (around zero), it is evident the concentrations about two values, referred to the two flow modes. This shows that the probability of occurrence of other flow modes is less than that for the wide near-wake and narrow near-wake flow modes (modes 1 and 2). However, analysis of other partitioned can provide new information about the phenomenon.

Through a reconstruction of the velocity signals with discrete wavelet transform (DWT) of level  $n = 9$  and function Db20, for a band of frequencies from 0 to 0.976 Hz, the filtered signals (Fig. 8) present about 39 switch modes for all observed time (131.072 seconds).

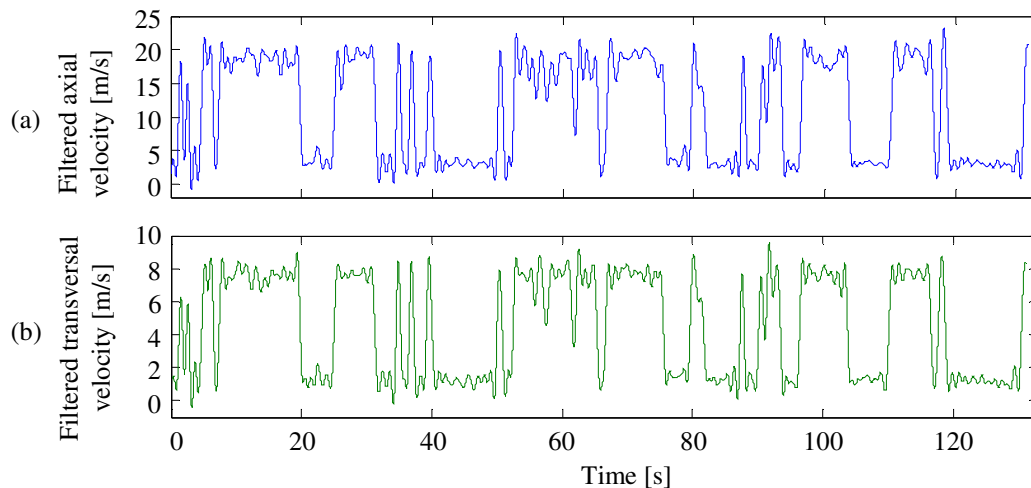


Figure 8. Reconstruction of the velocity signals with discrete wavelet transform of level  $n = 9$ .

The result of the transformation of the reconstructed signals (Fig. 8) in binary alphabet presents almost identical behavior for both velocity components (Fig. 9). The total number of states 0 (referred to the wide near-wake) is 64265 (49.03 % of the signal) and the total number of states 1 (referred to the narrow near-wake) is 66807 (50.97 % of the signal).

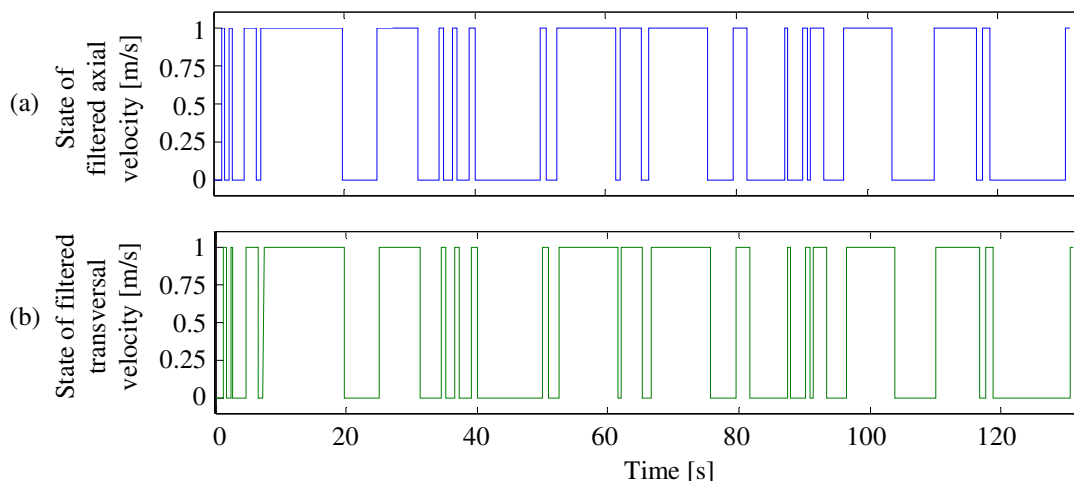


Figure 9. Transformation of the reconstructed signals in binary alphabet: (a) axial velocity and (b) transversal velocity.

Symbolic sequences of the filtered signals are shown in Fig. 10, for length ranging from 1 to 6, and the results of the relative frequencies of the time series also show concentrations about two values, referred to the two flow modes. The similar behavior of these symbolic sequences with those obtained for original time series is due to the partition around the mean velocity.

To analyze the stable flow modes of the bistable phenomenon, two alternative partitions are studied. From the velocity fluctuations are extracted the mean and standard deviation values, which were used to determine the positions of the partitions. The first partition (called P1) is determined by adding to the mean value of the time series the standard deviation divided by three. The second partition (P2) is determined by adding the mean value with two times the standard deviation divided by three, according to the scheme shown in Fig. 11.



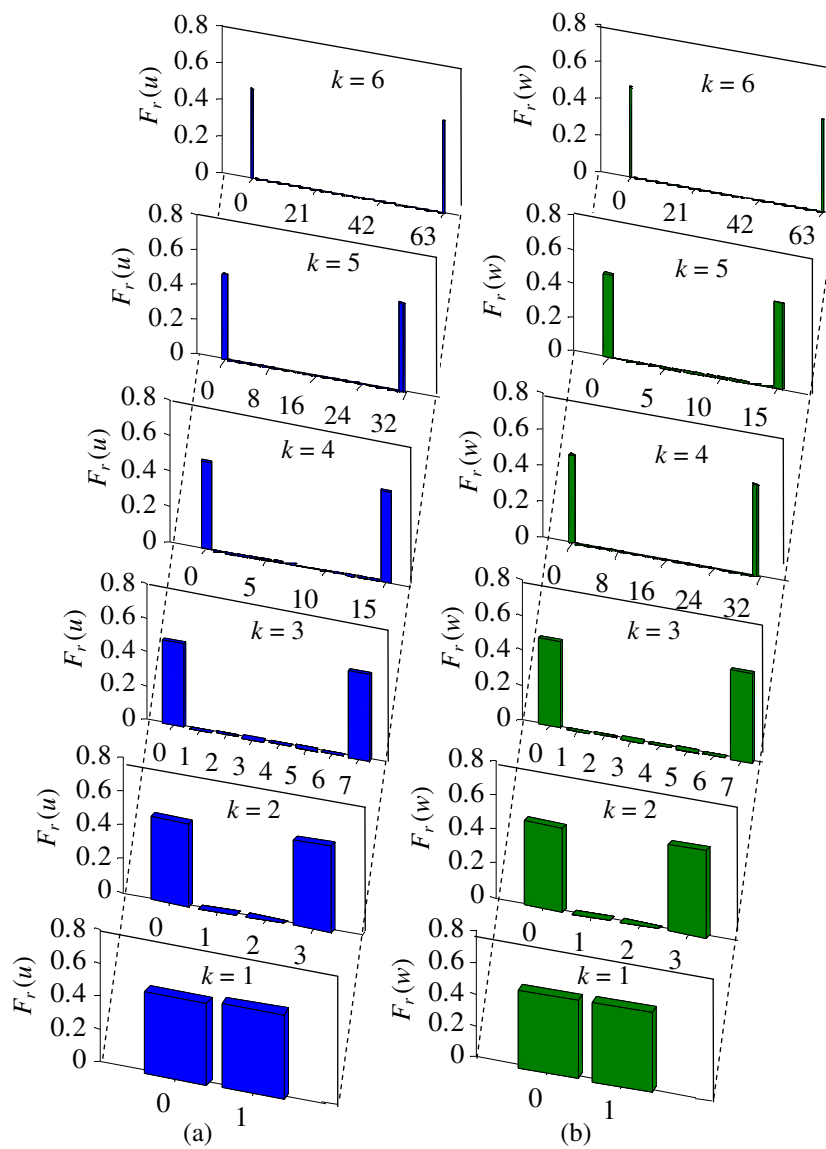


Figure 10. Relative frequencies ( $F_r$ ) of the filtered signals, extracting the average values for various lengths of symbolic sequence  $k$ : (a) axial and (b) transversal filtered velocity signal.

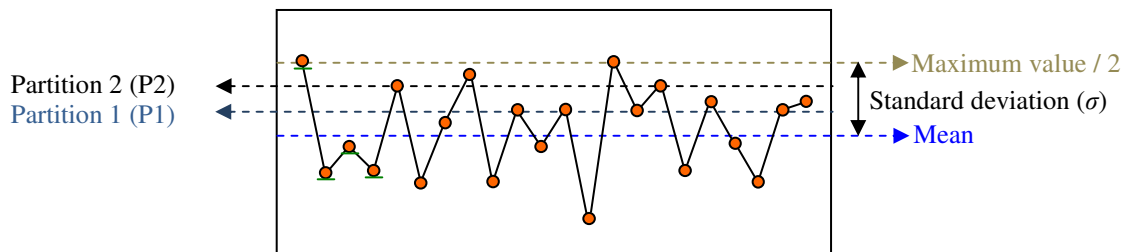


Figure 11. Alternative partitions to analyze the stable flow modes of the bistable phenomenon.

The results for the symbolic sequences of the alternative partition P1 of the filtered signals are shown in Fig. 12 and for alternative partition P2 in Fig. 13, for length ranging from 1 to 4. Results also show that the concentration of the flow modes is markedly in well-defined values.

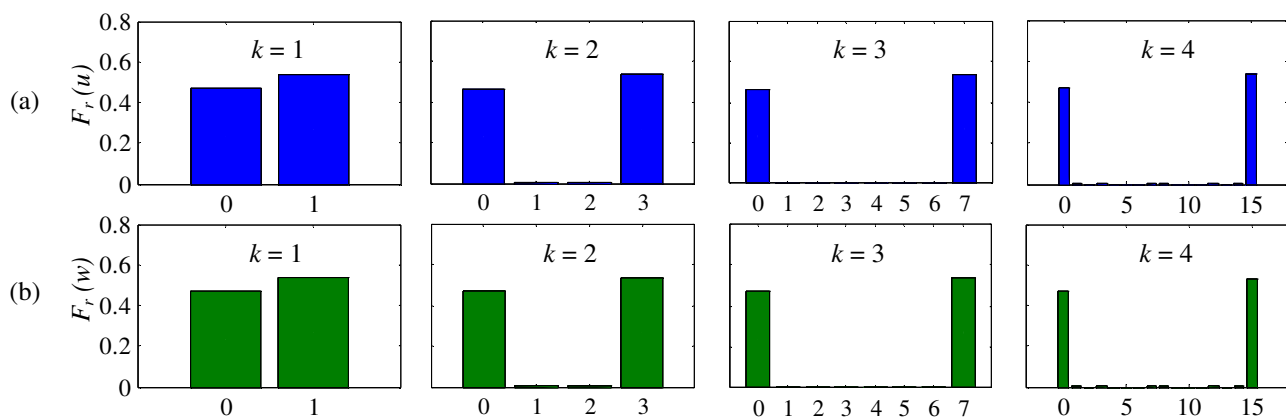


Figure 12. Relative frequencies ( $F_r$ ) of the alternative partition P1 of the filtered signals:  
 (a) axial and (b) transversal filtered velocity signal.

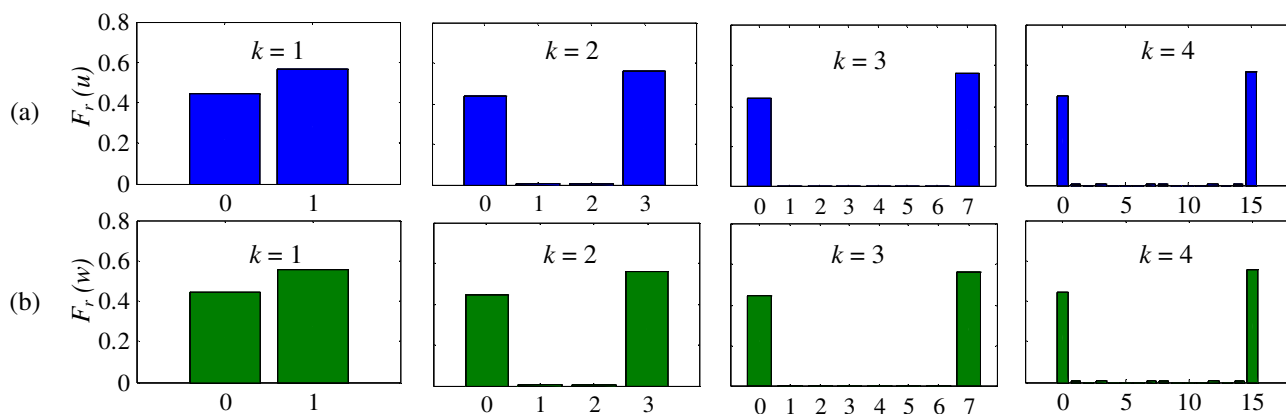


Figure 13. Relative frequencies ( $F_r$ ) of the alternative partition P2 of the filtered signals:  
 (a) axial and (b) transversal filtered velocity signal.

## 6. CONCLUSIONS

This work presents a study of the application of the symbolic dynamics in bistable flows on two cylinders placed side-by-side. By means of hot wire anemometry technique the presence of the phenomenon of the bistability is investigated in an aerodynamic channel. The results from hot wires were filtered by means of wavelet transform, so that the turbulence is isolated from the bistable phenomenon.

The application of symbolic dynamics allows the identification of the flow patterns where the time series are partitioned according to an alphabet. In this work, a binary alphabet was chosen.

The results of the histograms with decimal representation for the original experimental time series show that the concentration of the flow modes is markedly in well-defined values, for symbolic sequences chosen with length ranging from 1 to 6. For the filtered signals with discrete wavelet transform the results show similar behavior, as well as for the alternative partitions P1 and P2.

Future works contemplating the use of another alphabets or different partitions are intended. Also, this technique can be applied in the topological study of the bistable attractor, by the choosing of a convenient Poincaré section. This can better elucidate the bistable flow phenomenon in the simplified geometry of two circular cylinders placed side-by-size and in more complex cases, like those found in tube banks.

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