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AN INVESTIGATION OF THE MATHEMATICAL MODEL FOR THE VELOCITY PROFILE INTERNALLY TO A CONICAL DIFFUSER

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Abstract. *The use of diffusers around of the horizontal-axis wind turbines have been widely studied, since the diffuser causes an improvement in the power coefficient of a turbine and are often called Diffuser Augmented Wind Turbines (DAWT's). The DAWT's have the feature to make efficiency exceeding the Betz limit (maximum kinetic energy extracted of the flow = 59.26%), due to the increasing of the internal mass flow by influence of the diffuser. Thus, the present study shows a mathematical model describing the behavior of the velocity profile internally to a conical diffuser according to the characteristics of flow and diffuser. The mathematical model is based on the Biot-Savart law, which states that a vortex filament induces a velocity field at an arbitrary point. The results are compared with experimental data obtained for 3 diffusers with different geometries and show good agreement.*

Keywords: *DAWTs, Diffuser, Wind energy*

1. INTRODUCTION

The use of diffusers around of the horizontal-axis wind turbines aims at increasing the mass flow through the rotor. This effect causes a considerable improvement in the efficiency of the turbine, which extracts more energy of the flow, when compared with a turbine without diffuser (free-flow turbines). For comparison, the increment caused by the use of a diffuser may take a turbine to achieve a power coefficient between 4 to 5 times greater than free-flow turbine (Ohya and Karasudani, 2010). A diffuser is a device which causes a pressure drop in the output region (suction region downstream of the diffuser). The pressure drop causes an acceleration on the fluid particles within the diffuser, increasing the flow velocity near the entrance (Bussell Van, 1999). As the power in the wind is proportional to the cubic power of the wind velocity approaching a wind turbine, a small amount of acceleration gives a large increase in the energy output. Oman *et al*., (1975), Foreman and Gilbert (1979) show that increasing the velocity ratio between the plane of the rotor and undisturbed flow velocity can be two or more times, resulting in a proportional increase in the power coefficient of the turbine, exceeding the Betz limit (Betz, 1926), which is 59.26% in the case of the turbines without diffuser. Hansen et al., (2000) conducted a study on turbines with diffusers using Computational Fluid Dynamic (CFD), where the increasing of the velocity in the rotor plane was 1.83 for a case in which the geometry of the diffuser used was the NACA 0015 profile deformed.

The main limitation of the horizontal-axis wind turbines with diffusers design is not consider a formulation that is able to describe satisfactorily the influence of the diffuser geometry on the internal velocity profile. Thus, the present study shows a mathematical model which describes the velocity profile of the internal conical diffuser, using the Biot-Savart law to calculate the velocity induced by a vortex ring.

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2. MATHEMATICAL MODEL

In this paper, we described a mathematical model for the velocity profile internally to a conical diffuser, aiming its use in the efficient design of the horizontal-axis wind turbines. Therefore, to assess the velocity field in the diffuser flow is considered to be an overlap between the uniform flow and a flow caused by movement produced by a vortex ring. The movement in this case can be defined as the amount of rotation of the fluid acting on the spreader due to the flow in which is immersed. The movement modifies the velocity and pressure fields around the diffuser, resulting in a resultant force, which is accompanied by a vortex ring, which mathematical model is established by the Biot-Savart law, resulting in an increased velocity within the diffuser.

Figure 1 shows two rings vortices using the Biot-Savart formulation, in cylindrical coordinates, where it is necessary to define the vortex element and position vector in relation to the axis of symmetry.

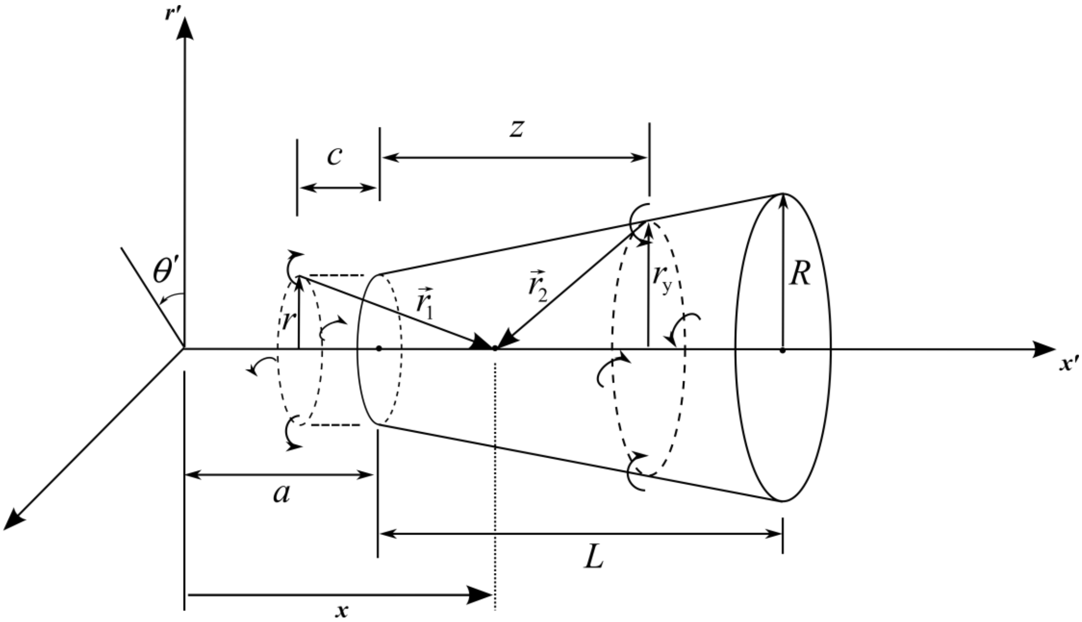

Figure 1. Representation of the geometry model and the vortex rings.

The Biot-Savart law is defined by:

$$d\vec{u}_i = \frac{K}{4\pi |r|^3} d\vec{l} \times \vec{r} \tag{1}$$

where $d\vec{u}_*$ is the velocity field induced, $d\vec{l}$ is an elemental length of the vortex ring, $\vec{r}$ is the position vector and K is the circulation.

The mathematical model considers the following assumptions: conical diffuser, the suction outlet of the diffuser is modeled using a vortex ring; the lock due to the presence of the diffuser on the flow is modeled considering a vortex ring invert, steady and unidirectional flow. The inner vortex ring is responsible for increasing the flow velocity along the diffuser, and the frontal vortex ring has the effect of reducing the free stream velocity which approaches the diffuser.

The total velocity induced by the two vortex rings is the sum of the individual velocities $d\vec{u}_{i,1}$ and $d\vec{u}_{i,2}$, then:

$$d\vec{u}_i = d\vec{u}_{i,1} + d\vec{u}_{i,2} \tag{2}$$

Applying the Biot-Savart law, Eq. (1), in Eq. (2):

$$d\vec{u}_i = \frac{K_1}{4\pi |\vec{r}_1|^3} d\vec{l}_1 \times \vec{r}_1 + \frac{K_2}{4\pi |\vec{r}_2|^3} d\vec{l}_2 \times \vec{r}_2 \tag{3}$$

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After calculation and taking only the velocity in the direction along the axis of the diffuser, there is the velocity induced by vortex rings:

$$u_i(x)=\frac{1}{2}\left\{\frac{K_1 r^2}{\left[\sqrt{r^2+(x-a+c)^2}\right]^3}+\frac{K_2\left(r+z.tg\phi\right)^2}{\left[\sqrt{(r+z.tg\phi)^2+(a+z-x)^2}\right]^3}\right\} \tag{4}$$

where R is the radius of the diffuser exit, r is the radius of the diffuser inlet, a is the distance from the origin to the input of the diffuser, c and z are the distances between the rings of the vortex entry and exit of the diffuser, respectively.

For calculation of the circulations K_1 and K_2, it is considered the geometry of the diffuser, the behavior of the flow around the diffuser and the structure of the vortex formed in the entrance and exit as it is shown in Fig. 2.

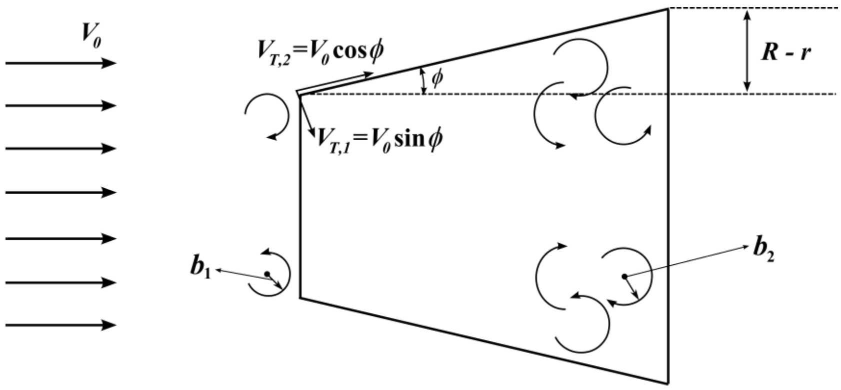

Figure 2. Vortex model for the diffuser.

Thus, for calculating the circulation K_2, it is released the hypothesis that the radius of the vortex b_2 shown in Fig. 2 is proportional to the difference between the radio at the outlet and the inlet of the diffuser $R-r$, and that its tangential velocities $V_{T,1}$ e $V_{T,2}$ are proportional the product of the undisturbed velocity V_0 by the cosine and sine of the opening angle ϕ of the diffuser, respectively.

Therefore, the circulation K_2 is given by:

$$K_2=\xi_2.V_0.\cos\phi.(R-r) \tag{5}$$

where ξ_2 is a proportionality constant and acts as a correction factor.

To determine the circulation K_1, it is used the same hypothesis, applied to the input of the diffuser, then K_1 becomes:

$$K_1=-\xi_1.V_0.\text{sen}\phi.(R-r)\frac{c}{L} \tag{6}$$

where ξ_1 is the proportionality constant for K_1.

Van Beveren (2008) shows that the velocity profile of the internal diffuser is given by the sum of the velocity induced by the ring vortex and undisturbed flow velocity:

$$u(x)=V_0+u_i(x) \tag{7}$$

The ratio between the effective velocity u (x) and the free stream velocity V_0 is called speed ratio, denoted by:

$$\varepsilon = \frac{u(x)}{V_0} \tag{8}$$

The parameters c, z, ξ_1 and ξ_2 are obtained empirically based on experimental data, as will be seen in the section later.

3. EXPERIMENTAL SURVEY

Model validation was performed in an experimental study of airflow around the conical diffusers from 3 different geometric configurations, in order to obtain the velocity profile along the axial length (axis) of the diffuser. The diffusers have a thickness of 0.5 mm (stainless steel plate), Fig. 3.

Figure 3. Diffusers used on the experimental study.

The geometric configurations of the diffusers were chosen to evaluate the behavior of the axial velocity along the diffuser for different opening angles and different lengths. Matsushima (2006) shows that the velocity ratio increases more steeply in angles of less than 4°, reaching a maximum when opening angle is 6°, and decreased at angles of more than 6°. The dimensions of the diffusers are shown in Fig. 4, and were used in the mathematical model proposed in this work.

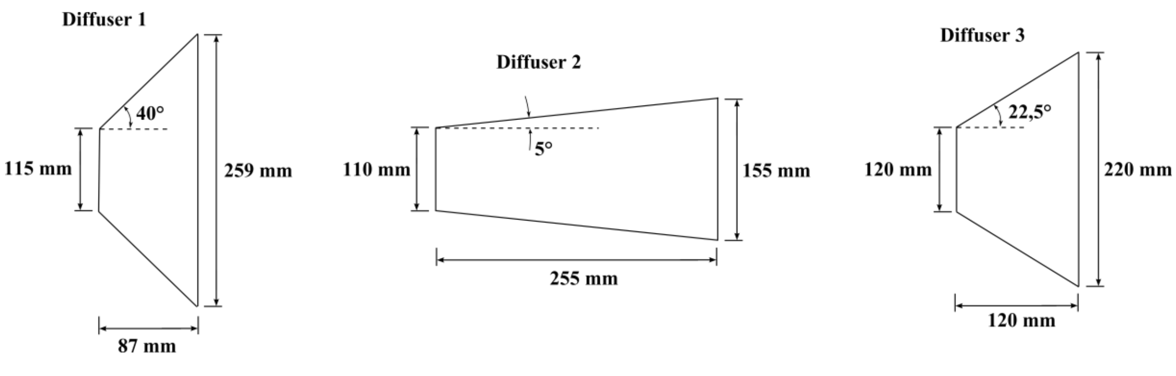

Figure 4. Dimensions of the diffusers.

The experiment was conducted in a wind tunnel (controlled by frequency inverter) operating at three different rotations, 400 rpm, 600 rpm, and 800 rpm, corresponding to speeds 4,05 m/s, 6,65 m/s e 9,05 m/s, respectively. The purpose of using the three speeds was to evaluate if speed ratio along the diffuser depends only on its geometry.

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Measurements of dynamic pressures were performed by means of a Pitot tube type L, with two pressure ports, having a diameter of 7.0 mm and a length of 300 mm. The apparatus also comprises a digital micro-manometer, a microcomputer, and the positioned Pitot tube installed on the wind tunnel and in front of the diffuser. The displacement of the Pitot tube on a linear positioner was controlled by a microcomputer. The choice of Pitot tube occurred due to the initial purpose of determining the velocity distribution along the axis only, leaving aside the quantification and characterization of the vortices generated, otherwise it could be applied to laser Doppler anemometry (LDA) or particle image velocimetry (PVI), commonly used in turbulent flows. The velocity distribution on the axis for a given diffuser is a function only of space and dimensions, thus, for any values of speed imposed on the diffuser, the velocity ratio curve has the same shape, as it will be shown subsequently.

Figure 5. Experimental Apparatus.

The measurements were performed with the diffuser positioned externally to the wind tunnel, under the action of a flow in a jet of air. This configuration was chosen due to the fact that the cross section of the wind tunnel (310 mm x 310 mm) is insufficient to hold the body of the Diffuser 1 (Exit's diameter $D = 259$ mm) and avoids wall effects on the flow around the diffuser. Figure 5 illustrates the experimental apparatus.

As the flow is similar an air jet, it was necessary to accomplish a mapping of the influence region of this jet (external to wind tunnel) to verify if the flow would maintain uniform as it move away from the exit of the wind tunnel.

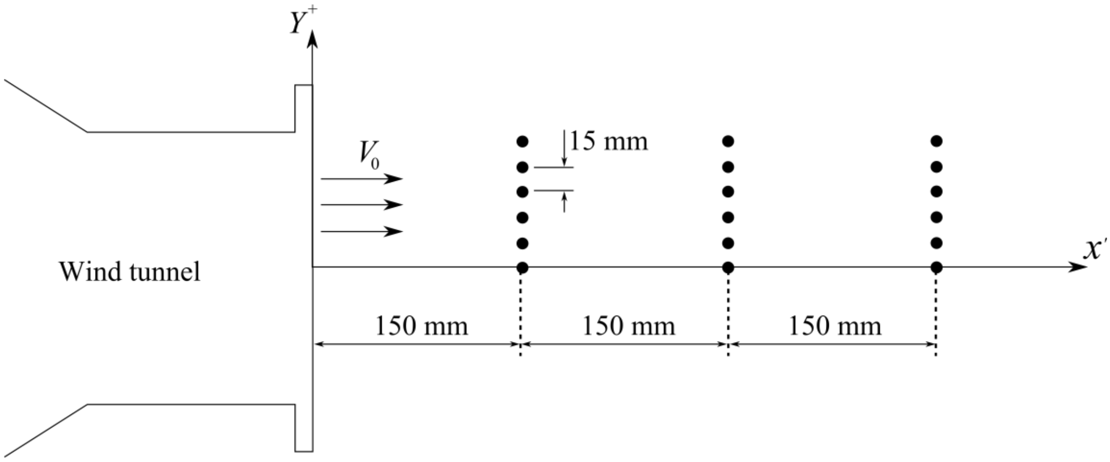

Figure 6. Mapping of te jet flow.

So, it were measured 3 velocity profiles from the exit of the wind tunnel, without the diffuser, with a spacing of 150 mm between each profile, totaling a distance of 450 mm. The choice of measurement to the point 450 mm away from the exit of the wind tunnel was made so that the diffuser of greater length, diffuser 2 ($L = 255\,mm$), was covered by the

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region of the flow. In each profile, measurements in vertical positions (Y^+) are equally spaced in 15 mm (5 mm), assuming that the origin of the Y^+ axis to be located in the center of the tunnel, Fig. 6.

The velocity distributions at the positions 150, 300 and 450 mm are shown below:

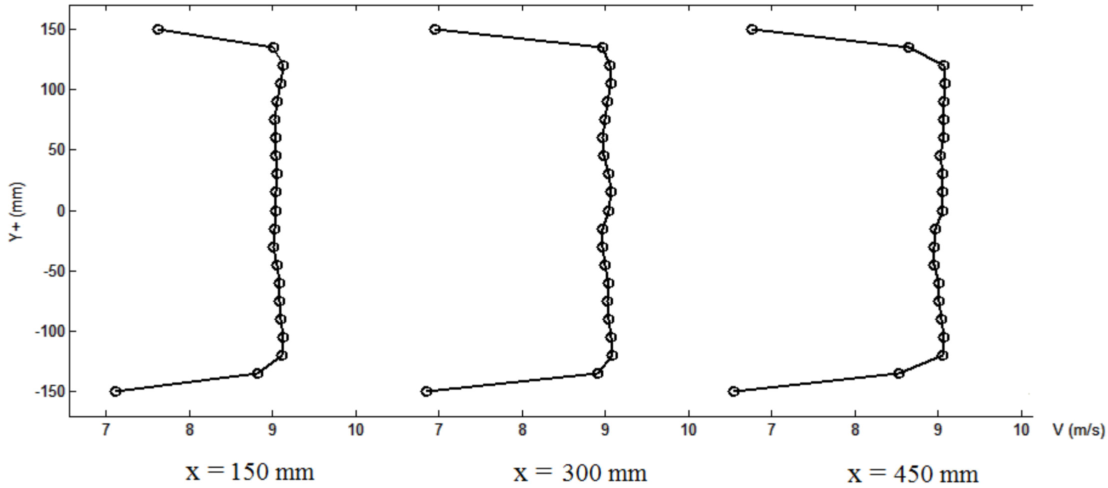

Figure 7. Results of the mapping of the jet flow.

These results indicate that the speeds measured at positions equidistant from the wind tunnel are very close, so that the profile remains constant from the exit of the wind tunnel at a distance of 450 mm. The results also indicate that on the region between $Y^+ = -120$ mm and $Y^+ = 120$ mm, there was the existence of intense velocity gradients in the vertical direction Y^+, so that such a configuration flow is satisfactory for the purpose of this work.

From verification that the air flow outside the wind tunnel was uniform until the distant region 450 mm, the experiments were performed with diffusers positioned in the region belonging to the air jet, estimated at approximately 250 mm in the vertical direction Y^+. An example of the speed measurements were conducted for each diffuser is shown Fig. 8 with the diffuser 3 positioned in front of the wind tunnel, with the Pitot tube installed on its positioner.

Figure 8. Diffuser positioned in front of the wind tunnel.

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4. RESULTS AND DISCUSSION

The velocity values for each point along the longitudinal axis of the diffusers were obtained from a sample of 500 measurements. The results have an uncertainty of 0.04 m/s at 95% of confidence level, for measurements of velocity, which ensures adequate repeatability. The x-axis is normalized by using $(x-a)/L$, thereby entrance of the diffuser is at point 0 and its exit at point 1 for the three diffusers. So, the diffusers are delimited by dashed lines (Inlet and Outlet).

The Figure 9, 10 e 11 shows the ratio velocities for the diffusers 1, 2 and 3, respectively. For each diffuser, it is confirmed that the curves of the velocity ratio depends only on its geometry.

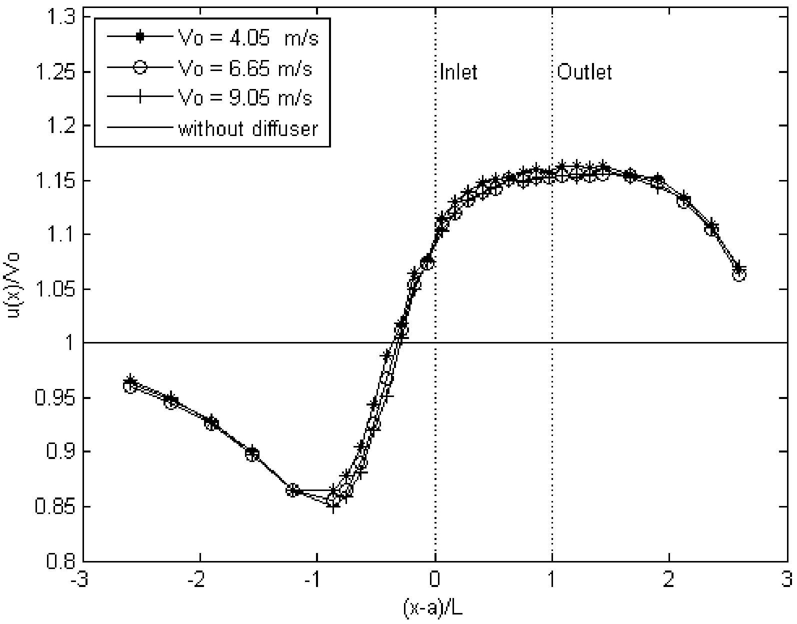

Figure 9 Experimental results - Diffuser 1.

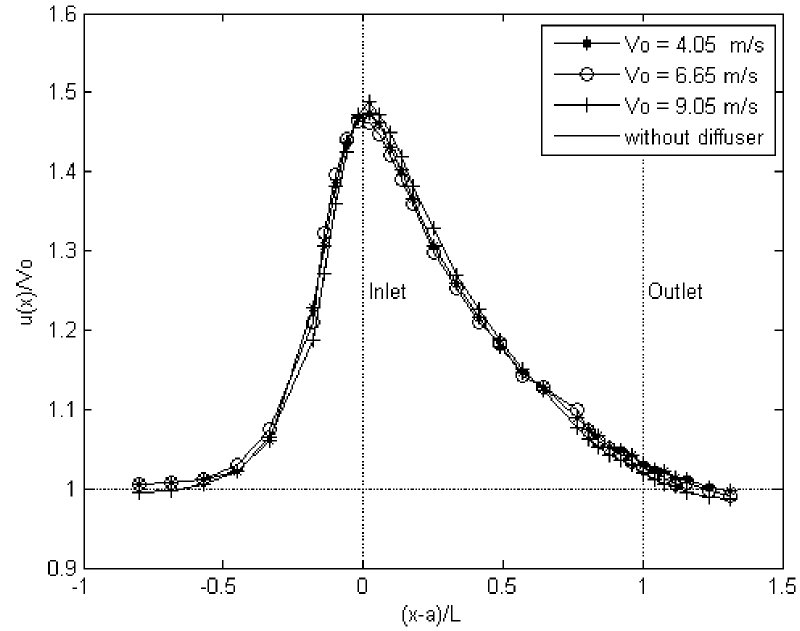

Figure 10. Experimental results - Diffuser 2.

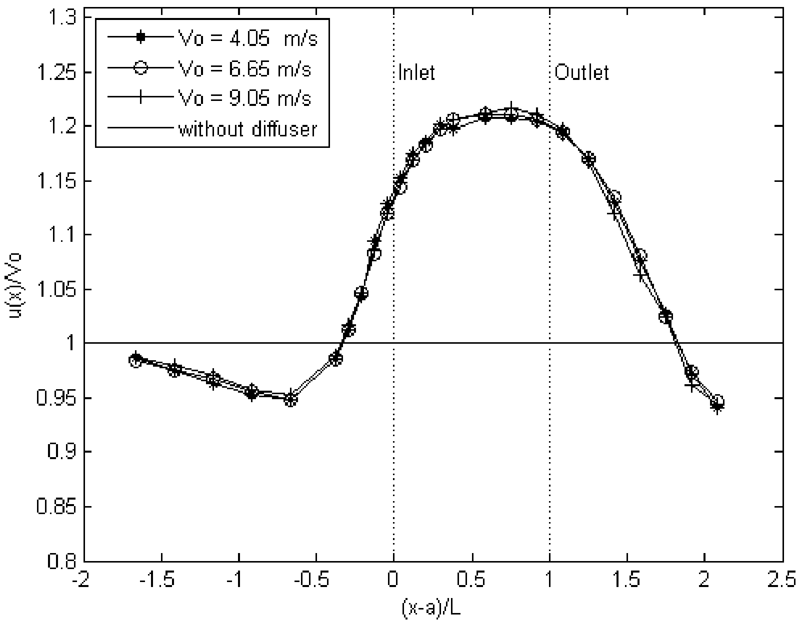

Figure 11. Experimental results - Diffuser 3.

Figure 12 shows, simultaneously, the velocity ratios for the three diffusers. In this graph, it is noted that the opening angle is responsible for reducing of the velocity ratios, and the region of maximum velocity heads toward the exit of the diffuser. It occurs also a reduction in wind speed before it approaches the entry of the diffuser.

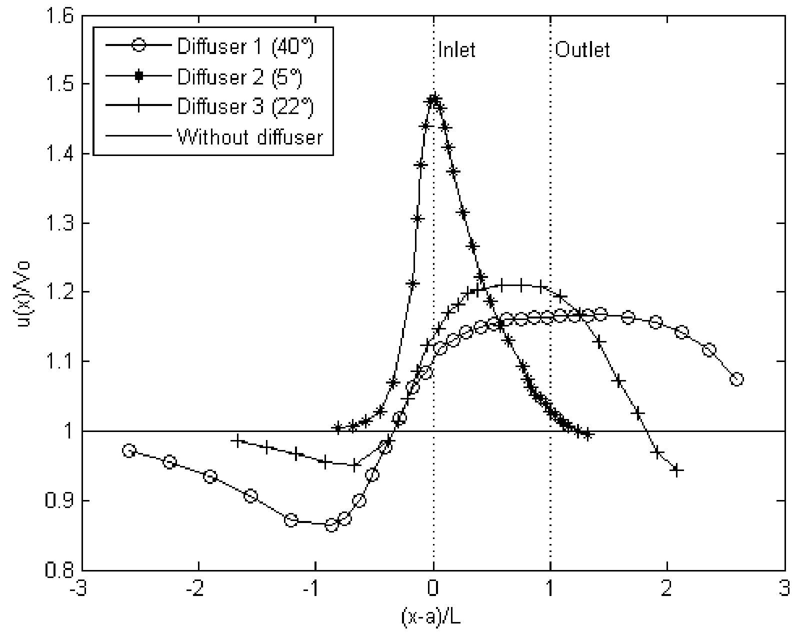

Figure 12. Comparison of the velocity ratios.

Based on the experimental results, from Eq. (4) e (8), the values for c, z, ξ_1 and ξ_2 were obtained by comparison method from the computer routines implemented in the Matlab software that solved the equations which rule the problem, so that the curve obtained by the mathematical model, approaching to the maximum of the experimental curve. It was observed that the parameter c admitted very close values for the three diffusers, so, its value was estimated as:

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$$c = 0.075 \tag{9}$$

This indicates that the frontal vortex ring has a fixed localization upstream the diffuser.

For the parameters z, ξ_1 and ξ_2, it was observed a relation with the ratio L/D. By using the quadratic interpolation of these values, we found the following empirical relations:

$$z = 0,003\left(\frac{L}{D}\right)^2 - 0,0503\left(\frac{L}{D}\right) + 0,0768 \tag{10}$$

$$\xi_1 = 0,8856\left(\frac{L}{D}\right)^2 + 1,688\left(\frac{L}{D}\right) - 0,1963 \tag{11}$$

$$\xi_2 = 0,8823\left(\frac{L}{D}\right)^2 - 0,2138\left(\frac{L}{D}\right) + 0,6507 \tag{12}$$

Finally, by using Eq. (4) and the Eq. (9), (10), (11) e (12), the mathematical model can be implemented for any diffuser, since this values depends only on its geometry. Figure 13, 14 and 15 shows the velocity distributions calculated by the Eq. (5) compared with values obtained experimentally.

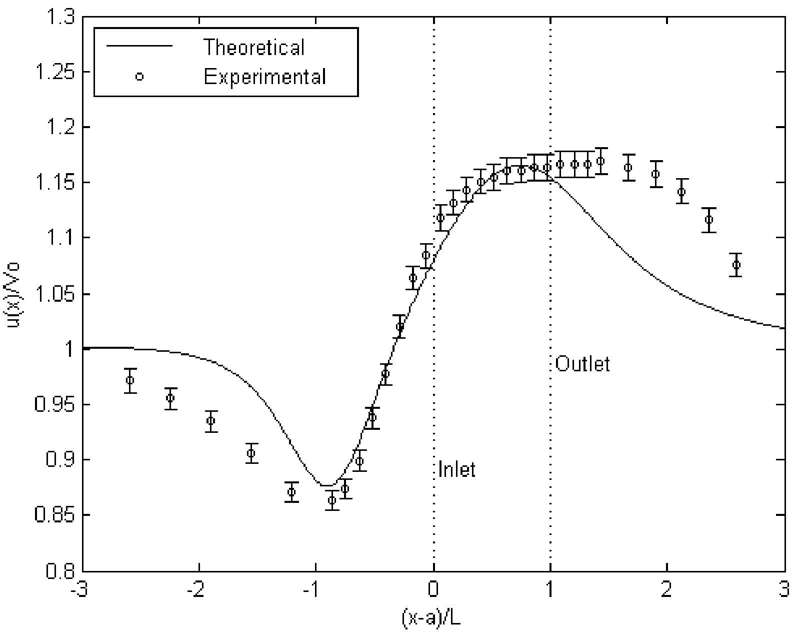

Figure 13. Comparative theoretical-experimental velocity ratios – Diffuser 1.

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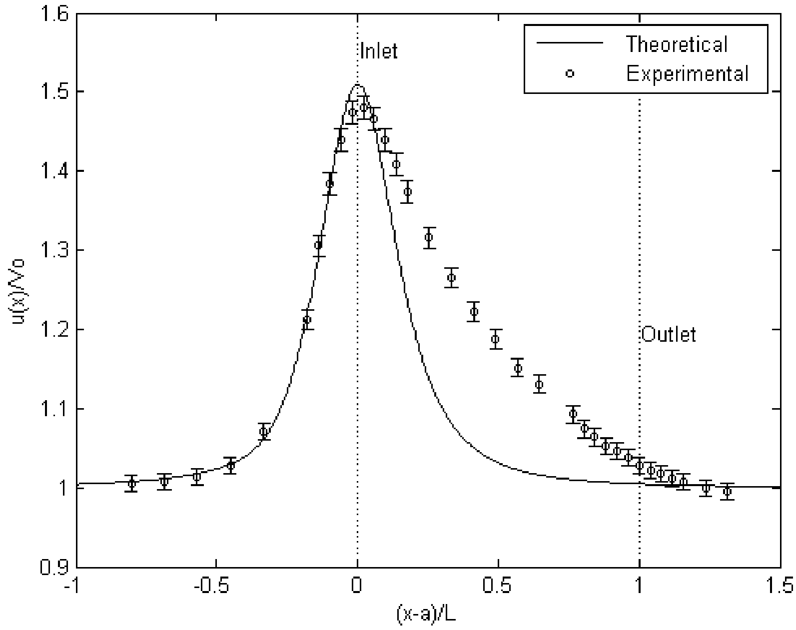

Figure 14. Comparative theoretical-experimental velocity ratios – Diffuser 2.

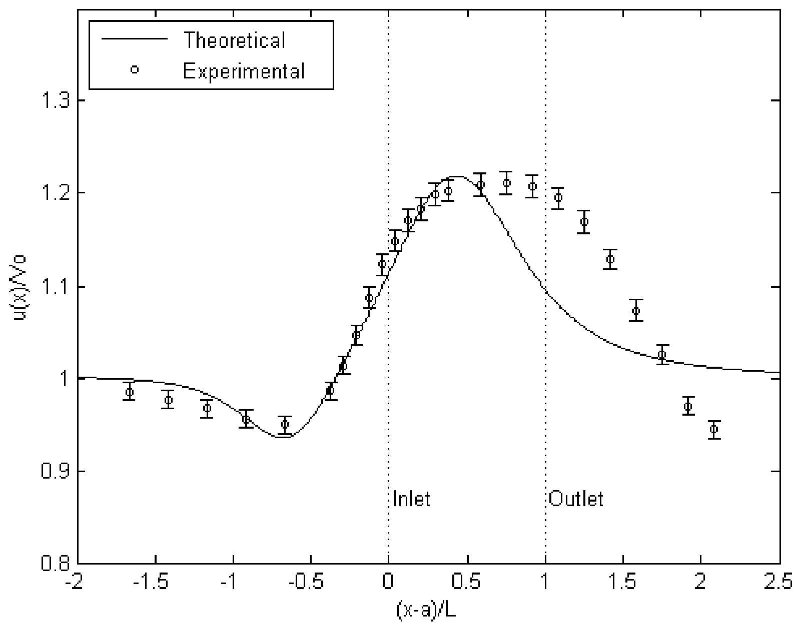

Figure 15. Comparative theoretical-experimental velocity ratios – Diffuser 3.

For the diffusers 1 and 3, the velocity ratios from the furthest region from the diffusers (non-disturbed flow) to its minimum value appears differently from the actual case. From this point of minimum to the maximum point, the velocity distribution shows good agreement with experimental results, diverging from the point of maximum to the region downstream of the diffuser. For the diffuser 2, there was no reduction for the velocity. This comparative result indicates that the mathematical model has limitations.

Comparing the diffusers 1 e 3 of this work with the work Ohya (2010) and Abe (2004), there is a difference in the velocity profile along the axis. The diffusers 1 e 3 tested here shows a reduction in speed before reaching its input (pressure increase) differed from the comparative cases and from the diffuser 2 which shows an acceleration of the flow from the free stream velocity to the entrance of the diffuser, followed by a deceleration, again reaching the free stream velocity. This difference probably is due to the opening angles of the diffusers 1 e 3 tested ($\phi = 22.5°$ and $\phi = 40°$), which causes a blocking effect on the flow, where it should be noted that the optimal value for this angle is adopted as $\phi = 4^{\circ}$ (Abe, 2004). In a diffuser, the flow expands along its inner wall, resulting in decreased speed and consequently an increase in the coefficient of pressure toward the exit of the diffuser (Abe, 2004), adverse pressure gradient. Based on this, even with a large opening angle, the velocity profile from the diffuser entry is similar to those diffusers which have L/d reduced, or increased followed by speed reduction.

4. CONCLUSION

The velocity distribution along the axis of the diffuser obtained with the proposed model present a good agreement with experimental data. It is intended that the elevated velocities occurs inside the diffuser, especially at the laminar region of the flow. The experimental data are important because it indicates the best position for positioning the wind turbine.

The blocking effect at the entrance of the diffuser opening angle is responsible for the elevated velocity profile (Fig. 4) which differs from cases with milder opening angles (Fig. 16). Therefore, it is necessary, in principle, to use small opening angles ($\phi \approx 4^{\circ}$) in order to smooth the flow in the entry region of the diffuser preventing the increase of pressure and the consequent reduction in speed. Subsequently it will be seen that such condition is of extreme importance for installation in the region of the turbine inlet diffuser.

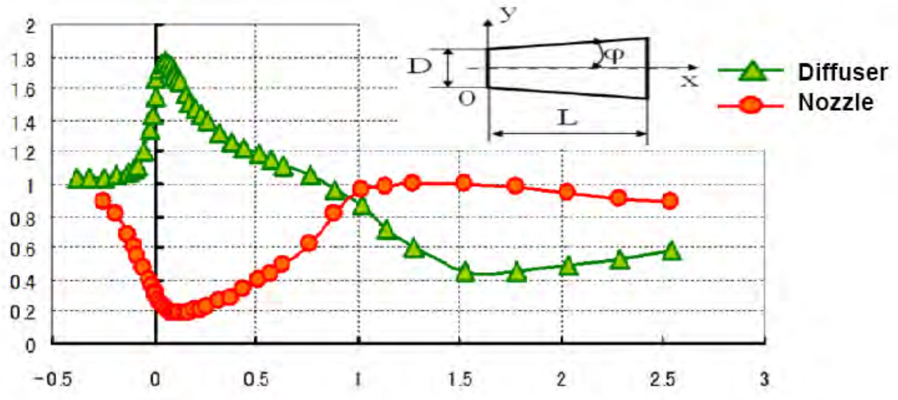

Figure 16. Velocity distribution in the diffusers with opening angle 4° (Ohya and Karasudani, 2010).

Ohya (2010) shows that for a diffuser with opening angle $\approx 4^{\circ}$, the peak of ratio velocities is located inside the diffuser near its entrance. Based on the diffuser of the present work, the opening angle 5° for the diffuser 2 reached the same results, with the maximum velocity point occurring for small opening angles near it entrance.

In principle, it would be more convenient to install the turbine in the region near the exit of the diffuser, where the speed is the greatest. However, Fig. 17 shows that in the region of the diffuser outlet, the flow is turbulent and the turbine, in case, is installed in this region, which coincides with the maximum speed ratio, the turbine would lose the necessary lift for the utilization of available energy in the wind, and consequently, there would be a decrease in the turbine power coefficient. It is necessary to conduct an experimental survey for other diffusers geometries to evaluate the parameters c and z, and refine the mathematical model.

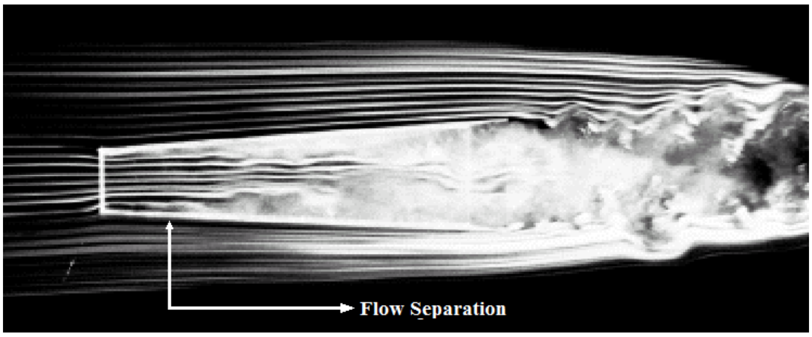

Figure 17. Streamlines of the flow around a diffuser (Ohya and Karasudani, 2010).

Therefore, the ideal position for installing turbine is near the entrance of the diffuser, because in this region the current lines are presented uniform (laminar flow), and avoids the turbulent flow formed by the separation of boundary

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layer on the inner wall diffuser, belt region, taking advantage of the speed gains without compromising the increased efficiency of the turbine.

In the design of wind turbines with diffusers, analytical models, as proposed in this work are considerably important, since these may be coupled to existing classical models, such as Glauert (1926), in order to supplement or improve them. Experimental data for other configurations of conical diffusers are currently being obtained to validate and refine the mathematical model proposed. It was also observed that the ratio between length and diameter of the diffuser outlet (length ratio) is closely related to the location of a maximum velocity ratio. Therefore, the proposed model has limitations, but it can be applied to DAWTs project, aiming at an improvement in the extraction of energy from the wind, resulting in increased efficiency.

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