

# EXPERIMENTAL STUDY ON THE APPLICABILITY OF ERGUN EQUATION IN BEDS WITH HOLLOW SPHERES

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Abstract. Many studies are conducted about the dynamics of fluids in porous media, which generates a number of factors and problems that are solved. In particular the phenomenon of pressure drop in flows on fixed bed, although fairly well in the form Ergun equation, still has certain applicability with regard to the shape of materials and characteristics of surface to be used in the packaging of the beds. The purpose of this paper is to realize a study of frictional pressure drop experimental results from fixed bed composed of hollow sphere, applying the Ergun equation with appropriate adaptations to the case of hollow sphere. The study will also raise properties for determining the frictional pressure drop as shape factor, porosity and tortuosity of the bed, as well as growth for various lengths bed. The main objective was to validate the experimental procedure and apparatus through data that were obtained according to the mentioned equation.

Keywords: pressure drop, fixed bed, hollow spheres.

## 1. INTRODUCTION

The growing expansion of systems that utilize porous beds in engineering is the subject of many studies. These studies that capture the nature of the process in laboratory scales in order to transmit what occurs in industry, the need to perform validation of the experimental apparatus has as main starting point for further studies that can be carried out in order to obtain more knowledge, whether in research as in engineering education.

Particularly the study of fixed beds come support and strengthen research in the wind tunnel of low turbulence, the application of the classical equations and grounds for new configurations that can be studied after the validation of the experimental apparatus.

A vast amount of information in the form of empirical and semi-empirical correlations which relate the pressure drop to the hydrodynamic conditions of the packed beds is available held a correspondence with the equation that governs this type of flow, widely known as Ergun equation (Ergun, 1952). The Eq. (1), has been widely used for design or treatment of raw experimental data extracted from certain geometrical configurations. Ergun's correlation accounts for viscous and inertial energy losses and relates them to the dynamic variable, velocity of the fluid, as well as the structure of the bed, as characterized by the bed mean voidage.

$$\frac{\Delta P}{L} = 150 \frac{\left(1-\varepsilon\right)^2}{\varepsilon^3} \frac{\mu U}{D_p^2} + 1.75 \frac{1-\varepsilon}{\varepsilon^3} \frac{\rho U^2}{D_p} \tag{1}$$

Carman (1937) shows that the first term on the right side represents the viscous flow and thus the change in pressure is proportional to  $(1-\epsilon)^2/\epsilon^3$  along with a 150 empirical proportionality constant at low flow rates. Blake and Plummer (1928) showed that the change of pressure in a turbulent flow, resulting in loss of kinematic energy, is proportional to  $(1-\epsilon)/\epsilon^3$ . A constant of 1.75 was found to be relevant to the turbulent flow. In order to verify the functional dependence on the porosity of the bed Ergun also varied density packaging of certain materials to verify the expression  $(1-\epsilon)^2/\epsilon^3$  for the loss of the viscosity and the term  $(1-\epsilon)/\epsilon^3$  part of the kinetic energy, and found that a small change in  $\epsilon$  had a great effect on the pressure drop.

Researches check the values of the empirical constants 150 and 1.75 in Ergun equation. A variety of empirical values have been encountered because of the use of different packaging materials such as spherical particles of regular shape or non-spherical particles of irregular shape. Universal values of the Ergun constants have been the subject of considerable speculation since 1952. The values of empirical constants in Ergun's correlation have been proposed by Leva (1947), who presented the values of 200 and 1.75, and Macdonald (1979) who presented the values of 180 and 1.8. The rationale for the variation in the constants was determined as the changes in regular spherical particle shape for nonspherical particles. Which leads to a theory held by most researchers is satisfied with the fact that the values of the Ergun constants must be determined empirically for each bed. This theory stems a belief that values are not only dependent on the geometry of the particle, but in addition they can vary from bed to bed macroscopic (made of the same particles) due to different structures of the pack in the bed after repackaging.

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Macdonald et al. (1979) studied two correlations factors Reynolds number friction, dimensionless Forchheimer equation of Ahmed and Sunada and modified Ergun equation. They concluded that the physical basis of the Forchheimer equation seems to be accurate. Also proposed a modified Ergun equation, while certainly not rigorous, can be expected to predict the experimental results for unconsolidated media with an accuracy of  $\pm$  50%. For a porosity range from 0.36 to 0.92, the function of the porosity of the Ergun equation is superior to others proposed in the literature, while others are better for small ranges of porosity. For non-spherical particles, it is necessary to measure the ratio of surface-to-volume or D<sub>n</sub> for use in sphericity.

The approach widely accepted by most researchers is the introduction of the form factor in Eq.1 and replace the traditional constant with new constants based on their experimental data, producing the following format of the pressure gradient in porous media. Thus we have the constant dependent on experimental results and the criterion of form factor or sphericity accounted Eq.2 in the letter  $\psi$ .

$$-\frac{\Delta P}{L} = K_1 \frac{\left(1-\varepsilon\right)^2}{\psi^2 \varepsilon^3} \frac{\mu U}{D_{VS}^2} + K_2 \frac{\left(1-\varepsilon\right)}{\psi \varepsilon^3} \frac{\rho U^2}{D_{VS}}$$
(2)

Li and Ma (2011) carried out experimental studies to determine the characteristics of friction and pressure drop of the fluid flow in bed packed with spherical porous particles and non-spherical. The aim is to examine the applicability of the Ergun equation flow resistance rating is packed beds of non-spherical particles. They found that the pressure drops in packed beds with opaque hollow spheres using the predictions from Ergun equation if the diameters of the balls are used in the equation. For this reevaluate inserted equivalent diameter of the particles, which is the product of Sauter mean diameter and particle shape factor, the flow resistance of the packed bed constituted by non-spherical particles can be predicted by the Ergun equation in the whole range flow rate.

The definition of equivalent particle diameter is based on the shape factor ( $\psi$ ) of the laws of friction and explicitly reflects the physical mechanism for the increased contribution of frictional drag due to the tortuous complex forms, particularly in the turbulent regime.

$$\psi = \frac{\text{surface of sphere of equal volume to the particle}}{\text{surface area of the particle}} = \frac{A_{sp}}{A_p} = \frac{\pi^{1/3} (6V_p)^{2/3}}{A_p}$$
(3)

Where  $V_p$  is volume of the particle,  $A_p$  the surface area of the particle, and  $A_{sp}$  the surface area of the equivalent-volume sphere.

The particle diameter defined in Eq.4 is equal to the volume-surface mean diameter as referred in literature (Ozahi et al. 2008; Dullien 1975).

$$D_{VS} = \frac{6V_p}{A_{sp}} = \frac{6V_p}{\psi A_p}$$
(4)

The analysis in the wind tunnel was made from different sizes of bed, with a decrease in arithmetic progressions corresponding to half the value of the diameter of the tube used in the test. Several bands of Reynolds number were also employed in order to be able to visualize how the phenomenon behaves on various tracks

The experimental results are analyzed in order to consider the analytical study of the pressure drop in porous media according to the equation of Ergun (1952), which has long governed this type of flow and shows up on the optics of some studies, the primary point to analyze this type of flow.

For the analysis of porous bed important factors should be raised as: porosity of bed, sphericity of particle and pressure drop caused by the fixed bed. To determine the bed porosity data were collected from a sample of diameters of hollow spheres, and taken their average value, as well as its specific gravity through weighting each seed and doing general survey to determine the average of these, always obeying the variation of the standard deviation caused by the total samples. In this study experiments were performed with a fixed bed of hollow spheres attached to a wind tunnel.

#### 2. METHODOLOGY

#### 2.1 Analytical solution according to the Ergun equation

Ergun (1952) combined the Carman (1937)-Kozeny (1927) equation and Blake Plummer (1928) in the form of the Forchheimer equation, which provided the equation 1.

The equation counts the energy losses viscous and inertial relates to dynamic variable, the fluid velocity, as well as the structure of the bed, as characterized by the average porosity of the bed. The specific equation for the case in this study is equation 2.

#### 2.2 Porosity

Porosity ( $\epsilon$ ) can be described as the dead space in a way which crosses a stream, and this would mean the fraction of empty space in relation to the total space in the porous medium occupies. When porosity is closer to 1 it would result in a reactor completely open without fills, as shown in fig.1.



Figure 1. Layout of porous bed.

This parameter is determined as a relation between the dimensionless spaces not occupied by particles ( $V_{empty}$ ) at a given total volume ( $V_{reactor}$ ) to be filled (Nield and Bejan, 1992). For the analysis of voidage has the relationship is:

$$\varepsilon = \frac{V_{empty}}{V_{reactor}} = \frac{V_{reactor} - V_{bed}}{V_{reactor}}$$
(7)

Where  $V_{bed}$  is the volume occupied by the hollow spheres in the reactor obtained by:

$$V_{bed} = V_{sphere} \frac{M_{bed}}{m_{sphere}} \tag{8}$$

To study the porosity it has been taken possession of the values obtained from measurements of the average diameter and average mass of the hollow spheres according to statistical described above.

The weight of each hollow sphere was obtained by grouping all the values in a sample of 58 weighing up to average, and always checking the standard deviation fit within a limit below 0.15 for the sample group. The porosity determination requires the diameter and mass averages of the hollow spheres. So to weight hollow sphere an analytical balance OHAUS model AL5000 with precision of 0.0001 cm, was used. The diameter sphere was measured with a caliper of 0.05 mm in precision.

#### 2.3 Data collection

The experiments were performed in a wind tunnel of low turbulence, at the outlet was interconnected a PVC pipe, fig.3. The pressure drop data were collected using a digital micromanometer. The data were all saved in files in spreadsheets by which facilitated the handling and processing of these data.

For the experimental analysis of the porous bed in the wind tunnel, the methodology was used to collect data of pressure drop for the analysis of loss. Before the drainage cover in the bed and after letting it flows. Performing various tests taken for different lengths of beds, and describe more clearly the process according to this variation.

In order to have the disposal of different beds, the beds were mounted on short lengths of half reactor diameter PVC (0.075 m) and separated only by steel bars, a total of three beds according to the diameter of the reactor at a rate 0.5 D.

The wind tunnel used is open circuit, straight-walled rectangular section of  $0.30 \times 0.30$  m reactor coupled to a PVC circular section of 0.15 m in diameter, which were carried out to data collection. The superficial velocity data are computed by measurement of flow stream before the bed. The flow is obtained through the velocity profiles measured with the pitot tube. The layout of the beds well as their pressure taps and arrangement of pitot tube for controlling the superficial velocity are shown in fig.2.

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Figure 2. Layout of beds in the PVC pipe.

The data acquisition was performed through a serial RS-232 and through the program FC487 Datalogger Version: 2.1.0. The data were saved in worksheets in XLS Microsoft Excell software.



Figure 3. View of the PVC pipe coupled to a wind tunnel.

The wind tunnel provides a uniform flow at the entrance of the test section, for this work, the velocity was varied according to frequency established in the inverter frequency and observing the corresponding reading velocity into the tube with the assistance of a pitot tube inserted into the flow at a distance of 195 cm coupling wind tunnel / PVC tube.

## 3. RESULTS

## 3.1 Results of porosity

The analysis of spheres followed for measuring various populations to the average diameter was used 10 spheres, for the mean weight evaluation 58 spheres were collected. The results are shown in Table 1.

Property	Value
Average diameter (m)	$0.013866347 \pm 0.001265$
Average volume (m <sup>3</sup> )	$1.396 \times 10^{-06}$
Average mass (g)	$1.417177586 \pm 0.11801686$

The void volume as determined above is the ratio of the total mass of the cores in the bed by spheres. After determination of the volume occupied by the spheres, and air voids caused by these through mathematical relationships presented above, the calculation to analyze the evolution of porosity was performed for different lengths of bed used in the experiment carried out in the wind tunnel. Then the calculations for the various bed porosity used in the experiment is presented in a curve shown in fig.4.

The curve shows a steep, and then rises to the top height of the bed, however, as will be increased bed height begins to stabilize, so as to form a line which leads us to conclude that the porosity remains stable after the region near the height 0.5D.





Figure 4. Evolution of Porosity.

### 3.2 Results of pressure drop

The results obtained from the pressure drop data collection were all subjected to statistical treatment in order to obtain the average behavior of the phenomenon. The use of Student's t test showed up to meet the requirements to minimize measurement error and obtain more accurate data. It was obtained about 300,000 data of pressure drop and superficial velocity. The experimental results and the values obtained by the Ergun equation for pressure drop are shown in Figures 5, 6 and 7.



Figure 5. Comparison of pressure drop experimental and Ergun equation, bed of 0,5D.

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Figure 6. Comparison of pressure drop experimental and Ergun equation, bed of 1,0D.



Figure 7. Comparison of pressure drop experimental and Ergun equation, bed of 0,5D.

The values obtained in data collection pressure drop in beds of hollow spheres are carried out for each bed length, and varying velocity, showing the flow behavior according to these variables. After experimentally verified, values for the pressure drop and velocity superficial on the bed was applied Ergun equation for the case of fixed bed flow-dimensional stationary.

Analyzing the graphs it can be seen that the theoretical values are very close to the experimentally measured values for the low velocity, which suggests that passes Ergun equation is valid for flow in the lower range of Reynolds numbers. A slight variation is observed as we increase the velocity, but none that is not with reliability assured.

#### 4. CONCLUSIONS

The determination of bed porosity obtained satisfactorily results with a uncertainty around 0.03. The validation of the experimental apparatus which is the main focus of this work achieved the objective so that the values obtained experimentally and analytically were within the margin of error calculated.

Values analyzed and tested the experimental apparatus for the study of fixed bed becomes totally safe for other configurations and materials of beds.

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## 6. REFERENCES

Burke, S.P. and Plummer, W.B., 1928, Ind Eng Chem, vol. 20(11): pp. 1196-1200.

Carman, P.C., 1937, Trans Inst Chem Eng, vol.15: pp. 150–166.

Churchill, S. W. Viscous Flow-The Practical Use of Theory Butter worth Publishers. 1988.

Ergun, S., 1952. Fluid flow through packed columns, Chem. Eng. Prog., vol. 48, pp. 89-94.

Keyser, M. J. et. Al, 2006. "Effect of coal particle size distribution on packed bed pressure drop and gas flow distribution". Fuel, vol.85, pp.1439-1445.

Kaviany, M. Principles of Heat Transfer in Porous Media, second ed. Springer, NewYork. .1995.

Kozeny, J., 1927, Sitzber Akad Wiss Wien, Math.-naturw. Klasse, 136 (Abt. IIa), 271.

Leva, M. Pressured drop through packed tubes, Chem.Eng.Prog.43, 549–554 (1947)

Macdonald,I.F.et al. Flow through Porous Media-the Ergun Equation Revisited, Ind. Eng. Chem. Fundam., Vol. 18, No. 3 1979.

Nield, D. A. and Bejan, 1992. A Convection in Porous Media, chapter 15, Springer, New York.

Niven, R. K. Physical insight into the Ergun and Wen & Yu equations for fluid flow in packed and

fluidized beds, Chem. Eng. Sci. 57, 527-534 (2002).

Sharma, A. Kr., 2007. "Modeling fluid and heat transport in the reactive porous bed of downdraft (biomass) gasifier". International Journal of Heat and Fluid Flow, vol. 28, pp.1518-1530.

Vafai, K., 2005. Handbook of Porous Media, Second Edition, Taylor & Francis Group, Boca Raton, Florida.

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