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USE OF UNIVERSAL INTEGRAL REGULATOR TO CONTROL THE FLIGHT DYNAMICS OF FLEXIBLE AIRPLANES

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Abstract. *This paper presents the use of universal integral regulator to control the flight dynamics of flexible airplanes. This technique of universal integral regulator is derived from the technique of sliding mode control. The results obtained demonstrate the efficacy and robustness of the universal integral regulator, once the same flight control law defined could command the maneuver desired in two airplanes with very different levels of structural flexibility. Other advantage of this control technique is the simplicity and easy of implementation, because it is not needed detailed information about the mathematical model of the system controlled.*

Keywords: *flight control, universal integral regulator, flexible airplane, nonlinear control, sliding mode control*

1. INTRODUCTION

The project and development of new airplanes begin with the definition of mission that the new vehicle must perform. Several project and certification requirements have to be fulfilled. Some new airplanes, that have been developed, have considerable structural flexibility. This is consequence of geometry, materials and structural properties defined during the development phase in order to guarantee that the vehicle developed will perform satisfactorily the mission for which the airplane was projected.

Higher structural flexibility implies in higher structural deformations. One immediate consequence of structural deformations is the modification in airplane geometry, aerodynamics and flight dynamics. The effects of structural flexibility in flight dynamics is one theme of research (Waszak and Schmidt, 1988; Silvestre, 2007; Guimarães Neto, 2008; Da Silva, 2012; Shearer, 2006; Su, 2008; Ribeiro, 2011; Sousa, 2013).

The development of flexible airplanes has required the implementation of more sophisticated, non linear flight control laws in order to compensate the effects of structural deformations.

There are different techniques studied in the literature to control the flight dynamics of flexible vehicles. One technique currently being analyzed by the authors is the universal integral regulator (Seshagiri and Khalil, 2005; Seshagiri and Prontum, 2008; Sousa and Paglione, 2012; Sousa, 2013).

This paper presents the results obtained with the universal integral regulator in the control of one commercial flexible airplane. The item 2 presents one brief explanation of the universal integral regulator. The item 3 presents the mathematical model of the airplane controlled. The item 4 presents the results obtained and the item 5 presents the conclusions.

2. UNIVERSAL INTEGRAL REGULATOR

The technique currently analyzed by the authors to control the flight dynamics of flexible airplanes is the universal integral regulator (Seshagiri and Khalil, 2005; Sousa and Paglione, 2012; Sousa, 2013).

This technique is derived from the sliding mode control technique. The difference between the universal integral regulator and sliding mode control are:

1) It is permitted one tolerance in the sliding surface, i.e., the dynamics must not exactly be in the surface s=0, but can be in the region ($-\mu<=s<=\mu$), where μ is the boundary layer.
2) The function sgn(s) in sliding mode control is changed to the function sat(s/ μ);

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3) The sliding surface s does not contain only the tracking error e and its derivatives, but also the integral of the error;
4) The integration of the error is conditional, i.e, it occurs only inside the boundary layer;
5) The control law is applied only to SISO systems;
6) The term related to the equivalent control u_e is considered to be zero. It means that it is not necessary to have detailed information about the mathematical model of the system controlled.

Detailed information about the theoretical development of the universal integral regulator can be found in (Seshagiri and Khalil, 2005). The Eq.(1),(2),(3) and (4) present the control law defined with the universal integral regulator (Seshagiri and Khalil, 2005).

$$u = -k.sat(\frac{s}{\mu}) = -k.sat(\frac{k_0\sigma + k_1e_1 + k_2e_2 + ... + e_\rho}{\mu}) \tag{1}$$

$$\dot{\sigma}_i = -k_0^i\sigma_i + \mu_i sat(\frac{s_i}{\mu_i}) \tag{2}$$

$$s_i = k_0^i\sigma_i + \sum_{j=1}^{\rho_i-1} k_j^i e_j^i + e_{\rho_i}^i \tag{3}$$

$$\begin{aligned} sat(s/\phi) &= s/\phi \qquad if \quad |s(t)| \le 1 \\ sat(s/\phi) &= \mathrm{sgn}(s/\phi) \qquad if \quad |s(t)| > 1 \end{aligned} \tag{4}$$

where:

u is the control law;
s is the value of sliding surface s;
σ is the variable that represents the conditional integrator;
e_1=y-r;
y is the output signal;
r is the reference signal;
e_ρ is the derivative of e_1 with order ρ-1;
ρ is the relative degree of output controlled;
μ is the boundary layer;
k is the gain of the controller;
k_0 is the gain of conditional integrator;
k_j^i it the value of the gain that multiplies e_j^i ;
sat is the saturation function.

Equation (1) shows that is necessary to know the derivatives of tracking error e_1 until the order ρ-1. Not all the derivatives can be measured during the flight. For this reason it is necessary to estimate these derivatives. Observers can be used for this purpose. The mathematical model of observer used to estimate these derivatives can be found in (Seshagiri and Khalil, 2005), (Sousa, 2013).

The Fig.1 shows the block diagram of the universal integral regulator used to control one system with relative degree ρ=4.

The Fig.2 presents the block diagram of flight control system defined. It can be seen that the elevator deflection δe is used only to make the airspeed V track the desired airpeed V_d. The thrust control δπ is used only to force the altitude H to track the desired altitude H_d. The aileron deflection δa is used to make the roll angle ϕ to track the desired angle ϕ_d. And the rudder deflection δr is used to damp the yaw rate oscillations r.

It should be said that the universal integral regulator demands stable internal dynamics (Seshagiri and Khalil, 2005). So, this control law only can present satisfactory results if the structural dynamics of the airplane is stable, i.e, in speeds lower than the flutter speed.

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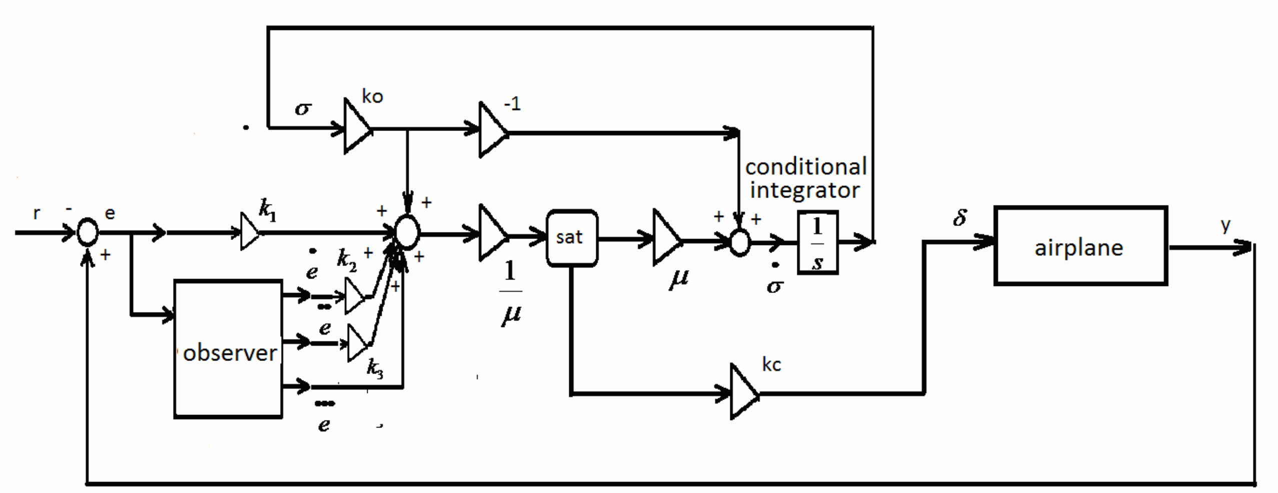

Figure 1. Block diagram of universal integral regulator for one system with relative degree 4.

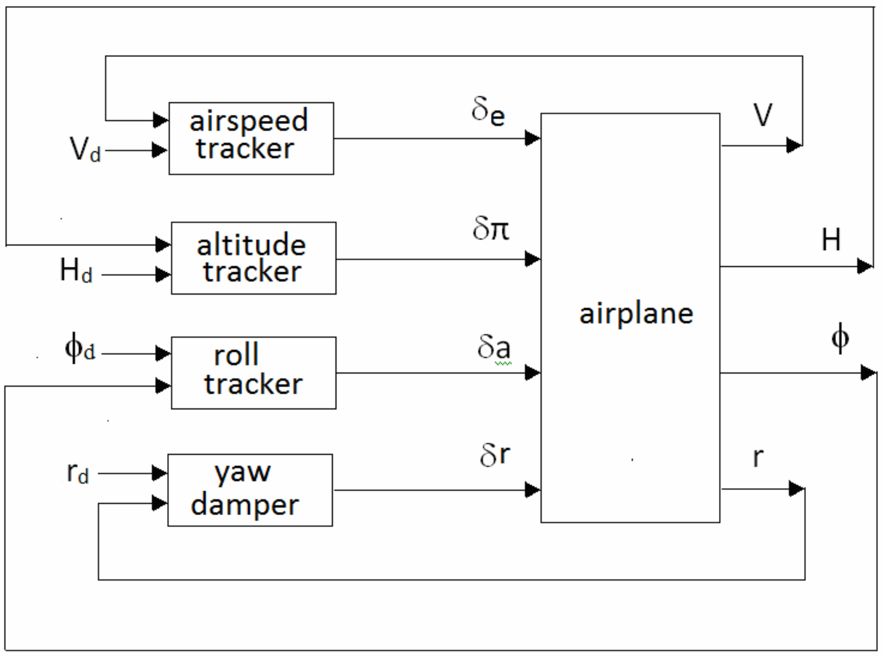

Figure 2. Block diagram of airplane flight control system

2.1 Airpeed Tracker

The Eq.(5) present the elevator flight control law used to perform the tracking of desired airspeed V_d.

$$\delta e = -k.sat(\frac{s}{\mu}) = -k.sat(\frac{k_0\sigma_V + k_1 e_V + k_2 \dot{e}_V + \ddot{e}_V}{\mu_V}) = -\frac{4.0}{57.3}sat(\frac{0.05\sigma_V + 1e_V + 6\dot{e}_V + \ddot{e}_V}{2}) \quad \textbf{(5)}$$

According to (Sousa and Paglione, 2012; Sousa, 2013) the relative degree ρ of airspeed is 3. So, it is necessary differentiate the airspeed two times. These parameters are not measured and must be estimated. The Eq.(6) presents the observer used to estimate these derivatives (Seshagiri and Khalil, 2005; Sousa, 2013).

$$\begin{bmatrix} \dot{\hat{e}}_{1V} \\ \dot{\hat{e}}_{2V} \\ \dot{\hat{e}}_{3V} \end{bmatrix} = \begin{bmatrix} -\frac{1}{0.05} & 1 & 0 \\ -\frac{6}{0.05^2} & 0 & 1 \\ -\frac{1}{0.05^3} & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{e}_{1V} \\ \hat{e}_{2V} \\ \hat{e}_{3V} \end{bmatrix} + \begin{bmatrix} \frac{1}{0.05} \\ \frac{6}{0.05^2} \\ \frac{1}{0.05^3} \end{bmatrix} e_{1V} \tag{6}$$

where: e_{1V} is the difference between the airspeed V and the airspeed commanded V_d;

$\hat{e}_{1V}$ is the estimated value of e_{1V}

$\hat{e}_{2V}$ is the estimated value of first derivative of e_{1V} ;

$\hat{e}_{3V}$ is the estimated value of second derivative of e_{1V} ;

The values of e_{1V} , $\hat{e}_{2V}$, $\hat{e}_{3V}$ were used in Eq.(5).

2.2 Altitude Tracker

The Eq.(7) presents the thrust flight control law used to perform the tracking of altitude desired H_d.

$$\delta\pi = -k.sat(\frac{k_0\sigma_H + k_1 e_H + k_2 \dot{e}_H + k_3 \ddot{e}_H + \dddot{e}_H}{\mu_H}) = -0.2sat(\frac{0.08\sigma_H + 1e_H + 3\dot{e}_H + 3\ddot{e}_H + \dddot{e}_H}{100}) \tag{7}$$

According to (Sousa and Paglione, 2012; Sousa, 2013) the relative degree ρ of altitude is 4. So, it is necessary differentiate the altitude three times. These parameters are not measured and must be estimated. The Eq.(8a),(8b) present the observer used to estimate these derivatives (Seshagiri and Khalil, 2005; Sousa, 2013).

$$\begin{bmatrix} \dot{\hat{e}}_{1H} \\ \dot{\hat{e}}_{2H} \\ \dot{\hat{e}}_{3H} \end{bmatrix} = \begin{bmatrix} -\frac{1}{0.05} & 1 & 0 \\ -\frac{6}{0.05^2} & 0 & 1 \\ -\frac{1}{0.05^3} & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{e}_{1H} \\ \hat{e}_{2H} \\ \hat{e}_{3H} \end{bmatrix} + \begin{bmatrix} \frac{1}{0.05} \\ \frac{6}{0.05^2} \\ \frac{1}{0.05^3} \end{bmatrix} e_{1H} \tag{8a}$$

$$\begin{bmatrix} \dot{\hat{e}}_{3Ho} \\ \dot{\hat{e}}_{4H} \end{bmatrix} = \begin{bmatrix} -\frac{1}{0.05} & 1 \\ -\frac{1}{0.05^2} & 0 \end{bmatrix} \begin{bmatrix} \hat{e}_{3Ho} \\ \hat{e}_{4H} \end{bmatrix} + \begin{bmatrix} \frac{1}{0.05} \\ \frac{1}{0.05^2} \end{bmatrix} \hat{e}_{3H} \tag{8b}$$

where: e_{1H} is the altitude error;

$\hat{e}_{1H}$ is the estimated value of e_{1H}

$\hat{e}_{2H}$ is the estimated value of first derivative of e_{1H}

$\hat{e}_{3H}$ is the estimated value of second derivative of e_{1H}

$\hat{e}_{3Ho}$ is the estimated value of $\hat{e}_{3H}$;

$\hat{e}_{4H}$ is the estimated value of third derivative of e_{1H}

The values of e_{1H} , $\hat{e}_{2H}$, $\hat{e}_{3H}$, $\hat{e}_{4H}$ were used in Eq.(7).

2.3 Roll Angle Tracker

The Eq.(9) presents the aileron flight control law used to perform the tracking of roll angle ϕ_d.

$$\delta a = -k.sat(\frac{k_0\sigma_\phi + k_1 e_\phi + \dot{e}_\phi}{\mu_\phi}) = \frac{40.0}{57{,}3} sat(\frac{0.005\sigma_\phi + 1.e_\phi + \dot{e}_\phi}{0.8}) \tag{9}$$

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According to (Sousa and Paglione, 2012; Sousa, 2013) the relative degree ρ of roll angle is 2. So, it is necessary differentiate the roll angle once. This parameter is not measured and must be estimated. The Eq.(10) presents the observer used to estimate this derivative (Seshagiri and Khalil, 2005; Sousa, 2013).

$$\begin{bmatrix} \dot{\hat{e}}_{1\phi} \\ \dot{\hat{e}}_{2\phi} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{0.05} & 1 \\ -\dfrac{1}{0.05^2} & 0 \end{bmatrix} \begin{bmatrix} \hat{e}_{1\phi} \\ \hat{e}_{2\phi} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{0.05} \\ \dfrac{1}{0.05^2} \end{bmatrix} e_{1\phi} \quad \textbf{(10)}$$

where: $e_{1\phi}$ is the difference between the airplane roll angle ϕ and the roll angle commanded ϕ_d;

$\hat{e}_{1\phi}$ is the estimated value of $e_{1\phi}$;

$\hat{e}_{2\phi}$ is the estimated value of derivative of $e_{1\phi}$;

The values of $e_{1\phi}, \hat{e}_{2\phi}$ were used in Eq.(9).

2.4 Yaw damper

The Eq.(11) presents the rudder flight control law used to damp the yaw rate oscillations.

$$\delta r = -k.sat\left(\frac{k_0\sigma_r + k_1 e_r}{\mu_r}\right) = \frac{-10.0}{57{,}3} sat\left(\frac{0.05\sigma_r + 1.e_r}{0.3}\right) \quad \textbf{(11)}$$

3. MATHEMATICAL MODEL OF AIRPLANE CONTROLLED

There are different methodologies to deduce the equations of motion of flexible airplanes. One of them is the methodology NFNS (**N**onlinear **F**light dynamics – **N**onlinear **S**tructural dynamics). This methodology is described in details in (Shearer, 2006; Su, 2008; Ribeiro, 2011; Sousa, 2013). The equations of motion are obtained with the use of Hamiltonian mechanics and considerations of arbitrary virtual displacements (Sousa, 2013). The Eq. (12) presents the equations of motion.

$$\begin{aligned}
&M_{FF}\ddot{\varepsilon} + M_{FB}\dot{\beta} + C_{FF}\dot{\varepsilon} + C_{FB}\beta + K_{FF}.\varepsilon = R_F \\
&M_{BF}\ddot{\varepsilon} + M_{BB}\dot{\beta} + C_{BF}\dot{\varepsilon} + C_{BB}\beta = R_B \\
&\dot{\phi} = p + \tan(\theta)(q\sin(\phi) - r\cos(\phi)) \\
&\dot{\theta} = q\cos(\phi) + r\sin(\phi) \\
&\dot{\psi} = (q\sin(\phi) - r\cos(\phi))/\cos(\theta) \\
&\dot{H} = U\sin(\theta) - V\cos(\theta)\sin(\phi) - W\cos(\theta)\cos(\phi)
\end{aligned} \quad \textbf{(12)}$$

where:

- β is one vector formed by the degrees of freedom of rigid body, i.e, the components of airspeed (u,v,w) and the components of the rotational speed (p,q,r). See Eq.(13).

$$\beta = \begin{bmatrix} v & u & w & q & p & r \end{bmatrix} \quad \textbf{(13)}$$

- The strain vector ε contains the deformations of all elements. Each element can suffer 4 deformations: one extensional deformation ε_x, one deformation in torsion k_x, and two bending deformations k_y and k_z. These deformations can be visualized on Figure 3.
- The matrices M_{FF}, M_{FB}, M_{BF}, M_{BF} are the components of generalized mass matrix (Su, 2008; Sousa, 2013);
- The matrices C_{FF}, C_{FB}, C_{BF}, C_{BF} are the components of generalized damping matrix (Su, 2008; Sousa, 2013);
- The vectors R_F, R_B are components of the generalized force vector (Su, 2008; Sousa, 2013);

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- The orientation of the airplane with relation to the earth is described by the euler angles ϕ,θ, ψ. These angles can be seen on Figure 4.
- The parameter H is the altitude of the airplane.

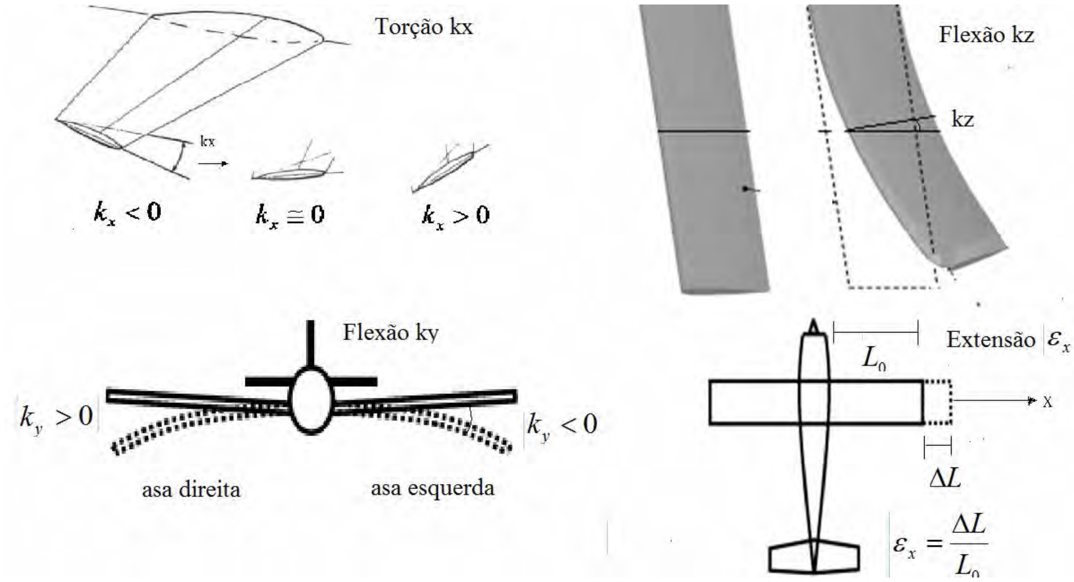

Figure 3. Deformations acting on the structural elements (Sousa, 2013).

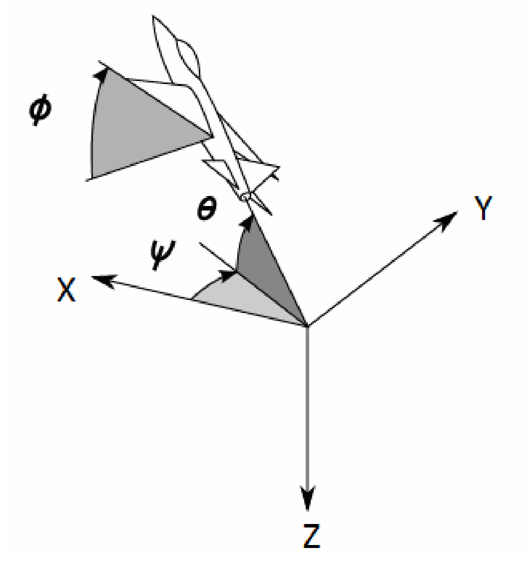

Figure 4. Euler Angles ϕ, θ, Ψ (Ribeiro, 2011)

The aerodynamic, structural and mass distribution data of the airplane simulated in this paper is described in details in (Sousa, 2013). The vehicle modeled has the properties similar to one commercial airplane.

4. RESULTS

This item presents the results obtained with the actuation of flight control laws defined in item 3 of this paper. Two identical airplanes with same geometry, mass distribution and aerodynamics were considered. The only difference between them is the different levels of structural flexibility. The very flexible airplane has the wing rigidity 6 times lower than the flexible airplane. The effect of smaller rigidity has the consequence of higher structural deformations. It can be seen on Figure 5. This figure presents the structural deformations on trimmed condition at 10000 m and 224,6

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m/s. It can be seen the bending deformations ky on the wing. The bending is considerable higher on the airplane with higher structural flexibility.

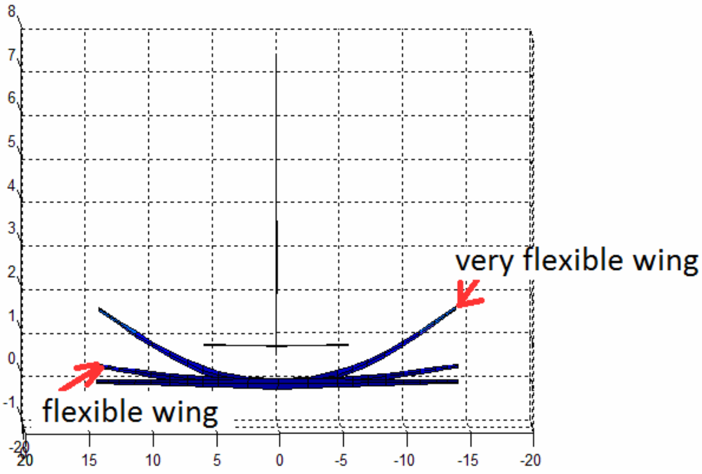

Figure 5. Structural deformations on flexible airplane and very flexible airplane on trimmed condition at 10000 m and 224,6m/s

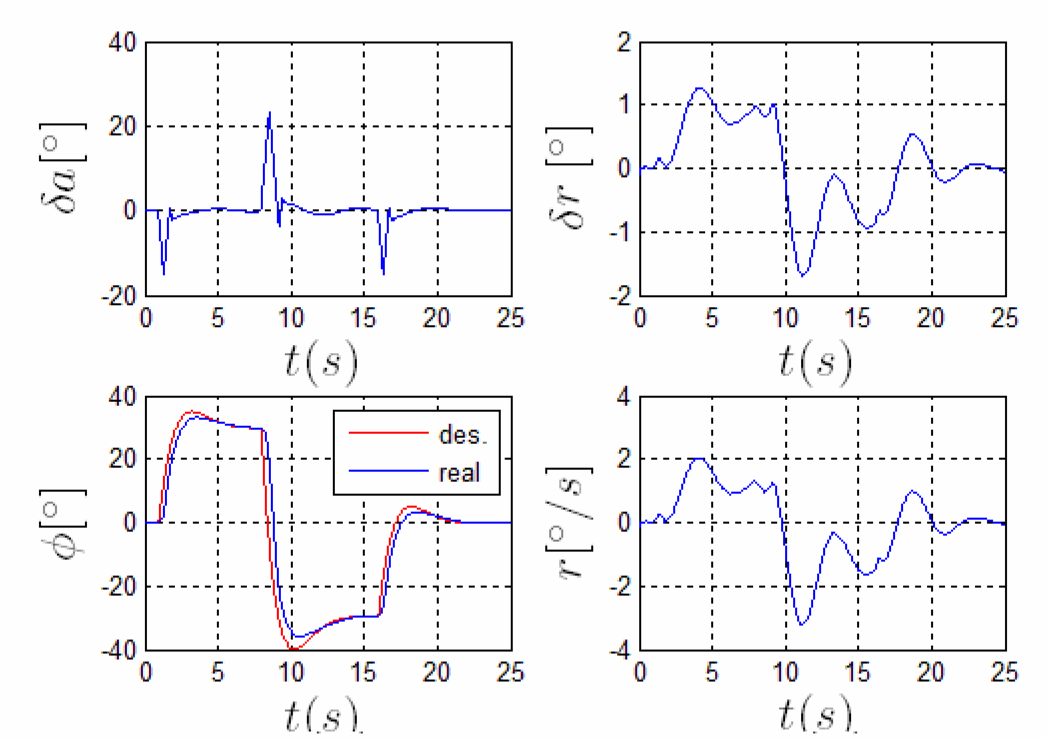

Figure 6. Doublet commanded in roll angle (flexible airplane)

The Fig. 6, 7, 8 present the results of the simulation of the flexible airplane. In this simulation, one doublet of roll angle was commanded. In the same time, the flight controller regulated the airspeed, altitude, and damped the yaw rate oscilations. The blue plots show the results obtained in the simulations, and the red plots presents the commanded (desired) values. The parameters presented in these plots are the airspeed V, altitude H, elevator deflection δe, thrust commanded π, roll angle ϕ, yaw rate r, aileron deflection δa, rudder deflection δr, bending deformations k_x and k_y of first and second elements of the right wing. In Fig.8, the red plots of ky and kx present the deformations in first element and the green plots present the deformations on second element. The mathematical model used in (Sousa, 2013) and in this paper uses five elements for each semi-wing. The elements 1 and 2 are closer to the fuselage (See Fig. 9). It was not shown all the deformations because the ones presented here is sufficient to show the effects of the aerodynamic forces on the structural deformations.

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The results obtained demonstrate the efficacy of the flight control law projected, once the doublet in roll angle was achieved, while the airspeed and altitude was maintained close to the initial values, and it was possible to damp the taw rate oscilations.

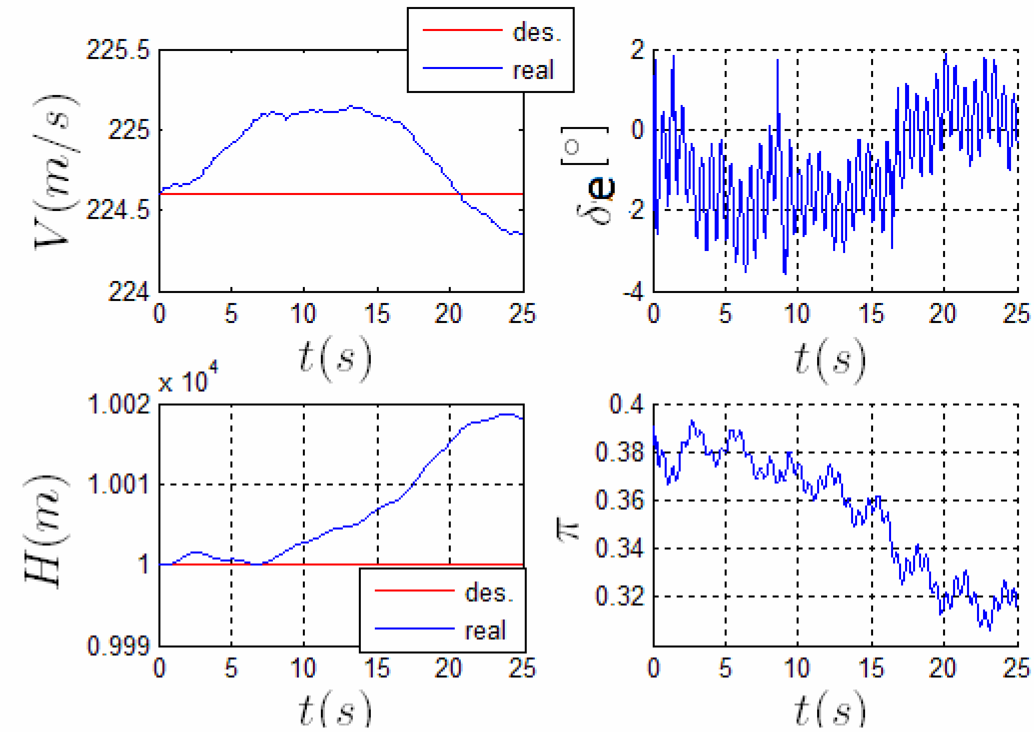

Figure 7. Regulation of airspeed and altitude (flexible airplane)

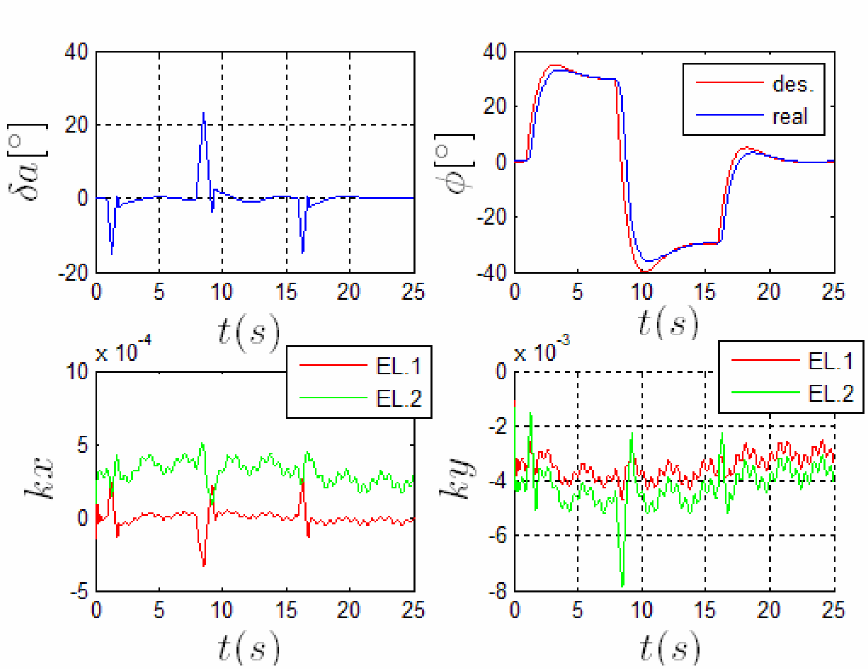

Figure 8. Doublet commanded in roll angle and structural deformations on the wing (flexible airplane)

Figure(8) presents higher values of structural deformations during the instants of time the aileron is being deflected. The reason to occur these structural deformations and its effects are explained in (Sousa, 2013). Basically it occurs due

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to the aerodynamic forces and decreases the aileron efficiency. Even with this fact, the flight control law compensates the effects of structural deformations.

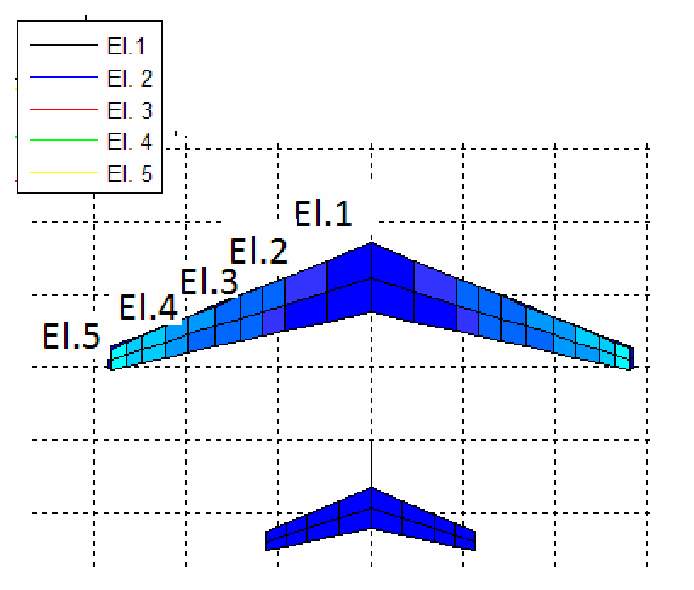

Figure 9 Structural elements on the wing.

The Fig.(10),(11),(12) present the same simulations of Fig.(6),(7),(8), respectively. The difference is that the simulations in Fig.(10),(11),(12) was performed with the very flexible airplane. It can be seen on Fig. (12) higher structural deformations and higher error on the tracking of bank angle (Fig.(10),(12)). Despite these facts, the controller projected for the flexible airplane presented satisfactory results in the control of very flexible airplane, that has very different dynamics due to the higher structural flexibility. It demonstrates the robustness of flight control law projected.

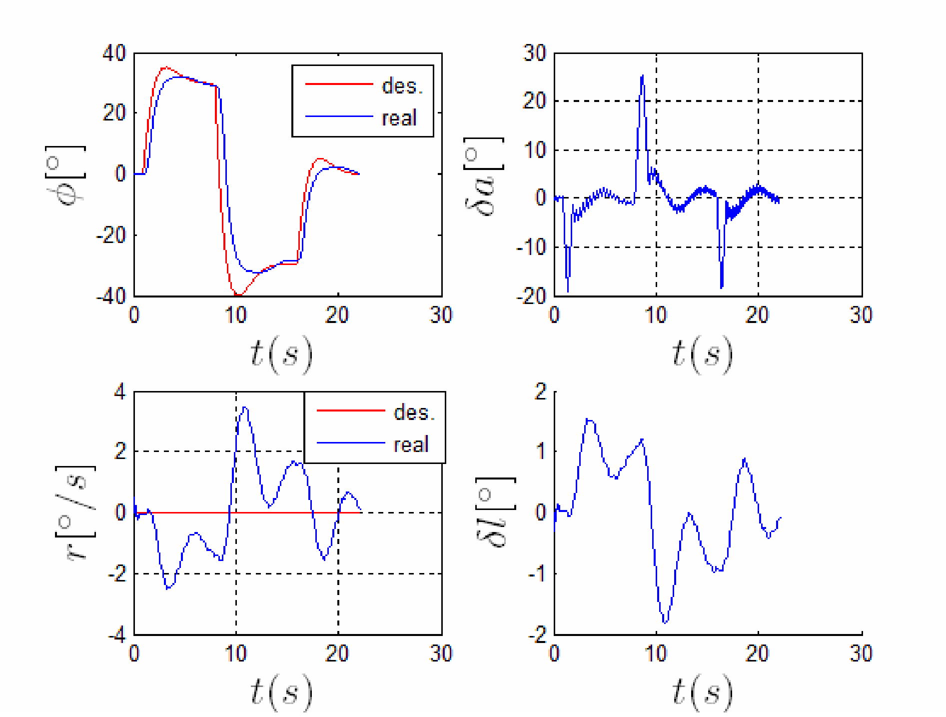

Figure 10. Doublet commanded in roll angle and (**very** flexible airplane)

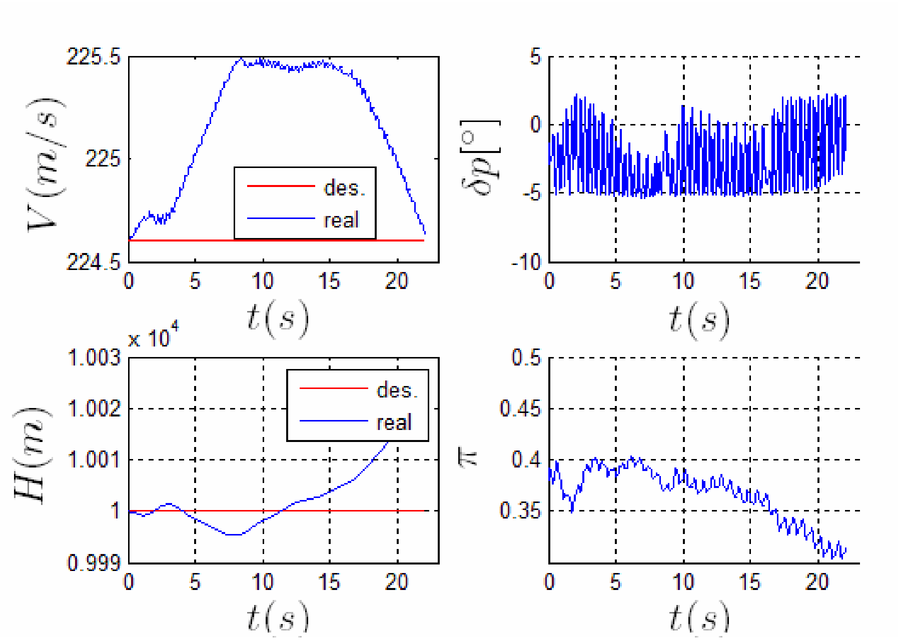

Figure 11. Regulation of airspeed and altitude (**very** flexible airplane)

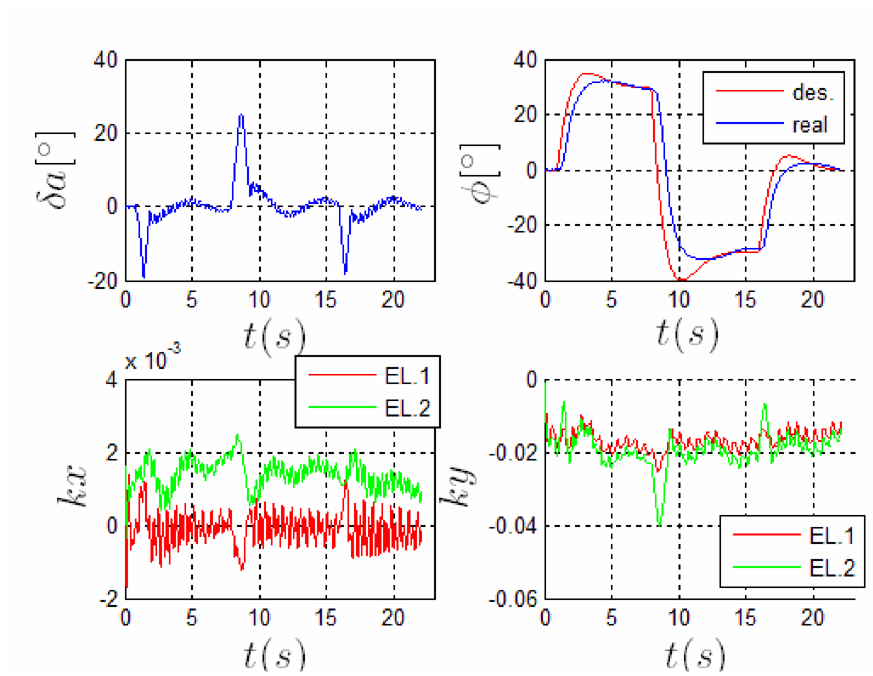

Figure 12. Doublet commanded in roll angle and structural deformations on the wing (**very** flexible airplane)

5. CONCLUSIONS

The results presented in this paper demonstrate the efficacy of the universal integral regulator to control the flight dynamics of flexible airplane, should the airplane has stable structural dynamics. The control can command the output

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desired while the perturbations due to structural deformations is minimized. The controller presented also this robustness when it was verified the controller could compensate the effects of structural flexibility. Naturally there is one limit to structural flexibility until the airplane control is possible. The robustness of universal integral regulator is on considerable advantage of this control technique. Other advantage is the fact that it is not necessary to have detailed information about the system controlled. These facts stimulate one more detailed study and analysis of the use of universal integral regulator on the flight control of flexible airplanes.

6. ACKNOWLEDGEMENTS

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