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HEAT FLUX INFLUENCE ON LAMINAR FLOW HEAT TRANSFER OF A WATER/ALUMINA NANOFLUID

P. Fariñas Alvariño
J.M. Sáiz Jabardo
J.D. Pena Agras
J. García del Valle
Escola Politécnica Superior. University of A Coruña. Mendizábal s/n, 15403 Ferrol. A Coruña, Spain.
pfarina@udc.es ; mjabardo@udc.es ; dpagras@udc.es ; javier.garciad@udc.es

Abstract. *The main objective of the reported study is the assessment of the effect of the wall heat flux in the internal laminar flow of nanofluids. A home made Computational Fluid Dynamics (CFD) solver has been developed for that purpose in order to include such unusually considered mechanisms in this kind of problem as the diffusion of nanoparticles due to both thermophoretic and Brownian effects. The solver assumes the nanofluid as being a single phase macroscopic mixture of base fluid (liquid) and nanoparticles. Effects of temperature and nanoparticles concentration on the transport properties of the nanofluid are considered in the present analysis. The obtained results include the nanoparticles distribution field along with the velocity, temperature and transport properties ones. It has been shown that a boundary layer like of nanoparticles concentration develops along the wall from the entrance cross section. The concentration boundary layer is closely related to the interplay of the thermophoretic and Brownian diffusion effects in the region close to the wall. A cross section average dimensionless parameter has been introduced that relates the Brownian diffusion with thermophoresis. It has been shown that this parameter is closely related with the concentration of nanoparticles in the wall region, which is clearly affected by the heat flux. A case study involving several heat fluxes at the wall has provided the needed information to conclude that the heat flux affects positively the heat transfer as a result of nanoparticles depletion of the wall region.*

Keywords: *Nanofluids, CFD, Particles field, Heat transfer enhancement*

1. INTRODUCTION

Thermal performance of the internal flow of nanofluids has been widely investigated in last decades, see, for example, the papers by (Pak and Cho, 1998; Xuan and Li, 2003; Wen and Ding, 2004; Williams *et al.*, 2008; Rea *et al.*, 2009). The heat transfer increments with respect to the base fluid generally observed with nanofluids is normally attributed to a significant increment of their thermal conductivity, a trend that has been extensively reported in the chemical and mechanical engineering literature. The reported results of a "benchmark exercise" by (Buongiorno *et al.*, 2009), and (Venerus *et al.*, 2010), though quantitatively questionable, are fully accepted from a qualitative point of view. It is not well stablished that the the thermal transport properties increment is the sole responsible for the observed nanofluids heat transfer enhancements. Such studies like the ones reported by (Wen and Ding, 2004; Buongiorno, 2006; Heris *et al.*, 2009; Williams, 2006) observed heat transfer enhancements in nanofluids as compared to those resuts predicted by convective heat transfer correlations. Another behavior has been proposed by (Fariñas Alvariño *et al.*, 2012, 2013) and (Prabhat *et al.*, 2012) according to which the thermal performance of nanofluids, on average, is comparable to that of pure fluids. In other words, the nanofluid would behave as a pure fluid with transport properties being determined from an adequate function of the corresponding volumetric concentrations of both the base fluid and nanoparticles.

The use of the field mass, momentum, and energy equations in the investigation of the role of the nanoparticles in the thermal performance of the nanofluid has been a challenge to the scientific community. For that purpose, both analytical and numerical solutions have been sought. In both approches the nanofluid has been considered as a mixture of the base fluid and nanoparticles, an assumption that leads to the concentration distribution of the nanoparticles in the flow field.

An interesting order of magnitude analysis was performed by Buongiorno (Buongiorno, 2006) in order to determine which of the mechanisms, among an extensive list, would potentially be the ones reponsible for the relative motion between nanoparticles and the base fluid. He concluded that, under common working conditions, the relative motion of nanoaparticles and the base fluid could only be promoted by brownian and thermophoretic difusion mechanisms. As a general conclusion, Buongiorno suggested to modifiy the classical turbulent flow heat transfer procedures to include the effect of the nanoaparticles concentration field. Following a similar procedure, Savithiri et al. (Savithiri *et al.*, 2011) investigated up to seven nanoparticles slip promoting mechanisms in several nanofluids. They concluded that besides brownian and thermophoretic diffusion, such mechanisms as lift, magnus and rotational effects should be considered in evaluating the global thermal performance of nanofluids. Wen et al. (Wen *et al.*, 2009) reported numerical solutions involving seven nanoparticles slip mechanisms. Their results regarding the nanoaparticles concentrations agreed with those from Boungiorno (Buongiorno, 2006) according to which the wall region is depleted of nanoparticles under heating conditions.

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There have been several attempts to solve the governing equations according to models that take into account both brownian and thermophoretic diffusion. The investigation developed by Avramenko et al. (Avramenko *et al.*, 2011) focused solutions of self-similar equations of the boundary layer, assuming the nanofluid as being a two component fluid. Velocity, temperature and concentration fields were obtained through numerical solution of the self-similar equations. Interestingly, they found nanofluid heat transfer enhancements with respect to the base fluid arguing that this was caused by the increment of the nanoparticles concentration in the wall region, a conclusion opposite to those by Buongiorno (Buongiorno, 2006) and Wen et al. (Wen, 2012).

Numerous experimental studies of the in flow of nanoafluids under turbulent and laminar regimes have been carried out during the last twenty years. Under turbulent flow conditions, the diffusion of nanoparticles in the base fluid can only occur in the laminar sublayer. On the other hand, in laminar flow, the development of momentum and thermal boundary layers along the tube wall promotes the corresponding development of a nanoparticle concentration boundary layer as a result of Brownian and themphoretic diffusion effects. Given the nature of the laminar flow, nanoparticles diffusion effects are more amenable to investigation in this flow regime, which is the objective of the reported study in this paper. Though a significant number of studies dealing with in tube laminar flow have been reported in the past, in the present paper just the most representatives according to the authors´ judgement will be introduced. One of such studies is the one by Wen and Ding (Wen and Ding, 2004) who reported experiments with tubes of 4.5 mm diameter and 970 mm length. Alumina nanoparticles presented volume concentrations that varied from 0.6% to 1.6%. Their results showed significant heat transfer enhancements of the nanofluid with respect to the base fluid what prompted them to suggest the operation in the entry length of laminar flow. Rea et al. (Rea *et al.*, 2009) performed experiments with 1,010 mm length tubes and the same diameter as the one of the Wen and Ding (Wen and Ding, 2004) research. They operated with two water based nanofluids with alumina and zirconia nanoparticles whose volumetric concentrations varying up to a maximum of 6%. Results from the investigation indicated that heat transfer enhancements with respect to pure based fluid (water) attained a maximum of 27% for the water/alumina nanofluid whereas much lower values were obtained with the water/zirconia nanofluid. Recently, an interesting investigation was reported by Ferrouillat et al. (Ferrouillat *et al.*, 2011) involving a water/SiO_2 nanofluid. Tests were performed in 4 mm diameter and 500 mm length tubes. Both turbulent and laminar regimes were investigated. According to these authors, heat transfer enhancements with respect to the base fluid for turbulent flow varied in the range between 10% and 50% for nanoparticles volumetric concentrations varying from 2.3% to 18.93%. They also claim that if the transport properties of the nanofluid were plugged into the dimensionless parameters, convective heat transfer correlations would fit experimental results within their range of validity, a conclusion that agrees with results from the numerical investigation by FariÃśas AlvariÃśo et al. (FariÃśas AlvariÃśo et al., 2013)

Numerical solutions of the nanofluids flow field equations could be divided into three categories: (i) the ones based on a Lagrangian approach for the nanoparticles Wen (Wen *et al.*, 2009); (ii) the Eulerian approach Mirmasoumi and Behzadmehr (Mirmasoumi and Behzadmehr, 2008); Kalteh et al. (Kalteh *et al.*, 2011); and (iii) a single phase fluid approach Oztop and Abu-Nada (Oztop and Abu-Nada, 2008); Izadi et al.(Izadi *et al.*, 2009). All the three approaches tend to justify, based on their own basic model assumptions, the observed heat transfer enhancements in the flow of nanofluids through the distribution of nanoparticles concentration in the flow field.

Fariñas Alvariño et al. (Fariñas Alvariño *et al.*, 2013) reported a numerical study (single phase approach) based on the model proposed by Buongiorno (Buongiorno, 2006). Solutions for laminar flow of a water/aluminia nanofluid were obtained. One of the conclusions draw from the investigation was the significant effect of the nanoparticles distribution on the rate of heat transfer though no significant heat transfer enhancements were observed. Further unpublished results from the investigation have shown a significant heat flux effect on the nanoparticles distribution. Since the nanoparticles distribution affects the transfer of momentum from the wall, see, for example, Fariñas Alvariño et al. (Fariñas Alvariño *et al.*, 2013); Hwang et al (Hwang *et al.*, 2009); and Sohn and Kihm (Sohn and Kihm, 2009), ultimately the wall heat transfer rate will be affected.

The present paper reports the results of an ongoing numerical investigation of the internal laminar flow of nanofluids. In the present paper, effects of the heat flux on the rate of heat transfer are the focus, using as a case study the test tube by Rea et al. (Rea *et al.*, 2009) but varying the heat flux within the range between 5 and 80 kWm^{-2}.

2. NANOFLUID AND NUMERICAL MODEL

The laminar flow of a water/Al_2O_3 (alumina) nanofluid in a straight 4.5 mm inner diameter pipe has been considered in the present investigation, Rea et al. (Rea *et al.*, 2009). Effects of heat flux on Nusselt values have been addressed in order to detect, in case it occurs, any kind of heat transfer enhancement. Two kinds of case studies have been run. The first one, corresponding to reported experimental investigations, has been run to validate the solver. The results from these case studies have been published in a previous paper Ref. (Fariñas Alvariño *et al.*, 2013). The second kind of case studies has been run to asses the effect over heat transfer of the wall heat flux. Results from these cases, listed in Tab. 1, are the basis for the analysis performed in the present paper. In order to investigate heat flux effects, a systematic change of the wall heat flux has been applied while keeping unaltered the other boundary conditions. Under these constrains any unusual heat transfer behavior must be attributed to the applied heat flux.

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The developed solver takes into account the base fluid properties dependence on temperature. Thus, the field equations solution depends both on the entrance temperature and the heat flux. An adequately low entrance temperature had to be imposed in order to cope with high heating rates (up to $80kWm^{-2}$) to prevent boiling in the test tube. Since the inlet temperature for the experimental/numerical case from Table 1 has been set to 299.2 K. this is the temperature that has been the imposed at the entrance in all the cases considered in the present study. The maximum temperature achieved in the test tube has been equal to 376.6 K, which is well below the boiling point of water at normal atmospheric pressure.

The nanoparticles size has been imposed according to the information reported in the experimental investigations (Wen and Ding, 2004; Rea *et al.*, 2009) used for the solver validation in (Fariñas Alvariño *et al.*, 2013). Wen and Ding, 2004 reported a nanoparticles size range between 27 nm and 56 nm Rea et al. (Rea *et al.*, 2009) reported particles size of the order of 50 nm. Since the Brownian diffusion coefficient, used in the present investigation, requires the nanoparticle diameter, its value has been assumed as being equal to 46 nm, which is in the range of the referenced experimental investigations. The velocity of the nanofluid at the test section entrance for all the cases of Tab. 1 has been assumed equal to 1.003 ms^{-1}, corresponding to a Reynolds number equal to 940.

Table 1. Numerical cases with different heat fluxes.
Second set of numerical cases.

Author (Exper./Numer.)	α_{BULK}	$T_{inlet}[K]$	$q''[Wm^{-2}]$	$L[mm]$	Tinlet
Numerical	6.00 %	299.20	0	1010	299.20
Numerical	6.00 %	299.20	5000	1010	299.20
Numerical	6.00 %	299.20	10000	1010	299.20
Numerical	6.00 %	299.20	20000	1010	299.20
Exper.(Rea *et al.*, 2009)/Numer.	6.00 %	299.20	45921	1010	299.20
Numerical	6.00 %	299.20	60000	1010	299.20
Numerical	6.00 %	299.20	80000	1010	299.20

2.1 Governing equations

The following are the set of governing equations which are based on the ones suggested by (Buongiorno, 2006) and implemented by (Fariñas Alvariño *et al.*, 2013).

$$\frac{\partial \rho_{nf}}{\partial t} + \nabla \circ \left(\rho_{nf}\vec{U}\right) = 0 \tag{1}$$

$$\frac{\partial \left(\rho_{nf}\vec{U}\right)}{\partial t} + \nabla \circ \left(\rho_{nf}\vec{U}\vec{U}\right) = -\vec{\nabla}P + \nabla \circ \left[\mu_{nf}\left(\vec{\nabla}\vec{U} + \vec{\nabla}\vec{U}^T\right)\right] \tag{2}$$

$$\frac{\partial \left(\rho_{nf}C_{nf}T\right)}{\partial t} + \nabla \circ \left(\rho_{nf}C_{nf}\vec{U}T\right) = \nabla \circ \left(\kappa_{nf}\vec{\nabla}T\right) + \rho_p C_p \left(D_B\vec{\nabla}\alpha\cdot \vec{\nabla}T + D_T\frac{\vec{\nabla}T\cdot \vec{\nabla}T}{T}\right) \tag{3}$$

$$\frac{\partial \alpha}{\partial t} + \nabla \circ \left(\vec{U}\alpha\right) = \nabla \circ \left(D_B\vec{\nabla}\alpha + D_T\frac{\vec{\nabla}T}{T}\right) \tag{4}$$

The nanofluid has been designated by the subscript “nf” and the base fluid and the nanoparticles, respectively, by the subscripts “bf” and “p”.

The brownian and thermophoresis diffusion coefficients are expressed as (Buongiorno, 2006):

$$D_B = \frac{k_B T}{3\pi\mu_{bf}d_p} \tag{5}$$

$$D_T = S_{coef}\frac{\mu_{bf}}{\rho_{bf}}\alpha = 0.26\frac{\kappa_{bf}}{2\kappa_{bf}+\kappa_p}\frac{\mu_{bf}}{\rho_{bf}}\alpha \tag{6}$$

The nanofluid transport properties depend on both the temperature and concentration. The correlations proposed by Williams et al. (Williams *et al.*, 2008) and Rea et al. (Rea *et al.*, 2009) expressing this dependence have been adopted in the present study. Detailed information can be found in (Fariñas Alvariño *et al.*, 2013). The boundary conditions are

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the common ones for this kind of problem (Fariñas Alvariño *et al.*, 2013). It must be noted that the thermal boundary condition at the wall requires a constant and uniform heat flux along the test tube. This condition does not correspond to a uniform temperature gradient at the wall since the thermal conductivity of the nanofluid is affected by both the temperature and the nanoparticles concentration. A worth mentioning condition is the one corresponding to the impermeability to nanoparticles of the heated wall, which requires a nil net diffusion of nanoparticles. In other words, the Brownian and thermophoretic diffusions act in opposite directions. Thus, this boundary condition can be written as:

$$\vec{\nabla}\alpha|_w = -\frac{D_T}{D_B}\frac{\vec{\nabla}T|_w}{T} \tag{7}$$

2.2 Numerical particulars

The discretization of the governing equations is performed in a 5^o axisymmetric wedge geometry longitudinally divided in two control volumes. The first one comprises the first 48.5 mm downstream the entrance section and contains 90 cells in axial direction with an increasing size rate of 2.0. The number of cells in the radial direction is equal to 250 with a rate factor of 15.0. The second block contains the rest of the duct up to the outlet section and the mesh is arranged with 1,286 equally longitudinally spaced cells. The radial discretization in the second block is the same as the one of the first block in order to allow the adequate link between them. Mesh details can be found in (Fariñas Alvariño *et al.*, 2013).

Two final notes regarding the numerical algorithm:

- The density of the nanofluid depends weakly on the temperature, an aspect that regularly allows one to assume the problem as incompressible. However, difficulties were found in the convergence of the numerical procedure by assuming the incompressibility. This led us to assume the problem as compressible, see Eq. 1, though, given that no pressure equation is available, the problem was assumed as incompressible in the pressure/velocity coupling.

- Two of the boundary conditions at the wall namely those related to the energy and nanoparticles concentration equations are dependent on the solution field, an aspect that makes the convergence more time consuming. As an example, the time elapsed for each case shown in this paper has been up to four weeks in a 8 parallel processor node of 1.2 Gz each.

3. RESULTS

The solver used in the numerical solutions of the present investigation has been validated against pure fluid experimental solutions and classical correlations for undeveloped laminar flow. A comparison of numerical results from the present solver with experimental ones obtained elsewhere performed in a previous investigation has revealed that numerical solutions are reasonably close to experimental ones, which include both pure base fluid (water) and a water/alumina nanofluid at several nanoparticles concentrations Fariñas Alvariño et al. (Fariñas Alvariño *et al.*, 2013). The adequate result of this comparison provided the needed confidence in the present numerical solver. From the same investigation reported by Fariñas Alvariño et al. (Fariñas Alvariño *et al.*, 2013) it was concluded that, under heating conditions, a nanoparticle concentration "boundary layer like" distribution sets in downstream of the tube entrance section shown in the plot of Fig. 1. A clear wall region depletion of nanoparticles can be noted in this figure in the downstream direction. This depletion is the result of the counteracting of Brownian and thermophoretic diffusion effects. In fact, under heating conditions, whereas Brownian diffusion effects tend to drive nanoparticles toward the wall region, down the concentration gradient, thermophoresis acts on the on the opposite direction, down the temperature gradient. The concentration boundary layer affects both the momentum and heat transfer at the wall. Heat transfer correlations for the undeveloped laminar flow of a pure fluid are generally expressed in terms of two dimensionless numbers namely the Nusselt and the Graetz number. No explicit effects of the wall heat flux have been reported in the literature. In a recently published paper, Fariñas Alvariño et al. (Fariñas Alvariño *et al.*, 2013) speculated about possible wall heat flux effects on the rate of heat transfer at the wall (Nusselt number) for a nanofluid in laminar flow. The main purpose of the present paper is to report the results of an investigation of these effects through numerical solutions based on one of the cases investigated by Rea et al. (Table 1) corresponding to a 6% nanoparticles volumetric concentration, but for a series of different heat fluxes.

The plot of Fig. 2 displays the variation of the local Nusselt number against the inverse of the Graetz number for different wall heat fluxes. The Graetz number is defined as usual:

$$Gz = \frac{RePr}{\frac{x}{d}} \tag{8}$$

Heat transfer correlations by Churchill and Ozoe (Churchill and Ozoe, 1973) and Shah (Shah, 1975) are overlaid in the same plot for reference purposes. It must be noted that the Churchill and Ozoe correlation fits reasonably well pure fluid numerical data. The effect of the wall heat flux is clearly observed in Fig. 2. A more explicit trend related to the effects of the wall heat flux can be observed in Fig. 3, where the nanofluid Nusselt number is plotted against the base fluid (water)

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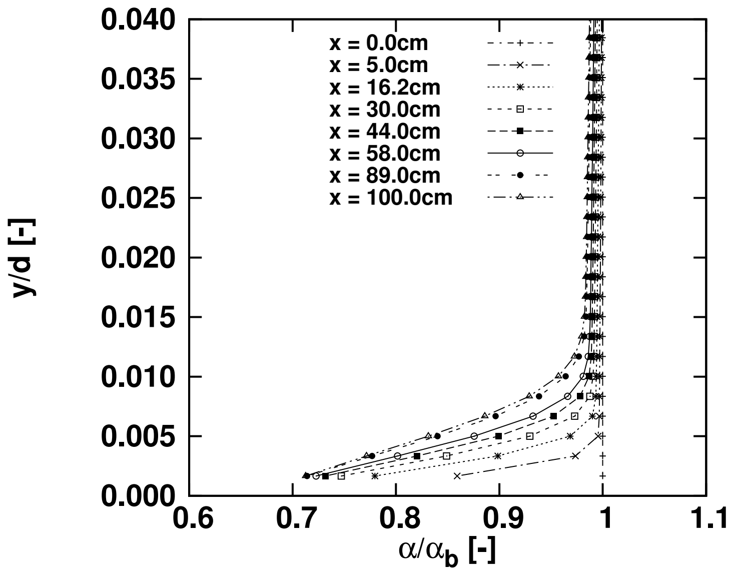

Figure 1. Particles distribution for the reference/experimental case shown in Tab. 1. Boundary layer pattern for particles from Ref. (Fariñas Alvariño *et al.*, 2013).

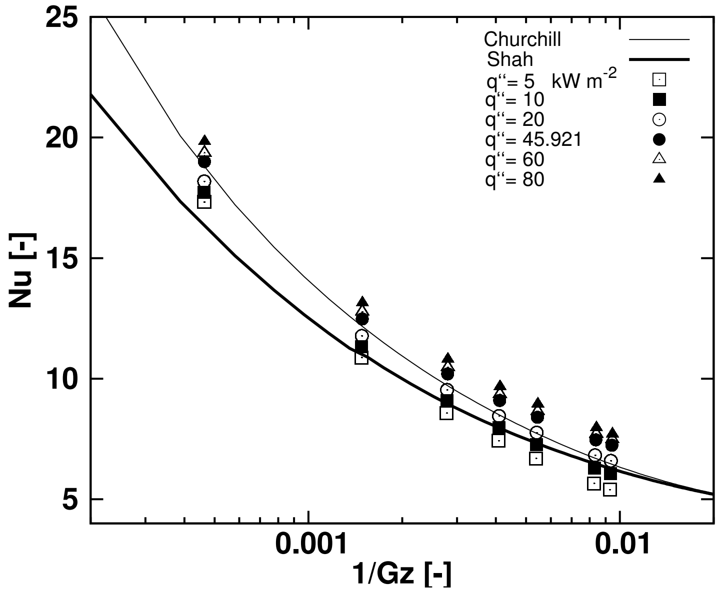

Figure 2. Variation of the local Nusselt number against the inverse of the Graetz number for cases of Tab. 1.

Nusselt number for the same flow conditions. Wall heat flux effects are clearly observed in this plot. Given that higher Nusselt numbers are typical of the tube entrance regions, it can be concluded that the level of heat transfer enhancement depends both on the position along the tube and the level of heat flux. It is interesting to note that, contrary to what is generally believed, decrements on the rate of heat transfer with respect to the base fluid are noted in entrance region of the tube, where the flow is fairly undeveloped. Even at downstream sections, heat transfer decrements are observed, depending on the heat flux. The trends observed in Figs. 2 and 3, as previously noted, must closely be related to the nanoparticles concentration in the wall region which results from the counteraction of brownian diffusion and thermophoresis, as will be demonstrated further on. The physical mechanism works like this for heating conditions. As the nanofluid enters the tube, it is submitted to a temperature gradient at the wall which induces a diffusion of nanoparticles away from it. Further downstream, as nanoparticles are driven away form the wall, a concentration gradient sets in and Brownian diffusion acts over the nanoparticles tending to drive them toward the wall. Thus, as mentioned before, thermophoresis and brownian diffusion counteract. The dominance of the former tends to deplete of particles the wall region whereas the last tends to increase the concentration there. The result of the counteraction between both effects is the development of a concentration boundary layer as the one in Fig. 1. Summing up, the concentration distribution depends directly on the relative dominance of Brownian and thermoporetic effects. The net effect could be numerically expressed by the dimensionless parameter N_{BT}, introduced by Buongiorno (Buongiorno, 2006), which is the ratio between brownian and thermophoresis diffusion. The dimensionless parameter N_{BT} is determined from the following expression Buongiorno (Buongiorno, 2006); and Fariñas Alvariño et al. (Fariñas Alvariño *et al.*, 2013):

$$N_{BT} = \frac{\alpha_b D_B T_\alpha}{\Delta T D_T} = \frac{\alpha_b D_B T_\alpha \kappa_{nf}}{q'' \delta_\alpha D_T} \tag{9}$$

Note that this is a parameter related to the local temperature gradient. In order to evaluate its effect over the nanoparticles concentration distribution along the tube, a cross section average parameter, $\overline{N_{BT}}$, has been introduced based on average values of both diffusion coefficients and temperature whose expression is as follows:

$$\overline{N_{BT}} = \frac{\alpha_b \overline{D_B} T_m}{(T_w - T_m)\,\overline{D_T}} \tag{10}$$

The cross section average temperature is determined in the present paper according to the usual expression. For a given heat flux, the concentration of nanoparticles in the wall region diminishes with the distance from the entrance section, as shown in the plot of Fig. 1. Given that the concentration gradient at the wall is rather high in cross sections close to the entrance, Brownian effects are significant and, as a result, the value of $\overline{N_{BT}}$ is high. In sections further downstream, concentration gradients diminish whereas the temperature gradient at the wall remains constant, causing a reduction in the value of $\overline{N_{BT}}$. This trend is clearly displayed in Fig. 4, where $\overline{N_{BT}}$ is plotted against the inverse of the Graetz number, which is directly proportional to the distance from the entrance. Thus it could be stated that the value of $\overline{N_{BT}}$ is closely related to the concentration of nanoparticles in the wall region. This leads us to the effect of the heat flux, clearly shown in the plot of Fig. 4. A higher heat flux implies higher thermophoresis effects (higher temperature gradient at the wall) tending to diminish the $\overline{N_{BT}}$ value what in turn results in a more intense depletion of nanoparticles of the wall region. Thus, for a given cross section along the tube, corresponding to a Graetz number, $\overline{N_{BT}}$ diminishes with the heat flux, with the reduction being more significant in the range of lower heat fluxes. The plot of Fig. 5 illustrates the depletion of nanoparticles of the wall region caused by increased heat fluxes at the exit section of the tube (x=1,010 mm). Note that the concentration of nanoparticles at a given distance from the wall diminishes with the heat flux confirming the trend of Fig. 4. The results of Figs. 4 and 5 along with those of Fig. 1 could be summarized as follows: (i) for a given heat flux, the wall region tends to be depleted of nanoparticles in the flow direction; and (ii) nanoparticles depletion increases with the heat flux.

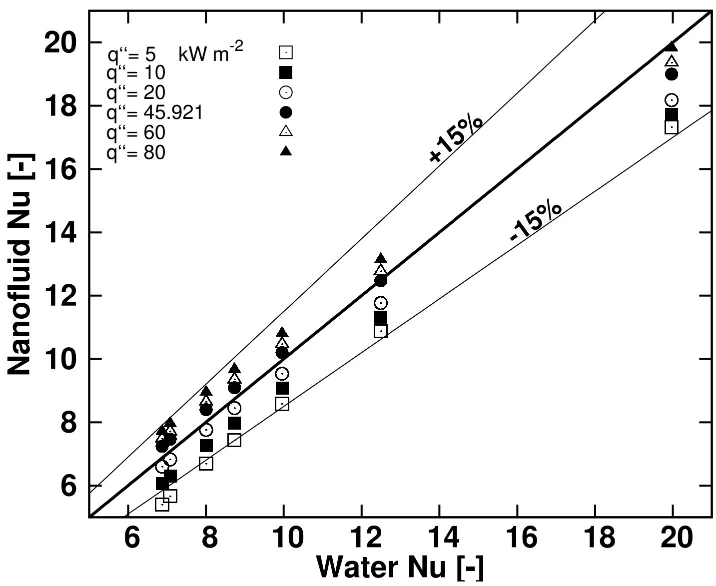

Figure 3. Nanofluid against pure water Nusselt numbers for the cases of Tab. 1.

Finally, it remains to relate the nanoparticle concentration at the wall with the observed heat transfer trends of Figs. 2 and 3. Higher concentration of nanoparticles in the wall region increases both the thermal conductivity and the viscosity, what in turn affects the velocity and temperature distribution. Though not shown in this paper, it has been found that the velocity and temperature gradients of the nanofluid at the wall increase with the heat flux, a trend that is closely related to Figs. 2 and 3 results. Thus, as a general conclusion, it can be stated that the lesser the concentration of nanoparticles in the wall region the higher the rate of heat transfer, a trend that is in accordance with a previously suggested one by Buongiorno (Buongiorno, 2006) for turbulent flow.

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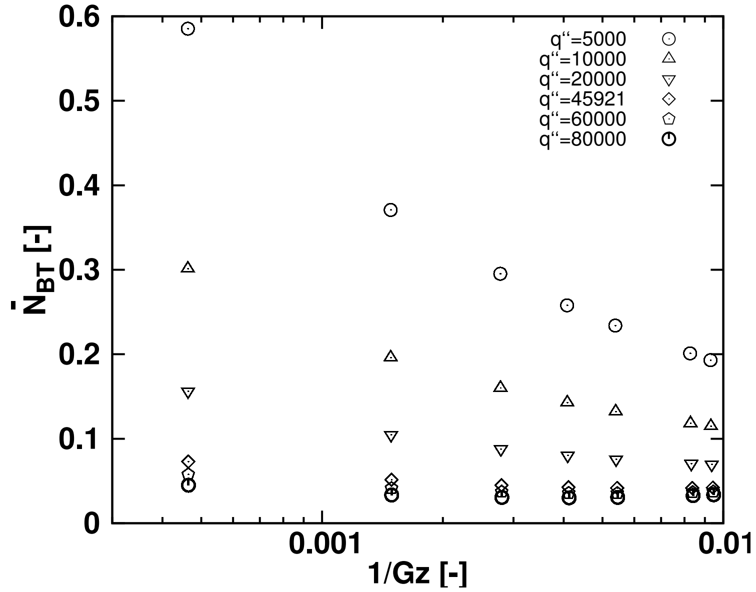

Figure 4. $\overline{N_{BT}}$ of the Tab. 1 cases against the inverse of the Gaetz number.

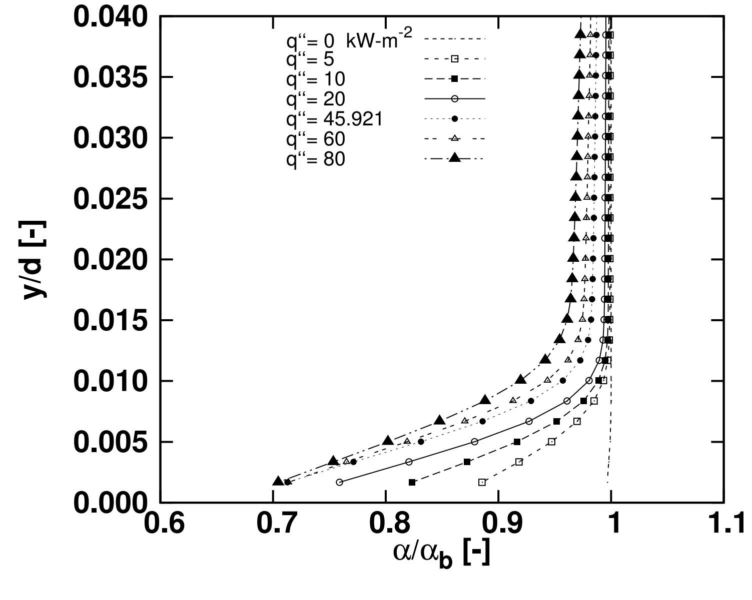

Figure 5. Boundary layer patterns at the exit cross section of the test tube.

4. CONCLUSIONS

The numerical investigation reported herein has shown the effect of the wall heat flux over rate of heat transfer. It has been demonstrated that the physical mechanism behind this effect is closely related to the effect of the heat flux over the concentration of nanoparticles in the heated wall region, a relation that has been made possible through the introduction of the dimensionless parameter $\overline{N_{BT}}$, defined as being the ratio between Brownian diffusion and thermophoresis.

The following are general conclusions drawn from the present investigation:

- Higher values of $\overline{N_{BT}}$ correspond to higher nanoparticles concentrations at the heated wall.
- The Nusselt number increases with the heat flux. For the range of heat fluxes of the present investigation, Nusselt number increments of the order of 30% were obtained.
- The Churchill and Ozoe (Churchill and Ozoe, 1973) correlation fits closely the nanofluid average heat flux solution. Deviations from this correlation are related to heat flux effects and varied in the range $\pm 15\%$ for the range of heat fluxes considered in the present investigation.
- The present study has found that nanofluids heat transfer enhancements with respect to pure base fluid might happen

under certain conditions, especially related to high heat fluxes. Heat transfer reductions with respect to the base fluid have also been obtained, especially in the lower heat fluxes range and in regions close to the entrance of the test tube.

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6. REFERENCES

Avramenko, A.A., Blinov, D.G. and Shevchuk, I.V., 2011. "Self-similar analysis of fluid dlow and heat-mass transfer of nanofluids in boundary layer". *Physics of fluids*, Vol. 23, pp. 082002–1–8.

Buongiorno, J., 2006. "Convective transport in nanofluids". *Journal of Heat Transfer*, Vol. 128, pp. 240–250.

Buongiorno, J., Venerus, D.C., Prabhat, N., McKrell, T., Townsend, J., Christianson, R., Tolmachev, Y.V., Keblinski, P., Hu, L.W., Alvarado, J.L., Bang, I.C., Bishnoi, S.W., Bonetti, M., Botz, F., Cecere, A., Chang, Y., Chen, G., Chen, H., Chung, S.J., Chyu, M.K., Das, S.K., Di Paola, R., Ding, Y., Dubois, F., Dzido, G., Eapen, J., Escher, W., Funfschilling, D., Galand, Q., Gao, J., Gharagozloo, P.E., Goodson, K.E., Gutierrez, J.G., Hong, H., Horton, M., Hwang, K.S., Iorio, C.S., Jang, S.P., Jarzebski, A.B., Jiang, Y., Jin, L., Kabelac, S., Kamath, A., Kedzierski, M.A., Kieng, L.G., Kim, C., Kim, J.H., Kim, S., Lee, S.H., Leong, K.C., Manna, I., Michel, B., Ni, R., Patel, H.E., Philip, J., Poulikakos, D., Reynaud, C., Savino, R., Singh, P.K., Song, P., Sundararajan, T., Timofeeva, E., Tritcak, T., Turanov, A.N., Van Vaerenbergh, S., Wen, D., Witharana, S., Yang, C., Yeh, W.H., Zhao, X.Z. and Zhou, S.Q., 2009. "A benchmark study on the thermal conductivity of nanofluids". *Journal of Applied Physics*, Vol. 106, No. 9, pp. 094312+. doi: 10.1063/1.3245330. URL `http://dx.doi.org/10.1063/1.3245330`.

Churchill, S. and Ozoe, H., 1973. "Correlations for laminar forced convection with uniform heating in flow over a plpate and in developing and fully developed flow in a tube". *Journal of heat transfer*, Vol. Transactions of the ASME, pp. 78–84.

Fariñas Alvariño, P., Sáiz Jabardo, J.M., Arce, A. and Lamas Galdo, I., 2012. "Heat transfer enhancement in nanofluids. a numerical approach." *Journal of physics: Conference series*, Vol. 395, p. 012116. ISSN 1742-6596.

Fariñas Alvariño, P., Sáiz Jabardo, J.M., Arce, A. and Lamas Galdo, I., 2013. "A numerical investigation of laminar flow of a water/alumina nanofluid". *International journal of heat and mass transfer*, Vol. 59, pp. 423–432. doi: dx.doi.org/10.1016/j.ijheatmasstransfer.2012.12.033.

Ferrouillat, S., Bontemps, A., Ribeiro, J.P., Gruss, J.A. and Soriano, O., 2011. "Hydraulic and heat transfer study of sio2/water nanofluids in horizontal tubes with imposed wall temperature boundary conditions". *International journal of heat and fluid flow*, Vol. 32, pp. 424–439.

Heris, S.Z., Etemad, S.G. and Esfahany, M.N., 2009. "Convective heat transfer of a cu/water nanofluid flowing through a circular tube". *Experimental heat transfer*, Vol. 22, No. 4, pp. 217–227. ISSN 0891-6152.

Hwang, K.S., Jang, S.P. and Choi, S.U., 2009. "Flow and convective heat transfer characteristics of water-based al_2o_3 nanofluids in fully developed laminar flow regime". *International journal of heat and mass transfer*, Vol. 52, pp. 193–199. doi:10.1016/j.ijheatmasstransfer.2008.06.032.

Izadi, M., Behzadmehr, A. and Jalali-Vahida, D., 2009. "Numerical study of developing laminar forced convection of a nanofluid in an annulus". *International journal of thermal sciences*, Vol. 48, pp. 2119–2129.

Kalteh, M., Abbas Abbassi, Saffar-Avval, M. and Harting, J., 2011. "Eulerian–eulerian two-phase numerical simulation of nanofluid laminar forced convection in a microchannel". *International journal of heat and fluid flow*, Vol. 32, pp. 107–116.

Mirmasoumi, S. and Behzadmehr, A., 2008. "Effect of nanoparticles mean diameter on mixed convection heat transfer of a nanofluid in a horizontal tube". *International journal of heat and fluid flow*, Vol. 29, pp. 557–566.

Oztop, H.F. and Abu-Nada, E., 2008. "Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids". *International journal of heat and fluid flow*, Vol. 29, pp. 1326–1336.

Pak, B.C. and Cho, Y.I., 1998. "Hydrodynamic and heat transfer study of dispersed fluid with submicron metallic oxide particles". *Experimental heat transfer*, Vol. 11, No. 2, pp. 151–170.

Prabhat, N., Buongiorno, J. and Hu, L.W., 2012. "Convective heat transfer enhancement in nanofluids: Real anomaly or analysis artifact?" *Journal of nanofluids*, Vol. 1, No. 1, pp. 55–62. doi:http://dx.doi.org/10.1166/jon.2012.1003.

Rea, U., McKrell, T., Hu, L.W. and Buongiorno, J., 2009. "Laminar convective heat transfer and viscous pressure loss of alumina-water and zirconia-water nanofluids". *International journal of heat and mass transfer*, Vol. 52, pp. 2042–2048.

Savithiri, S., Pattamatta, A. and Das, S.K., 2011. "Scaling anlysis for the investigation of slip mechanisms in nanofluids". *Nanoscale research letters*, Vol. 6, No. 1, p. 6:471.

Shah, R.K., 1975. "Thermal entry length solutions for the circular tube and parallel plates". *Proceedings of 3rd National*

Heat and Mass Transfer Conference, Vol. 1, pp. HMT–11–75.

Sohn, C.H. and Kihm, K.D., 2009. "Nonhomogeneous modeling of nanofluidic energy transport accounting for thermophoretic migration of nanoparticles inside laminar pipe flows". *Journal of korean physical society*, Vol. 55, No. 5, pp. 2200–2208. doi:10.3938/jkps.55.2200.

Venerus, D.C., Buongiorno, J., Christianson, R., Townsend, J., Bang, I.C., Chen, G., Chung, S.J., Chyu, M., Chen, H., Ding, Y., Dubois, F., Dzido, G., Funfschilling, D., Galand, Q., Gao, J., Hong, H., Horton, M., wen Hu, L., Iorio, C.S., Jarzebski, A.B., Jiang, Y., Kabelac, S., Kedzierski, M.A., Kim, C., Kim, J.H., Kim, S., McKrell, T., Ni, R., Philip, J., Prabhat, N., Song, P., Van Vaerenbergh, S., Wen, D., Witharana, S., Zhao, X.Z. and Zhou, S.Q., 2010. "VISCOSITY MEASUREMENTS ON COLLOIDAL DISPERSIONS (NANOFLUIDS) FOR HEAT TRANSFER APPLICATIONS". *APPLIED RHEOLOGY*, Vol. 20, No. 4. ISSN 1430-6395. doi:10.3933/ApplRheol-20-44582.

Wen, D., 2012. "Nanofluids for heat transfer applications: state-of-art and beyond". Eurotherm Keynote.

Wen, D. and Ding, Y., 2004. "Experimental investigation into convective heat transfer of nanofluids at entrance region under laminar flow conditions". *International journal of heat and mass transfer*, Vol. 47, pp. 5181–5188.

Wen, D., Zhang, L. and He, Y., 2009. "Flow and migration of nanoparticle in a single channel". *Heat mass transfer*, Vol. 45, No. 8, pp. 1061–1067. doi:10.1007/s00231-009-0479-8. URL `http://dx.doi.org/10.1007/s00231-009-0479-8`.

Williams, W.C., 2006. *Experimental and theoretical investigation of transport phenomena in nanoparticle colloids*. Ph.D. thesis, Massachusetts Institute of Technology.

Williams, W.C., Buongiorno, J. and Hu, L.W., 2008. "Experimental investigation of turbulent convective heat transfer and pressure loss of alumina/water and zirconia/water nanoparticle colloids (nanofluids) in horizontal tubes". *Journal of heat transfer*, Vol. 130, pp. 042412–1–7.

Xuan, Y. and Li, Q., 2003. "Investigation of convective heat transfer and flow features of nanofluids". *Journal of Heat Transfer*, Vol. 125, pp. 151–155.

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