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THE MEASUREMENT OF THE ACOUSTIC PARTICLE VELOCITIES INSIDE A RECTANGULAR DUCT USING PARTICLE IMAGING VELOCIMETRY TECHNIQUE

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Abstract. *The overall finality of this work is to contribute to the understanding of combustion process under acoustic field. Previous experiments showed that under strong acoustic waves, at certain conditions, non-premixed combustion behaves like the premixed combustion, which includes the advantages of the premixed combustion regarding the reduced flame length and pollutants emission and keep the safety of non-premixed combustion regarding flashback ignition. The main restriction of the particle image velocimetry (PIV) available equipment is its maximum double frame acquisition rate of 15 Hz, what is not enough to get more than one velocity data during an acoustic wave period. To overcome this limitation it was developed a methodology based on random distributed acquisition data using the probability density function of sine waves. This methodology was used to figure out the acoustic particle velocity inside a rectangular duct at resonance fundamental frequency and forced oscillation. The reliability of the method is validated by comparison of the acoustic particle velocities measured using PIV with the acoustic particle velocities inferred through the measurements of the acoustic pressure along the duct. The method showed to be a reliable tool to measure acoustic particle velocities under the equipment data acquisition rate restriction.*

Keywords: *combustion, pollutants emission, PIV, acoustic particle velocity*

1. INTRODUCTION

The present work has as main motivation the necessity of the comprehension of the dynamic of the thermo acoustic phenomena. Literature about this issue extends back over a century and the quantity of studies is enormous. In fact, several areas in engineering in some moment had to face off one thermo acoustic problem, and it is common that the solution involves innumerous hours of the hard work and if one definitive solution is achieved is in general one motive of general delight. For example, classical problems involving instabilities, like in rocket engine, albeit have been well known has some solution which are neither simple nor easy. In 1990 decade one serious problem was elicited to gas turbine engineering (Keller, 1995). These engines demand one particular configuration that includes one efficiency burner jointly to minimal pollutants reduction without forget of the compactness. One configuration which includes these exigencies is those with swirl and burning lean premixed charge of fuel and oxidizer. In fact, this kind of flames yields high compactness and high volumetric heat release rate. In the other hand, these condition lead to a great facilitator to start thermo acoustic instabilities, actually these operation condition are the perfect ambiance to existence of thermo-acoustic instabilities (Peracchio and Proscia (1998), Lieuwen et al. (1998), Lieuwen et al., 1999).

Considering modeling studies in gas turbine one simple system or even some more complexes has been achieved some kind of success (Cronemyr et al. (1998), Kruger et al. (1999), Fannin (2000)). But, the solution of the problem necessarily collides with the necessity of the knowledge of dynamic of flame, crucial point to solving the problem. It is essential the comprehension of the response of the dynamic flame in terms of the fluctuating heat release, q' and its relationship with velocity fluctuation, u'. This importance is based in the fact that the coupling between them has been pointed as the main responsible for thermo acoustics instabilities. The physic parameter which couple both is the fluctuating mass flow rate, m', once that this mass fluctuation will lead to one fluctuation in mass flow rate in flame zone, which in turn will oscillate the heat release. Thus, u' oscillations are analogous to m' oscillations.

And lastly, equations which describe the combustion are inherently non linear. This means that the construction of the one system with equations which include the term of the combustion and acoustic is a huge challenge. In nowadays is usual the utilization of the simplified models that in general are able to capture only one qualitative scenario of the flame. Considering this limitation is of paramount importance provide reliable experimental results to scientific community that then can develop more reliable numerical models. For this, is necessary the accurate knowledge of the fluctuation of the velocity. This measurement can be made both with intrusive and non intrusive technical experimental. In the non intrusive techniques that as general rule has high resolution and high spatial resolution there is the necessity of the one laser device and one sophisticate processing software, obviously this leads to highest cost. The present work has how main goal contribute to acquire reliable measurements of fluctuations of the velocity using one simple PIV system, frame acquisition rate of 15 Hz, and pressure transducers. These measured are not direct and here will be showed the methodology and the development until achieve values of the fluctuations velocities. In future, these data can be utilized into one model to better describe of the relation input-output (velocity fluctuation – heat release fluctuation).

2. EXPERIENCE DESCRIPTION

The present section starts with the description of whole system experimental. The experimental assembly specification established to acquiring the measurements through PIV technique included the following main requirements:

a) High transparency of the duct wall where the PIV pictures are taken;
b) Low light distortion through the duct walls;
c) Means to flow the working fluid into the duct without leaks;
d) Means to control the seed particles concentration in the working fluid
e) Acoustic pressure plugs along the duct;

One good solution for the experimental assembly is compounded by the loudspeaker with a convergent channel followed by the transparent duct as illustrated in "Fig. 1 b)". The speaker case and the convergent channel are made with plywood while the transparent duct is made with high transparence acrylic. The acrylic duct has constant square cross section with internal side of 68 mm; 1000 mm long and 4 mm of wall thickness. To avoid air leaks in the flanged couplings, between the case and the convergent channel and between this and the acrylic duct it was applied rubber tape of 5 mm thickness and about several screws 30 mm apart along the flange perimeter. Five (5) pressure plugs were installed along to the duct; they follow the British Standard Pipe (BSP) size 3/8". The loudspeaker model WPU 1309 made by Selenium was selected due its high power of 450W RMS and it is a professional woofer of 305 mm external diameter and accurate frequency answer from 47 to 3000 Hz.

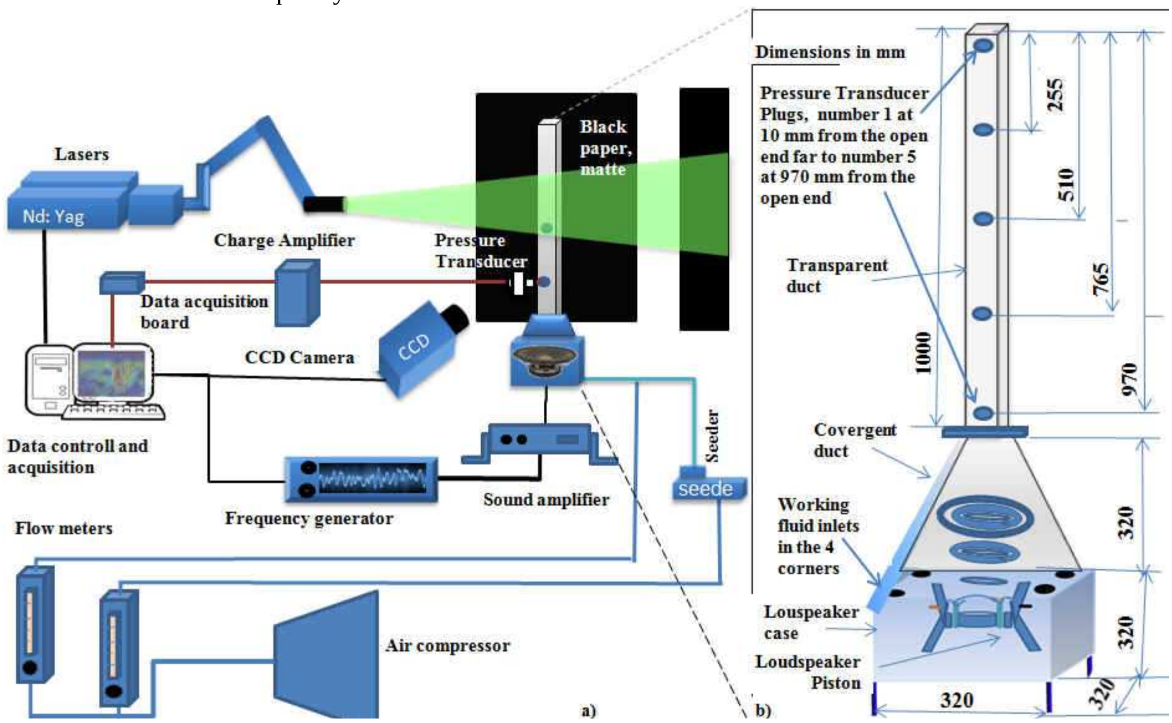

Figure 1. a) General assembly and b) Main duct assembly design

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The general experience assembly accounts with the air supply by an air compressor, the supply air is divided in two lines: one dedicated to the seeder air supply and the other dedicated to the majority of the air flow. Figure 1 a) shows that just after the seeder the lines are splice in one line again and the mixing of the seed particles and the air takes place inside the line, so when this working fluid arrives to the main duct its concentration is already homogeneous. The line that supplies air to the seeder is controlled by a needle flow meter in order to get a fine control of the seed concentration in the working fluid.

On the other hand a computer controls the frequency input and receives the acoustic pressure data. The system that generates the acoustic waves and reads the acoustic pressure is the same used in the Corá (2010) thesis; its main characteristic are: one quartz low pressure sensor with his suitable charge amplifier using a high acquisition rate of 5000 samples per second; the acoustic pressure data is analyzed by fast Fourier transform in the frequency domain from zero up to 2500 Hz.

The PIV equipment installed ran independently of the sound generation and acoustic pressure acquisition. Supplied by LaVision GmbH with an embedded DaVis 7.2 software, the main characteristics of this apparatus are the special CCD camera that accounts with an parallel CCD device that memorizes the data of the first picture, and after to take the second frame, send both to the computer, this device reduces the time between the frames, however the pictures discharge to the memory is limited in about 14.72 double frames per second. The two lasers pulses are synchronized with the pictures shots to obtain the better light intensity in the camera CCD.

Other simple device that showed to be very useful was two opaque black paper sheets, as showed in "Fig. 1 a)", one is used to prevent the laser light to be reflected or scattered in the lab walls and the other is used to maximize the contrast in the image between scattered light by the seed particles and the background.

3. METODOLOGY

3.1 Acoustic particles velocities measurements intrinsic reliability

The first analysis that needs to be performed when acoustic particles velocity are been measured by PIV is to know how closed they are from the instantaneous particle velocity. Suppose that the time between the two pictures in the "Fig. 2" is the equal to the acoustic wave period, then someone will say that the particle position doesn't change and the velocity is null, what is a absurd if the sound is heard and actually the particle velocity is any velocity between the interval from zero up to the maximum acoustic velocity.

It is reasonable to predict that the intrinsic reliability of the PIV technique is inversely proportional to the time between the two frames (dt); however the better way is to develop the relationship that includes de angular acoustic velocity. Starting with the particle position (η') in a sinusoidal wave one can write:

$$\eta' = \frac{u'_{\max}}{w} sen\ wt \tag{1}$$

Where $u'_{\max}$ is the maximum acoustic particle velocity; w is the wave angular velocity and t is the time elapsed since the particle passed in the maximum position.

Using the PIV technique to measure the velocity means to calculate it by the ratio between the displacement and the time elapsed between the two pictures, as showed in the "Fig. 2"; then the acoustic particle velocity measured using PIV technique is:

$$u'_m = \frac{\eta'_2 - \eta'_1}{dt} = \frac{u'_{\max}}{wdt}[sen\ (w(t+dt/2)) - sen\ (w(t-dt/2))] = \frac{u'_{\max}}{wdt}[2\cos(wt) sen(wdt/2)] \tag{2}$$

Where u'_m is the acoustic particle velocity measured using PIV technique.

The instantaneous velocity is $u'_i = u'_{\max}\cos(wt)$ then there is a relative error between the measured particle velocity and the instantaneous velocity quantified by the following relation:

$$Erru'_{m/i} = \frac{u'_i - u'_m}{u'_i} = \frac{\cos(wt) - \dfrac{2\cos(wt)\ sen\ (wdt/2))}{wdt}}{\cos\ (wt)} = 1 - \frac{2sen\ (wdt/2))}{wdt} \tag{3}$$

To approach this error to zero is needed that the ratio of the second term will tend to 1, what means that:

$$sen(\frac{wdt}{2}) \cong \frac{wdt}{2} \tag{4}$$

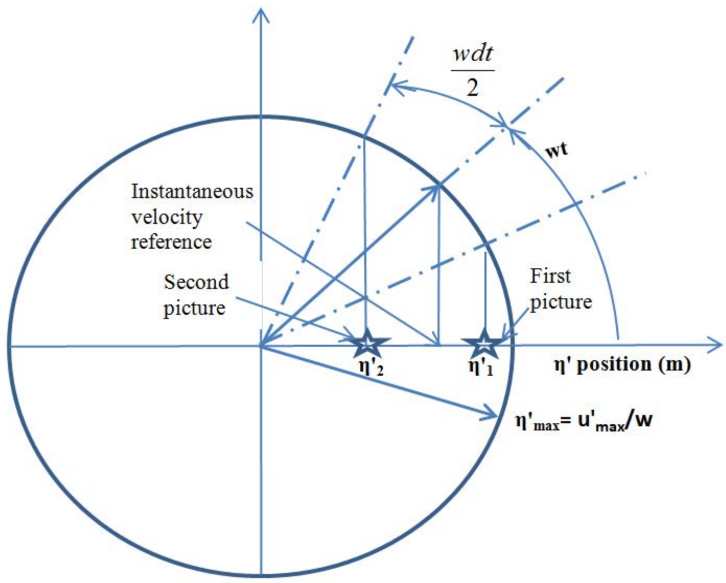

Figure 2. How PIV technique captures an oscillatory movement

This relation expressed in “Eq. 4” is close to true for small angles. For an angle of 30 degrees the error approaches 5%, which means an intrinsic reliability of 95%. Inside this reliability margin the time between pictures “dt” will be limited by:

$$\frac{wdt}{2} \leq \frac{\pi}{6} \Leftrightarrow dt < \frac{1}{6f} \quad (5)$$

Where f is the acoustic wave frequency.

It is notable that the tolerance of the PIV technique regarding the time between pictures is quite good. Equation (5) says that it may be as big as one sixth of the wave period and the intrinsic reliability will be bigger than 95%. This is very important because de variable “dt” has others restriction in the experiences; one is that the seed particles must be allowed to displace between one quarter up to one third of the interrogation window and this time must be enough to achieve the proper light intensity to be detected by the camera CCD in the first picture. For non reactive flows this practical lower limit is about 50 μs, applying this value in the Eq. (4) it means that measurements with 95% of intrinsic reliability will be achieved for acoustic waves up to 3.34 KHz.

3.2 Treatment of the raw particle velocity measurements

If the measured velocities are the instantaneous particle velocities with a high confidence as discussed above they can be handled to indicate the flow and the acoustic particle velocities. The raw measurements are presented as the total particle velocity in a rate that follows the acquisition rate of the two frames; this rate is restricted mainly due the time elapsed to unload the camera CCD data to a file in order to be able to acquire the next double frame. The total particle velocity (u_t) applied to sinusoidal waves can be viewed in this experiment being compounded by a steady flow velocity ($\overline{u}$) and an acoustic oscillation (u') contribution that depends on the time and space by the formula:

$$u_t = \overline{u} + u' = \overline{u} + A\cos(\frac{nx\pi}{L})\cos(\omega t) \quad (6)$$

How the measurements are related to a defined place in the space the acoustic overall amplitude and the spatial variation can be resumed into a local maximum velocity as $u_{\max} = A\cos(nx\pi / L)$, then the local total velocity may be written as:

$$u_t = \overline{u} + u' = \overline{u} + u'_{\max}\cos(\omega t)) \quad (7)$$

If the raw measurements could be performed in a rate bigger than the double of the wave frequency then the Nyquist theorem assures that the acoustic frequency can be obtained accurately, or if the acquisition rate exceeds widely the acoustic wave frequency then the sinusoidal curve could be reconstructed interpolating the measured points; however the PIV equipment available has a maximum double frame acquisition rate of about 14,7 Hz, which is not enough to get 2 points in the same wave period since the acoustic frequencies tested are bigger than 47 Hz. To face this problem it is possible to synchronize the acoustic wave signal with the frames acquisition time as done by Nabavi *et al*. (2007) to get the raw velocity measurement always in the same phase. Other strategy is to accept the random phase shift of the velocity measurement regarding the acoustic wave and treat the measurements by probabilistic means, this strategy was first bring to light by Hann and Greated (1997), in spite of in their experience was used film cameras and autocorrelation was used to figure out the displacements the method can be adapted for this experience context.

The raw measurement set can be handled by using the probability standpoint looking for the mean and variance expectations. The only restriction here is to avoid the synchronicity or entire multiples between the acoustic wave frequency and acquisition rate.

The trivial part is the steady flow velocity, by definition its mean is always the same ($\overline{u}$) regarding the time and its variance is null. The simple sinusoidal part depends on time and its mean and variance are known as null and $u'^{2}_{max}/2$ respectively.

The goal is to extract the flow velocity and the acoustic particle velocities from the raw velocities measurements; this can be made developing the mean and the variance of the probability law, which definitions are, respectively:

$$E[u_t] = \int_{-u'_{max}}^{u'_{max}} z f_U(z) dz = \overline{u}_t \quad \text{and} \quad E[(u_t - \overline{u}_t)^2] = \int_{-u'_{max}}^{u'_{max}} (z - \overline{u})^2 f_U(z) dz = \sigma_t^2$$

Where f_U is the density probability function of the acoustic velocity and z is an integration variable.

The flow velocity can be related to the mean of the raw acoustic velocities readings by

$$\overline{u}_t = E[u_t] = E[\overline{u} + u'] = E[\overline{u}] + E[u'] = \overline{u} \Leftrightarrow \overline{u} = \overline{u}_t \tag{8}$$

The acoustic particle velocity in its turn is recognized by the variance of the raw reading as per:

$$\sigma_t^2 = E[(u_t - \overline{u}_t)^2] = E[(\overline{u} + u' - \overline{u})^2] = E[(u')^2] = \frac{u'^{2}_{max}}{2} \Leftrightarrow u'^{2}_{max} = 2\sigma_t^2 \tag{9}$$

It's practical to use the root mean square (rms) of the raw velocities measures; it can be related to the acoustic velocity rms, sometimes called acoustic velocity effective value as well, by:

$$u'_{rms} = \frac{u'_{max}}{\sqrt{2}} = \frac{\sqrt{2\sigma_t^2}}{\sqrt{2}} = \sigma_t = u_{t\,rms} \tag{10}$$

Therefore the conclusion is that flow velocity is equal to the mean of the raw measurement and the effective acoustic particle velocity value is equal to the root mean square of the raw measurements.

3.3 Acoustic waves in ducts, acoustic velocities inferred from acoustic pressures along the strait duct

The finality of this analysis is to infer the acoustic velocities inside the duct from the measured acoustic pressures measured along the duct. Afterwards these deduced acoustic velocities will be used as a reference to evaluate the reliability of the measured acoustic velocities using PIV technique.

In the subtitle 3.2 it is seen the acoustic wave in its temporal movements, it is time to look at its space characteristics. This space must be a stated space into the world where the acoustic wave takes place. In general this place is recognized by its environment and the boundary condition. The simplest environment is a homogeneous element, and the simplest wave characterization is a wave radiating spherically without barriers to reflect, absorb or diffract the acoustic waves.

In this experience as showed in the "Fig. 1 b)" the environment where the acoustic waves take place is for practical effects the air, the lithe amount of particulate is negligible. The boundary conditions are the rigid walls of the duct which has its end open to free space.

The assumption to carry out this analysis is that wavelengths (λ) of the acoustic waves in the duct is much greater than the cross-sectional dimensions of the duct, the duct walls are perfectly rigid and the energy losses from fluid viscosity and thermal conductivity are negligible, therefore it is possible to assume that the acoustic velocity is uniform across a duct cross section, i.e., the acoustic waves are plane waves; therefore its equation depends on longitudinal length (x) only. So the expression for the acoustic pressure along the duct, in one plane wave of wavelength λ after Morse and Ingard (1968), is written out as:

$$P'_i(x) = A_i sin\left(\frac{2\pi(x-x_{si})}{\lambda_i}\right) \tag{11}$$

Where $P'_i(x)$ is the acoustic pressure due the contribution of the wave whose wavelength is λ_i; the coordinate "x" has its origin at the duct open end as illustrate in the "Fig. 3". A_i is the acoustic pressure amplitude and x_{si} is the wave phase shift in meters regarding the origin of the coordinate "x" , with is used for computation convenience and defined by its proportionality with the phase shift angle in radians $\frac{2\pi x_{si}}{\lambda_i}$.

With the above assumptions about the phenomena Morse and Ingard (1968) states, "the air in the open end may be considered, for long wavelengths, to be a piston of zero mass, radiating sound out into the free space as well as reflecting some back down to the duct. To this approximation it's possible to analyze the acoustic impedance at the open end, the ratio of the mean pressure to the mean acoustic velocity at the open end. The reactive part of this impedance would produce reflection back to the driving loudspeaker; the resistive part would represent energy lost from the duct by being radiated out of the open end". Herein won't be developed the acoustic impedance of the open end, but taking the hint about radiation and reflection the total acoustic pressure will be build up by the sum of the waves being radiated by the loudspeaker plus the waves being reflected at the open end. In the experiences performed the process to infer the acoustic velocity was qualified in two cases:

a) In the experiences where the input frequency doesn't match the resonance frequency – in this case the raw and the contribution due the input frequency to the total acoustic pressure are very close; therefore the total acoustic pressure can be represented only by one wave being radiated by the loudspeaker and other being reflected by the open end back down to the duct through the equation:

$$P'_t(x,t) = A_1 sin\left(\frac{2\pi(x-x_{srad})}{\lambda_i}\right)\cos(w_i t) + A_2 sin\left(\frac{2\pi(x-x_{sref})}{\lambda_i}\right)\cos(w_i t + \Phi_r) \tag{12}$$

Where Φ_r is the wave phase shift due the reflection.

Considering that it isn't the resonance condition then the phase shift is different than zero. The temporal mean square acoustic pressure a distance x from the open end is:

$$\begin{aligned}&\left(P'_{t_{rms}}\right)^2 = E[P'_t(x)^2] = \\ &E\left[\left(A_1 sin\left(\frac{2\pi(x-x_{srad})}{\lambda_i}\right)\right)^2 (cos(w_i t))^2 + \right.\\ &\left. 2A_1 sin\left(\frac{2\pi(x-x_{srad})}{\lambda_i}\right) A_2 sin\left(\frac{2\pi(x-x_{sref})}{\lambda_i}\right) cos(w_i t)\, cos(w_i t + \Phi_r) + \left(A_2 sin\left(\frac{2\pi(x-x_{sref})}{\lambda_i}\right)\right)^2 (cos(w_i t + \Phi_r))^2\right]\end{aligned} \tag{13}$$

Taking into consideration that $\mathrm{E}[(\cos(\mathrm{w_i t}))^2] = \mathrm{E}[(\cos(\mathrm{w_i t} + \Phi_r))^2] = \frac{1}{2}$ and $\mathrm{E}[\cos(\mathrm{w_i t})\cos(\mathrm{w_i t} + \Phi_r)] = \frac{1}{2}\cos(\Phi_r)$; substituting these means into "Eq. 13" it yields:

$$\left(P'_{t_{rms}}\right)^2 = \frac{1}{2}\left|A_1 \sin\left(\frac{2\pi(x-x_{srad})}{\lambda_i}\right)\right|^2 + \frac{1}{2}\left|A_2 \sin\left(\frac{2\pi(x-x_{sref})}{\lambda_i}\right)\right|^2 + A_1 \sin\left(\frac{2\pi(x-x_{srad})}{\lambda_i}\right) A_2 \sin\left(\frac{2\pi(x-x_{sref})}{\lambda_i}\right)\cos(\Phi_r) \tag{14}$$

The unknown amplitudes and phase shift are determined by measuring the rms acoustic pressure in the duct as a function of x.

Once the total acoustic pressure is achieved in "Eq. 12" then the total particle acoustic velocity can be obtained by the space derivation of the acoustic pressure, which uses the first order equation of motion, leaving:

$$u_t{}'(x,t) = \frac{\delta\left(\frac{1}{w\bar{\rho}} P'_t(x,t)\right)}{\delta x} \tag{15}$$

Where $\bar{\rho}$ is the average mass density and w is the angular wave velocity.

Substituting "Eq. 13" in "Eq. 12" and afterwards in the "Eq. 15" the total acoustic particle velocity can be inferred through the equation:

$$u'_t(x,t) = \frac{1}{c\bar{\rho}}\left(A_1 cos\left(\frac{2\pi(x-x_{srad})}{\lambda_i}\right)\cos(w_i t) + A_2 cos\left(\frac{2\pi(x-x_{sref})}{\lambda_i}\right)\cos(w_i t + \Phi_r)\right) \tag{16}$$

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Where c is the speed of the sound in the medium.
The mean square particle acoustic velocity as a function of x is:

$$\left(u'_{t\,rms}(x)\right)^2 = \frac{1}{c\bar{\rho}}\left\{\frac{1}{2}\left|A_1\cos\left(\frac{2\pi(x-x_{srad})}{\lambda_i}\right)\right|^2 + \frac{1}{2}\left|A_2\cos\left(\frac{2\pi(x-x_{sref})}{\lambda_i}\right)\right|^2 + A_1\cos\left(\frac{2\pi(x-x_{srad})}{\lambda_i}\right)A_2\cos\left(\frac{2\pi(x-x_{sref})}{\lambda_i}\right)\cos(\Phi_r)\right\} \quad (17)$$

In the “Fig. 3” all these steps were performed for an input frequency of 144 Hz, the raw and input frequency rms acoustic pressure values are coincident, the radiated (P’1(x)) and the reflected (P’2(x)) waves are showed individually, the rms of the total acoustic pressure was calculated for the best fit with the rms pressure measured values, afterwards the rms acoustic particles velocities curve was calculated through “Eq. 17” for future comparison with the acoustic particle velocities measured using PIV.

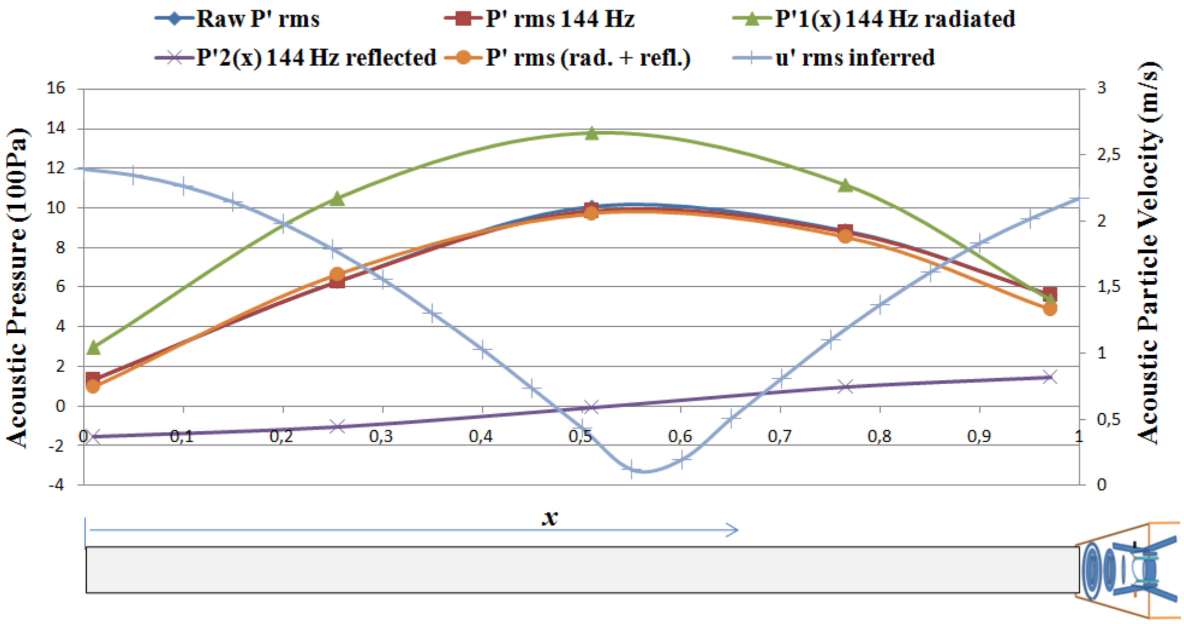

Figure 3. Raw acoustic pressure rms measured along the duct and inferred acoustic velocity related to an input frequency of 144 Hz

In the experiences where the input frequency matches the resonance duct frequency the procedure is quite the same, if in one hand the number of wave terms increases according with the relevant contributions of the harmonics on the other hand there are no temporal phase shift due the resonance condition. So the total acoustic can be written out as:

$$P'_t(x,t) = \sum_{i=1}^{n}\left(A_{i1}sin\left(\frac{2\pi(x-x_{israd})}{\lambda_i}\right)\cos(w_i t) + A_{i2}sin\left(\frac{2\pi(x-x_{isref})}{\lambda_i}\right)\cos(w_i t)\right) \quad (18)$$

The temporal mean square acoustic pressure a distance x from the open end is:

$$\left(P'_{trms}\right)^2 = E[P'_t(x)^2] = E\left[\left(\sum_{i=1}^{n}\left(A_{i1}sin\left(\frac{2\pi(x-x_{israd})}{\lambda_i}\right)\cos(w_i t) + A_{i2}sin\left(\frac{2\pi(x-x_{isref})}{\lambda_i}\right)\cos(w_i t)\right)\right)^2\right] \quad (19)$$

Taking into consideration that $E[(\cos(w_i t))^2] = \frac{1}{2}$ and that the mean of the cross products $E[\cos(w_i t)\cos(w_j t)] = 0$; these means are substitute into “Eq. 19” to yield the mean square acoustic pressure equation:

$$\left(P'_{trms}\right)^2 = \frac{1}{2}\sum_{i=1}^{n}\left|A_{i1}sin\left(\frac{2\pi(x-x_{israd})}{\lambda_i}\right) + A_{i2}sin\left(\frac{2\pi(x-x_{isref})}{\lambda_i}\right)\right|^2 \quad (20)$$

The unknown amplitudes and phase shifts are determined by measuring the rms acoustic pressure in the duct as a function of x.

Once the total acoustic pressure is achieved in "Eq. 18" then the total acoustic particle velocity can be obtained by the space derivation of the acoustic pressure, which uses the first order equation of motion, leaving:

$$u_t{}'(x,t) = \frac{\delta\left(\sum_{i=1}^{n}\left(\frac{1}{w_i\bar{\rho}}P'_t(x,t)\right)\right)}{\delta x} \tag{21}$$

Where $\bar{\rho}$ is the average mass density and w_i is the angular wave velocity.

Substituting "Eq. 18" in "Eq. 21" and afterwards performing the derivation then the total acoustic particle velocity can be inferred through the equation:

$$u'_t(x,t) = \frac{1}{c\bar{\rho}}\sum_{i=1}^{n}\left(A_{i1}cos\left(\frac{2\pi(x-x_{israd})}{\lambda_i}\right) + A_2cos\left(\frac{2\pi(x-x_{isref})}{\lambda_i}\right)\right)\cos\left(w_i t\right) \tag{22}$$

Where c is the speed of sound in the medium.

The mean square acoustic velocity as a function of x is:

$$\left(u'_{t\,rms}(x)\right)^2 = \frac{1}{2c\bar{\rho}}\sum_{i=1}^{n}\left|A_1cos\left(\frac{2\pi(x-x_{srad})}{\lambda_i}\right) + A_2cos\left(\frac{2\pi(x-x_{sref})}{\lambda_i}\right)\right|^2 \tag{23}$$

4. RESULTS AND DISCUSSION

4.1 Experience results

The first task was to characterize in time and space the acoustic pressure response along de duct regarding the input frequency in the loudspeaker. For each input frequency the pressure transducer acquires the acoustic pressure in a defined position. This reading is performed during 2 or 3 seconds and it is the raw information about acoustic pressure, which shall be considered the more valuable information than any other transformed or derived pressure characteristic, such as the root mean square (RMS) or others. In each of the 5 position prepared to receive the dynamic pressure sensor the acoustic pressure answer was captured for input frequencies ranging from 47 up to 500 Hz with an interval of 1 Hz. The sound amplifier was set in a level 6 of load which can be 10 at maximum.

The results for the second position, beginning with the first position close to the open end, showed that the resonance fundamental frequency in the duct is 161 Hz, due the clearly maximum summit in the fundamental resonance frequency showed in the "Fig. 4 a)", and that its 2 first harmonics, in 322 and 483 Hz, achieve secondary summits as well.

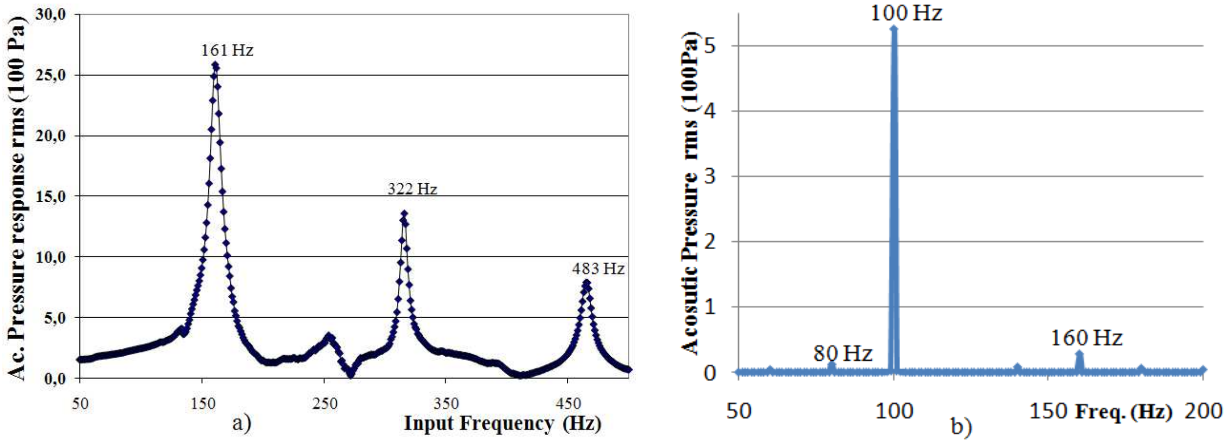

Figure 4. a) Acoustic pressure response for input frequencies ranging from 47 up to 500 Hz, position 2 in the duct; and b) Fourier transform of the acoustic pressure response from the time domain to frequency domain to an input frequency of 100 Hz

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Along the duct the acoustic pressure measurement showed that the acoustic pressure tends to zero at the open end while increases in the direction of the loudspeaker. Nearby the duct resonance frequency the acoustic pressure have its maximum values close to the middle of the duct as expected and showed in "Fig. 5".

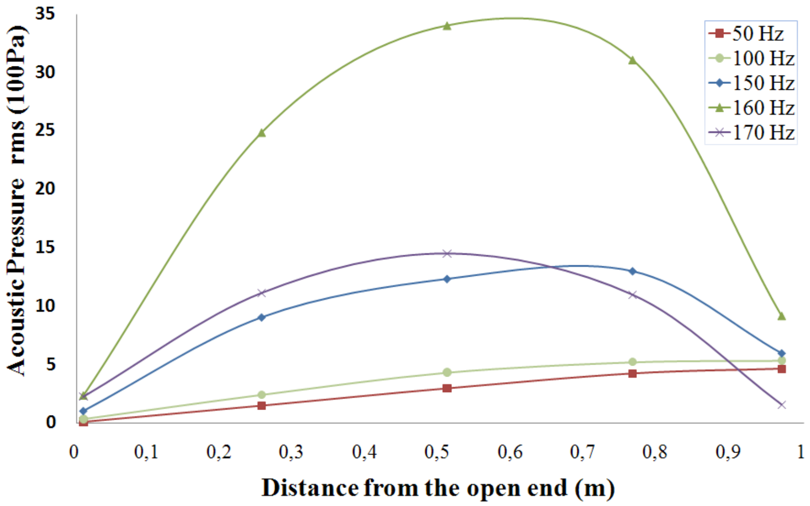

Figure 5. Acoustic Pressure Root Mean Square (rms) along the duct due input frequencies of 50 up to 170 Hz

Taking into consideration that the objective of this paper is evaluate the reliability of the PIV acoustic velocities measures, it is reasonable to choose the frequency that showed the simplest behavior as it is for the 100 Hz input frequency as showed in "Fig. 5". The Fourier transform of the raw pressure readings for input frequency of 100 Hz shows the dominance of the response in the same frequency, however same traces of 160 and 80 Hz, the fundamental and half the fundamental resonance frequencies, appear as well as showed in "Fig. 4 b)"; they will be considered in the acoustic wave.

The measurement of the acoustic velocity using PIV used the random strategy to acquire the instantaneous velocities. The particle movement can be qualified as an oscillatory movement; therefore the root mean square of the velocities was computed to represent it. Figure 6 shows a typical rms acoustic velocity result, the zero vertical position in the figure is located 35 mm below the open end; it's notable that the cross section velocities, along the horizontal position, are uniform as presumed in the methodology development.

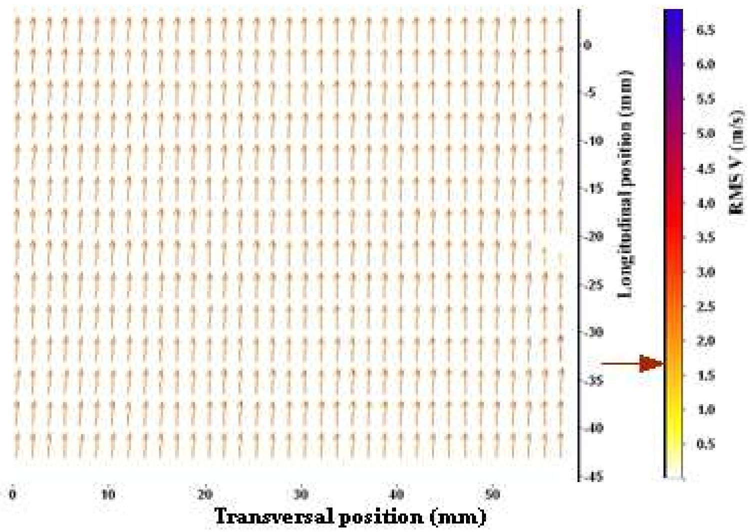

Figure 6. Typical result of the acoustic particle measurement performed as the root mean square of a data set of 40 instantaneous velocities measurement. The vertical position (mm) is a coordinate along the duct.

Following the methodology it's needed to deduce the acoustic velocity from this acoustic pressure response. The inferred acoustic velocity was done for the frequency of 100 Hz plus 160 and 80 Hz, which contributions are in the order of 5% and 3% respectively. Figure 7 shows the raw pressure rms and the 100 Hz contribution almost coincident, due the low contributions of 160 and 80 Hz in the pressure, however 160 Hz contribution regarding acoustic velocities in the duct extremities are more relevant because the in these location the 160 Hz wave achieves its maximum particle velocities.

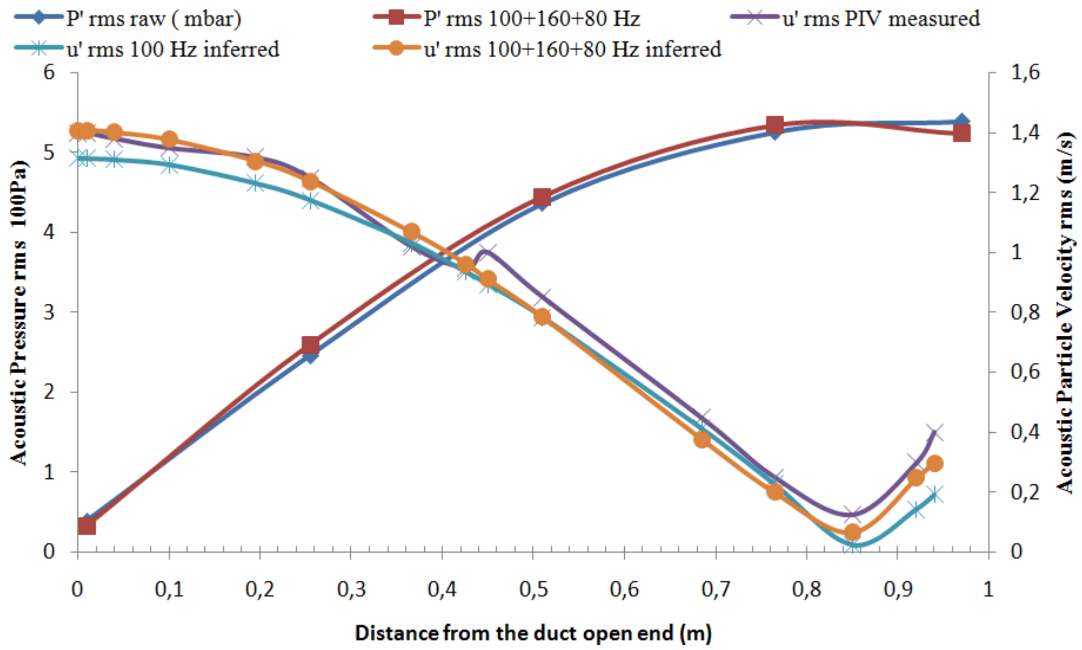

Figure 7. Measured PIV versus inferred acoustic particle velocities at 100 Hz input frequency

4.2. Discussion about the reliability of the acoustic particle velocities measurements using PIV

The reliability of the measurement using PIV as put in the methodology has the contribution of the intrinsic reliability, which depends on the frequency and the time between pictures as stated in "Eq. 3":

$$Erru'_{m/i} = 1 - \frac{sen\,(wdt/2))}{wdt/2}$$

The measurements were performed using time between pictures of the order of 150 µs, applying this value in the "Eq. 3" the result is: 0.0004; therefore the error is insignificant as expected for low frequency waves.

The reliability compared with the analytic acoustical solution can be figured out by the mean of the differences between them. Using the 3 main waves contributors (100, 160 and 80 Hz) as showed in the "Fig. 9" it's visible that for the higher velocities the results are very close while for the smaller velocities the differences tend to increase. The mean of the differences between the measured and inferred values of the acoustic velocity for the first half of the duct is 0.030, indicating 97% of reliability; while for the second half the mean of the differences is 0.34, indicating a poor reliability of 66%.

There is a third contribution on the reliability of the PIV measurements that takes into account the number of double images up to the stabilization of the rms acoustic velocity measured. In this experience the stabilization of the rms acoustic velocity occurred after about 40 double images, therefore in order to obtain reliable results it was acquired 60 double frames for each set of measures.

5. CONCLUSIONS

The present work has showed that acquiring acoustic pressure data, from some points along a duct, and using it in the light of the acoustical theory to infer the acoustic particle velocities makes the measurements of the acoustic particle velocities more reliable. The main achievements were:

- It was possible to assess the reliability of the acoustic particle velocity measurements using PIV technique in despite of the complexity regarding the acoustic waves behavior.

- The convergent duct between the loudspeaker and the strait transparent duct used in the configuration increased the difficulties to achieve the analytical solution for the acoustic pressure and particle velocity pressure, to the purpose of assess the reliability of the PIV measurement system could be better to use a strait duct direct coupled to the loudspeaker.
- The acoustic particle velocities values were measured and are in good concordance with the acoustical theory.
- The main achievement was the good quality of the images, which was essential during all the process to figure out the acoustic particle velocity.
- It is possible to visualize the particle oscillatory movement even taking pictures in an acquisition rate much smaller than the wave frequency.

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7. RESPONSIBILITY NOTICE

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