



DIMENSIONLESS HIGH TEMPERATURE SUPERCONDUCTING (HTS) DC CABLE MODEL

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Abstract. *The recent increase in distributed power generation and the need to transfer large amounts of power in applications where cabling weight and space are a major issue has increased the interest in High Temperature Superconducting (HTS), transmission DC Cables. HTS DC Cables, have the potential to address the need for more efficient transmission and their usage is expected to increase in the future. Thermal modeling of HTS DC Cables is a critical tool to have in order to better understand and characterize the operation of such transmission lines. A physical model, (VEM) Volume Elements Methodology, is employed to obtain a system of ordinary differential equations with time as the independent variable which combines principles of classical thermodynamics, momentum equation and heat transfer. Writing all the materials properties as temperature and pressure local functions in volume elements, dividing the problem domain in N small volumes, in both R and Z directions, generated a $9 \times N$ and $2 \times N$ differential equations to temperature and pressure solved for Runge – Kutta 5th order method, discretized in space. As results, dimensionless temperature, pressure and puning power profiles are determined along a Superconducting HTS DC Cable, Based on Realistic Correlations and the model accounts heat transfer by conduction, convection and radiative transfer, useful tool for simulation, design and your optimization.*

Keywords: *High Temperature Superconducting DC Cables, Volume Element Methodology, Dimensionless Model, Thermodynamic Analysis.*

1. INTRODUCTION

Superconducting High Superconducting Transmission HTS - DC Cables are very important study, through the increase the demand to power generation and more efficient distribution grids. Conventional power distribution grids have a high costs effectiveness when compared with the use of superconductor transmission cables. However it is expected that in a near future, difficulties associated with the construction of new cable tunnels or the retrofit of larger diameter cables in the existing underground of very populated areas will overcome the cost disadvantages associated with the installation of superconductor lines as related in “Cho, *et al.*, 2006”. The high power density in a superconducting cable is a necessary feature for solving above stated challenges, however, according to “Chowdhuri, *et al.*, 2005”, another important advantage of a superconducting cable is that it can transport the same amount of electric power as conventional lines, but at lower voltage level. Since superconducting lines must operate at cryogenic temperatures, the design of such transmission grids requires the installation of cryogenic cooling units capable of providing sufficient cooling to keep the transmission superconductor cables at its optimum operating temperature. For long transmission lines, the thermal losses become an important parameter that must be predicted and controlled, however there is a limited number of available models in the literature as “Demko, *et al.*, 2001”, to predict the thermal behavior of superconducting cables.

In a recent paper “Ordonez, *et al.*, 2013”, present a methodology to pressure drop calculations, based on realistic correlations that account for the wavy nature of coolant channels, “Hammons, *et al.*, 2012”, described the DC Cables as an efficient solution for bulk power transmission especially of renewable energy, “Rodrigo, *et al.*, 2012”, employed a 1m long model cable rated at 1 kV DC Cable. “Souza, *et al.*, 2011”, proposed a mathematical model to predict the temperature profile in DC Cable with nine volume elements, “Demko and Hassenzahl (2011)”, proposed a nitrogen refrigeration stations positioned every 10 and 20 Km to 23kW and 30 bar pressure DC Cable, “Hamabe, *et al.*, 2011”, constructed a 20m – class DC SC – PT with thirty nine layers BI 2223 HTS, “Wang, *et al.*, 2011”, present a new approach for design of DC HTS cable for minimizing the loss as small as possible, “Yamaguchi, *et al.*, 2011”, utilized a

iron – steel cryogenic pipe power transmission line applied to 200m DC Cable, “Kephart, *et al.*, 2011”, applied high temperature superconducting to degaussing system in USS HIGGINS Ship. “Golebiowski and Zareba, (2011)”, proposed a transient thermal field analysis in a futuristic polymeric DC Cable; “Jonhson, *et al.*, 2011”, study the impact of superconducting cables on the dynamic response of current transformers, further the papers “Ciazynski and Turck, (1993)”, “Cho, *et al.*, 2006” and “Grant, *et al.*, 2007”, and the outthers references.

In present work, thermodynamic analysis, combination first thermodynamics law and momentum equation, is proposed for the determination of the dimensionless temperature, pressure and pumping power profiles along a superconducting cable, useful tool for optimization of HTS DC Cable. The solution domain is based on the Volume Elements Methodology (VEM), described in “Vargas, *et al.*, 2005”, in both r and z direction and the energy equation is applied to each volume element. Each layer is modeled with a different volume element, generated a $9 \times n$ and $2 \times n$ differential equations to solved for Runge – Kutta 5th order method and all materials properties are described as temperature and pressure functions, based on realistic correlations. Two Helium cooling channels are accounts in model analysis by convective heat transfer, “Hendricks, *et al.*, 1975”, “McCarty and Stewart, (1962)”, and NIST home page. The results containing temperature, pressure and pumping power profiles analysis to one cell and array DC Cable length, useful tool optimization HTS DC transmission Cables.

2. MATHEMATICAL MODEL

Based on DC Cable model, “Figure 1”, as described in “Souza, *et al.*, 2011”, “Buiar and Vargas, (2012)”, “Ordenez, *et al.*, 2013”, and “Vargas, *et al.*, 2005” (Volume Element Method – VEM), the solution domain is divided in small Volume Elements (VE), in both r and z direction where the energy equation (first law of thermodynamics), is applied to each VE. Momentum equation is applied in helium cooling channels gas (VE1 and VE4), by pressure drop calculus as showed in Fig. (1) the schematic diagram of the problem geometry. Each layer is modeled with a different VE. In the current version of the model, nine layers are considered. These layers are: internal Helium channel (VE1), stainless steel structural pipe (VE2), superconducting cable (VE3), external annular Helium channel (VE4), stainless steel (VE5), Mylar insulation (VE6), vacuum (VE7), stainless steel (VE8) and Mylar (VE9).

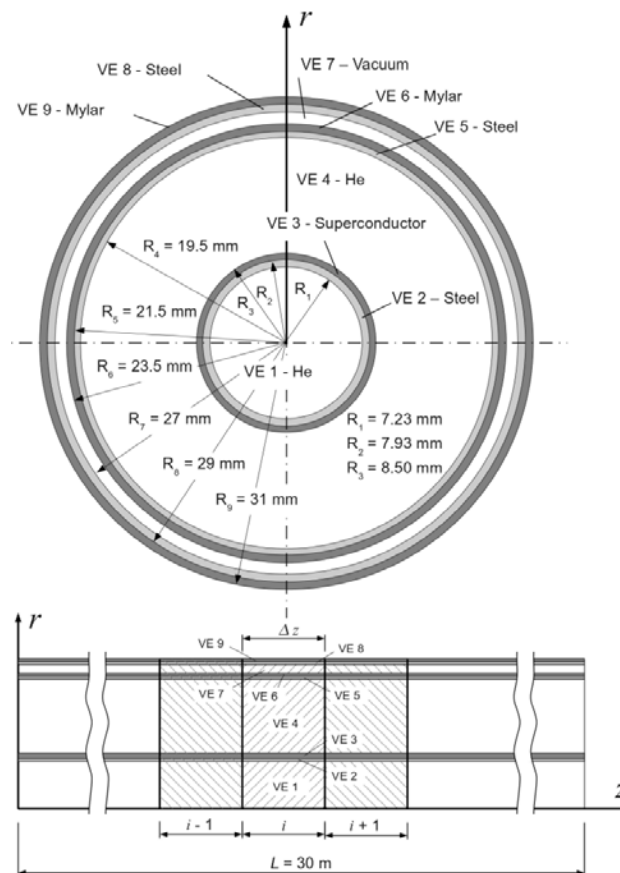


Figure 1. Schematic diagram DC Cable. “Souza, *et al.*, 2011”

2.1 Dimensionless Model

Based on dimensional HTS DC Cable Model, are defined dimensionless variables as specific heat, heat transfer coefficient, thermal conductivity, mass, mass flow rate, scale time and global heat transfer and writing dimensionless groups to time, temperature and pressure, given by:

$$\tau = \frac{t}{t_{esc}} \quad (1)$$

$$\theta = \frac{T_i}{T_0} \quad (2)$$

$$P = \left[\frac{2p}{\rho V^2} \right] \quad (3)$$

Where t is the time, t_{esc} is the scale time, τ is the dimensionless time, T is the temperature, T_0 is the ambient temperature, θ is the dimensionless temperature, ρ is the density, p is the pressure, V the mean velocity in channel section and P is the dimensionless pressure. To obtain the dimensionless temperature profile, using the Eq. (1) and Eq.(3), applied in DC Cable model, generate a differential equations system to temperature profile, given by:

- Helium Cooling Channels:

$$\frac{d\theta_1}{d\tau} = H_1[\theta_1^i - \theta_2^i] + \dot{M}_1[\theta_1^{i-1} - \theta_1^i] \quad (4)$$

$$\frac{d\theta_4}{d\tau} = \{H_4[\theta_3^i - \theta_4^i] - H_5[\theta_4^i - \theta_5^i] + \dot{M}_4[\theta_4^{i-1} - \theta_4^i]\} \frac{1}{M_4} \quad (5)$$

Where θ is dimensionless temperature, τ is the dimensionless time, H is the dimensionless heat transfer coefficient, \dot{M} is the dimensionless mass flow rate and M is the dimensionless mass.

- Stainless Steel:

$$\frac{d\theta_2}{d\tau} = \left\{ -H_1[\theta_1^i - \theta_2^i] - \tilde{U}_{23}[\theta_2^i - \theta_3^i] + K_2[2\theta_2^i - \theta_2^{i-1} - \theta_2^{i+1}] \right\} \frac{1}{M_2 C_2} \quad (6)$$

$$\frac{d\theta_5}{d\tau} = \left\{ H_5[\theta_4^i - \theta_5^i] - \tilde{U}_{56}[\theta_5^i - \theta_6^i] + K_5[2\theta_5^i - \theta_5^{i-1} - \theta_5^{i+1}] \right\} \frac{1}{M_5 C_2} \quad (7)$$

$$\frac{d\theta_8}{d\tau} = \left\{ \tilde{Q}_{7,rad} - \tilde{U}_{89}[\theta_8^i - \theta_9^i] + K_8[2\theta_8^i - \theta_8^{i-1} - \theta_8^{i+1}] \right\} \frac{1}{M_8 C_2} \quad (8)$$

Where H , θ , C , K , M are the same dimensionless variables, \tilde{U} is the global heat transfer coefficient, and $\tilde{Q}_{7,rad}$ is the dimensionless radiative heat transfer, given by:

$$\tilde{Q}_{7,rad} = \frac{\dot{Q}_{7,rad}^i}{\dot{m}_{ref} c_{p,He} T_0} \quad (9)$$

- Conductor:

$$\frac{d\theta_3}{d\tau} = \left\{ -H_4[\theta_3^i - \theta_4^i] + \tilde{U}_{23}[\theta_2^i - \theta_3^i] + K_3[2\theta_3^i - \theta_3^{i-1} - \theta_3^{i+1}] - \tilde{G}_3 \right\} \frac{1}{M_3 C_3} \quad (10)$$

Where H , θ , C , K and \tilde{U} are the same dimensionless variables and \tilde{G} is the dimensionless generation contribution term.

- Mylar:

$$\frac{d\theta_6}{d\tau} = \left\{ \tilde{Q}_{6,rad} + \tilde{U}_{56}[\theta_5^i - \theta_6^i] - K_6[2\theta_6^i - \theta_6^{i-1} - \theta_6^{i+1}] \right\} \frac{1}{M_6 C_6} \quad (11)$$

$$\frac{d\theta_9}{d\tau} = \left\{ \tilde{U}_{89}[\theta_8^i - \theta_9^i] - \tilde{U}_{9w}[\theta_9^i - \theta_w^i] - K_9[2\theta_9^i - \theta_9^{i-1} - \theta_9^{i+1}] \right\} \frac{1}{M_9 C_6} \quad (12)$$

Where θ , C , K , \tilde{U} , are the same dimensionless variables in others Eqs. $\tilde{Q}_{6,rad}$, the dimensionless radiative is described as:

$$\tilde{Q}_{6,rad} = \frac{\dot{Q}_{6,rad}^i}{\dot{m}_{ref} c_{p,He} T_0} \quad (13)$$

2.2 Pumping Work

The parametrical analysis was performed in gas channels, using mass flow as the changing in process, obtained:

$$\dot{W}_p = \frac{\dot{m}}{\rho} \Delta p \quad (14)$$

Integrating the pressure profile are obtained as:

$$p = p_0 - \frac{2fL\rho^i(v^i)^2}{D_h} \quad (15)$$

Using the pressure gradient profile and the fluid mass flow rate given by $\dot{m} = \rho v \pi D^2 / 4$, the pumping power is calculated as follows:

$$\dot{W}_p = \frac{\pi \rho f L v^3 D}{2} \quad (16)$$

Where p is obtained from Eq. (15) along the length dc cable, ρ the density, f is the friction factor, L is the DC Cable length and V the mean velocity in channel section.

2.2.1 Dimensionless Pumping Work

Consider the VE1 configuration shown in “Figure 1”. The Helium channel has a length L in z direction, is surrounded by VE2 at a temperature T_2 . A stream of single-phase fluid is pumped into the tube at a known inlet temperature, the transient balance of energy in the control volume defined by the internal surface of the duct requires is described in Eq. (4). The pressure drop along the tube is described in pressure gradient profile and the pumping power in Eq. (14). Using Eqs. (15) and pressure gradient profile, the dimensionless pumping power constraint becomes:

$$\tilde{W}_{p,i} = \dot{W}_p \frac{\rho^2 L_{ref}^4}{\dot{m}_{ref}^3} = \frac{32}{\pi^2} \dot{M}^3 f \frac{\tilde{L}}{\tilde{D}_i^5} \quad (17)$$

Where $(\tilde{L}, \tilde{D}) = (L, D) / D_{ref}$. The dimensionless fluid mass flow rate is defined as $\dot{M} = \dot{m} / \dot{m}_{ref}$, where L_{ref} and \dot{m}_{ref} are a reference length and a reference mass flow rate, respectively. The index i , present the local channel analysis, in this case $i=1$ or $i=4$.

2.3 Electrical Work:

Based on Ohm's law and Wiedemann – Franz law, for the relationship between thermal conductivity and electric resistivity, see (Bejan, 1982), the heat generation associated with the current transmission is given by:

$$\dot{Q}_{gen}^i = \frac{y^2 k_c A_b}{L_0 \Delta z T_3^i} \quad (18)$$

Where y is the voltage, T is the temperature, A_b is the sectional area, k_c the conductor thermal conductivity, constant $L_0 = 2.45 \times 10^{-8} \text{ (W A}^{-1} \text{K}^{-1})^2$ and L the DC Cable length. The units for Voltage (volt), current (ampere) and resistance (ohm), has the similar signification with power (watt), energy and work (joule), in units, given by:

$$\Omega = \frac{W}{m^4 K^2}; V = \frac{W}{m^2 K} \quad (19)$$

Where Ω - ohm, W – watt, K – Kelvin, m – meter and V – volt unit. Create the dimensionless variable $\tilde{W}_{electric}$ is defined as:

$$\tilde{W}_{electric} = \frac{\dot{W}_{electric}}{\dot{m}_{ref} c_p T_0} = \left[\left(\frac{y^2 k_c A_b}{L_0 L T_3^i} \right) \left(\frac{1}{\dot{m}_{ref} c_p T_0} \right) \right] \quad (20)$$

Where y , k_c , A_b , T and constant L_0 , are the same parameters showed in Equation (18). Create in the Eq.(20) the dimensionless groups $\Psi = \left(\frac{y^2}{L_0 T_0 T_3^i} \right)$; $\Xi = \frac{\dot{m}_{ref} c_p L}{k_c A_b}$, the dimensionless electric power is given by:

$$\tilde{W}_{electric} = \frac{\dot{W}_{electric}}{\dot{m}_{ref} c_p T_0} = \left[\frac{\Psi}{\Xi} \right] \quad (21)$$

3. RESULTS – BY FORTRAN

Insert my results in fortran program with modifications.

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