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A NUMERICAL STUDY OF THE INFLUENCE OF ELASTICITY ON EXTERNAL FLOWS OF VISCOPLASTIC MATERIALS

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Abstract. *The main goal of the current work is the numerical investigation of external flows of viscoplastic materials taking into account the influence of elasticity in the development of material unyielded regions. Such an elastic addition to the classical purely viscous modeling of viscoplastic flows can be viewed as an attempt to better fit some viscoplastic experimental data recently published in the literature. According to this works, elastic effects can be detected in material regions subjected to very low shear-rates, a fact that a priori limits the capability of classical viscoplastic models to describe accurately those unyielded regions. The mechanical modeling herein considered makes use of the incompressible continuity and motion equations, coupled with the elasto-viscoplastic material equation introduced by De Souza Mendes, 2012. This constitutive equation is based on a modification of the Oldroyd-B model making both the fluid relaxation time and the viscosity function sensitive to the break-down of the material microstructure – a collapse that occurs once the stress level exceeds the material yield limit. This is accomplished incorporating to the modeling a scalar structure parameter that measures the structuring level of the material. In fully-structured (elastic) material regions, it assumes the value one, while in fully-unstructured (viscous) material regions, it is equals to zero. The mechanical model briefly described above is approximated by a stabilized finite element methodology, a multifield GLS-type method in terms of the extra-stress tensor, the velocity vector and the pressure field. Taking advantage of the good stability features of the GLS method, all computations are performed using an equal-order combination of bi-linear Lagrangian interpolations – a very attractive choice from the computational point-of-view. Preliminary results are obtained investigating the external flow of viscoplastic materials over a flat plate, focusing the investigation on the development and topology of the material yield regions around the plate.*

Keywords: *elasto-viscoplastic fluid flow, viscoplastic materials, external flow, finite element approximations, GLS method*

1. INTRODUCTION

The study of non-Newtonian fluids flows is of great interest in industrial processes, since most industrial fluids exhibit this behavior. Can be cited as examples the petroleum, cosmetics, paints, drilling muds, food industry products, among others. To represent the nonlinear behavior of the viscosity of these fluids it is adopted a viscoplastic model proposed by De Souza Mendes, 2011, which is a modified version of the Oldroyd-B model. This model is approximated numerically by the Galerkin least squares method in terms of the extra-stress, pressure and velocity. This methodology – introduced by Hughes *et al.*, 1986, for the Stokes problem, and later extended to mixed and multi-field Navier-Stokes equations in Franca and Frey, 1992, and in Behr *et al.*, 1993, respectively – does not need to satisfy the compatibility conditions arisen from finite element sub-spaces for extra-stress-velocity and pressure-velocity fields. It enhances the stability of the classical Galerkin method adding mesh-dependent terms, which are functions of the residuals of flow governing equations, evaluated element-wise.

This article aims the study of inertialess steady state flows of elasto-viscoplastic fluids around a flat plate. In order to investigate the influence of the dimensionless flow-rate in the morphology of the yielded surfaces, numerical simulations were performed and U^* is varied from 0.002 to 0.01. In all simulations, the power-law index n, the number of jump J

and the structural elastic modulus G_0 remained constant and respectively equal to 0.5, 10,000 and 10.

2. MECHANICAL MODELING

This work considers that the fluid is incompressible in steady flow and the governing equations are expressed in a fixed Eulerian system. The continuity and the momentum conservation equations can be respectively expressed as:

$$\begin{aligned} \nabla \cdot \mathbf{u} = 0 \quad &\in \Omega \\ \nabla \cdot \boldsymbol{\tau} - \nabla P = 0 \quad &\in \Omega \end{aligned} \tag{1}$$

where $\mathbf{u}$ is the velocity vector, $P = p + \rho\phi$ is the modified pressure and $\mathbf{g} = -\nabla\phi$ is the gravitational force per unit mass. The non-tixotropic elasto-viscoplastic model used herein originates from the one proposed in De Souza Mendes, 2011, when the equilibrium time t_{eq} equals zero. The equation for the extra-stress tensor has the same form of the one for the Oldroyd-B model,

$$\boldsymbol{\tau} + \theta_1(\dot{\gamma})\check{\boldsymbol{\tau}} = 2\eta_{eq}(\dot{\gamma})(\mathbf{D}(\mathbf{u}) + \theta_2(\dot{\gamma})\check{\mathbf{D}}(\mathbf{u})) \tag{2}$$

where $\mathbf{D}$ is the strain rate tensor, $\dot{\gamma} = \sqrt{2\mathrm{tr}(\mathbf{D}(\mathbf{u}))^2}$ is its magnitude, and $\check{\boldsymbol{\tau}}$ and $\check{\mathbf{D}}$ are the upper-convected derivatives respectively:

$$\check{\boldsymbol{\tau}} = (\nabla\boldsymbol{\tau})\mathbf{u} - (\nabla\mathbf{u})\boldsymbol{\tau} - \boldsymbol{\tau}(\nabla\mathbf{u})^{\mathrm{T}} \quad \text{and} \quad \check{\mathbf{D}} = (\nabla\mathbf{D})\mathbf{u} - (\nabla\mathbf{u})\mathbf{D} - \mathbf{D}(\nabla\mathbf{u})^{\mathrm{T}} \tag{3}$$

The quantities that appears in Eq. (2) are respectively the relaxation and the retardation times. They are defined as

$$\theta_1 = \left(1 - \frac{\eta_\infty}{\eta_{eq}}\right)\frac{\eta_{eq}}{G_{eq}} \quad \text{and} \quad \theta_2 = \left(1 - \frac{\eta_\infty}{\eta_{eq}}\right)\frac{\eta_\infty}{G_{eq}} \tag{4}$$

This definitions involves the equilibrium viscosity η_{eq}, the equilibrium elastic modulus G_{eq}, and the infinite-shear-rate viscosity η_∞. The equilibrium viscosity and the equilibrium elastic modulus are assumed to be functions of the equilibrium structure parameter λ_{eq}. The relation between λ_{eq} and η_{eq} are given by

$$\lambda_{eq}(\dot{\gamma}) = \frac{\ln\eta_{eq}(\dot{\gamma}) - \ln\eta_\infty}{\ln\eta_0 - \ln\eta_\infty} \tag{5}$$

where η_0 is the zero-shear-rate viscosity. The equilibrium structure parameter is thus a scalar quantity that varies whitin the range [0,1] and gives a measure of the structuring level of the microstructure, such that $\lambda_{eq} = 0$ when the structuring level is minimum and $\lambda_{eq} = 1$ when the material is fully structured. The equilibrium elastic modulus, G_{eq}, is also a function of the structure parameter, which is taken as in De Souza Mendes and Thompson, 2012,

$$G_{eq} = G_0 e^{m\left(\frac{1}{\lambda_{eq}} - 1\right)} \tag{6}$$

In this equation, G_0 is the structural elastic modulus of the fully structured material and m is a positive scalar parameter that dictates the sensitivity of G_{eq} with λ_{eq} variation. The viscosity function employed in this work is dependent of the shear rate and is modified version of the model proposed by De Souza Mendes and Dutra, 2004, by the addition of an infinite shear rate viscosity,

$$\eta_{eq}(\dot{\gamma}) = \left[1 - \exp\left(-\frac{\eta_o\dot{\gamma}}{\tau_y}\right)\right]\left\{\frac{\tau_y}{\dot{\gamma}} + K\dot{\gamma}^{n-1}\right\} + \eta_\infty \tag{7}$$

where τ_y is the yield stress, K the consistency index and n the power-law index.

In order to obtain the dimensionless governing parameters, the rheological dimensionless normalization introduced by De Souza Mendes, 2007, is applied. Therefore, the following set of dimensionless quantities are introduced:

$$\mathbf{x}^* = \frac{\mathbf{x}}{L_c}, \quad \mathbf{u}^* = \frac{\mathbf{u}}{\dot{\gamma}_1 L_c}, \quad \dot{\gamma}^* = \frac{\dot{\gamma}}{\dot{\gamma}_1}, \quad P^* = \frac{P}{\tau_0}, \quad \boldsymbol{\tau}^* = \frac{\boldsymbol{\tau}}{\tau_0}, \quad {\eta_{eq}}^* = \frac{\eta_{eq}\dot{\gamma}_1}{\tau_0}, \quad \theta_1^* = \theta_1\dot{\gamma}_1, \quad \theta_2^* = \theta_2\dot{\gamma}_1 \tag{8}$$

Using these definitions, the dimensionless version of the governing equations for steady creeping flows are given by

$$\begin{aligned} \nabla^* \cdot \mathbf{u}^* = 0 \quad &\in \Omega^* \\ \nabla^* \cdot \boldsymbol{\tau}^* - \nabla^* P^* = 0 \quad &\in \Omega^* \\ \boldsymbol{\tau}^* + \theta_1^*(\dot{\gamma}^*)\check{\boldsymbol{\tau}}^* = 2\eta_{eq}^*(\dot{\gamma}^*)(\mathbf{D}^*(\mathbf{u}^*) + \theta_2^*(\dot{\gamma}^*)\check{\mathbf{D}}^*(\mathbf{u}^*)) \quad &\in \Omega^* \end{aligned} \tag{9}$$

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an in addition, the dimensionless viscosity function runs

$$\eta_{eq}^*(\dot{\gamma}^*) = \left[1 - \exp\left(-(J+1)\,\dot{\gamma}^*\right)\right]\left(\frac{1}{\dot{\gamma}^*} + \dot{\gamma}^{*n-1}\right) + \eta_\infty^* \tag{10}$$

The following governing parameters may be identified from the above equations:

$$U^* = \frac{u_c}{\dot{\gamma}_1 L_c}, \quad J = \eta_0 \left(\frac{\tau_0^{(n-1)}}{K}\right)^{1/n} - 1 \tag{11}$$

where the dimensionless velocity U^* arises from the boundary condition – the free stream velocity – which controls the flow intensity at the flat plate. The parameter J gives a relative measure of the colapse of the material microstructure – see De Souza Mendes *et al.*, 2007, for more details.

3. NUMERICAL APPROXIMATION

The finite element method consists of a numerical approximation of differential equations based on the concept that the solution of a differential equation can be represented as a linear combination of degrees of freedom incognito and selected approximation functions throughout the problem domain [Reddy and Gartling, 1994]. To solve the mechanical model of this work, the finite element method is employed using a multifield Galerkin least-squares (GLS) formulation in terms of velocity, pressure and extra-stress.

The numerical formulation employs the usual finite element sub-spaces for incompressible multi-field problems: $\boldsymbol{\tau} \in C^0(\Omega)^{N\times N} \cap H^2(\Omega)^{N\times N}$, $P \in C^0(\Omega) \cap L_2^0(\Omega)$ and $\mathbf{u} \in H^1(\Omega) \in H^1(\Omega)^N$, and uses stability parameters to control the conservation equations and the constitutive equations. The discretization of the GLS formulation results in a non-linear system of equations, which is solved using a quasi-Newton method. The GLS method produces stable approximations for both elastic and viscous dominated flow regions, using simple combinations of finite element interpolations, and allowing the use of equal-order combinations of Lagrangian finite elements.

The problem studied in this article is the flow of elasto-viscoplastic fluids around a flat plate with length *L*=50 and infinitesimal thickness. This geometry is depicted in Fig. 1. To solve numericaly this problem, the computational domain is partitioned into a finite element mesh, with bi-linear Lagrangian elemnts, for all primal variables. After a mesh independence test, the mesh adopted for the simulations has 10,300 Q1xQ1xQ1 elements and the computational domain have a length of 3.5 *L* upstream and downstream the flat plate. As is assumed horizontal symmetry for the flow, only half of the domain is simulated, thus, the distance from the plate to the upper computational boundary is 4*L*.

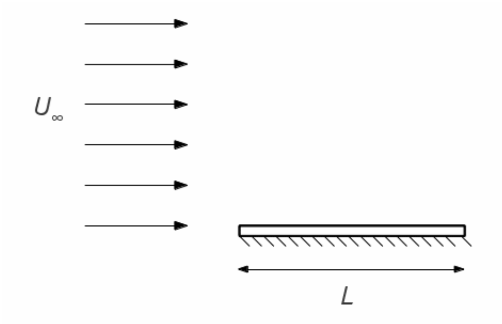
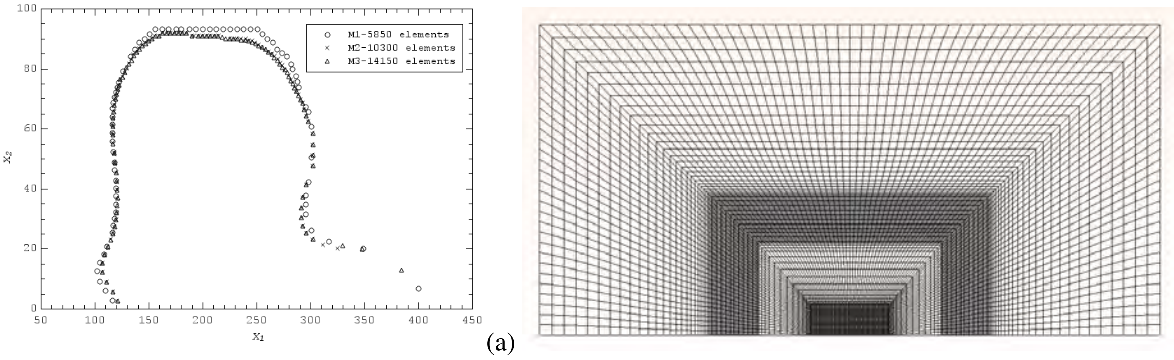


Figure 1. Problem statment

The mesh independence test consists in an investigation of the geometric locus of points of the yilded surface – the transition from the yilded to the unyilded regions. Three meshes are employed for this investigation, one with 5,850 elements (mesh M1), another with 10,300 (mesh M2) and a third with 14,150 elements (mesh M3). The Fig. 2 depicts the comparation of the results obtained with this three meshes. It can be observed that the results obtained with the meshes M2 and M3 virtually do not differ, that is, it can be considered that the mesh M2 generates results independent of the domain discretization.

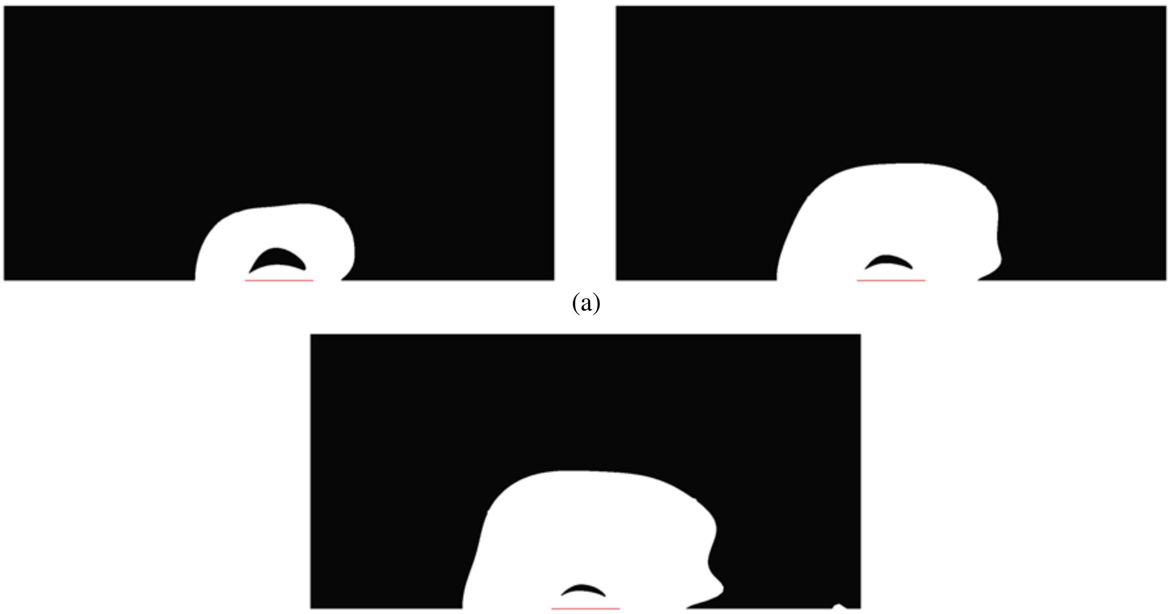
4. NUMERICAL RESULTS

The objetive of the numerical simulations in this article is to analyze the influence of the yield stress on the morfology of the unyilded zones. The boundary conditions employed are impermeability and non-slip on the plate, prescribed free stream velocity, U_∞, and symmetry condition ($u_2 = 0$) at the line that follows, horizontally, the flat plate. Another point to mention is the criterion employed to characteryze the unyield regions – it is used the shear rate criterion ($\dot{\gamma} \leq \dot{\gamma}_0$ for the unyielded zones – see Santos *et al.*, 2011.



(a) (b)

Figure 2. Domain discretization: (a) mesh independence test; (b) adopted mesh



(a) (b) (c)

Figure 3. Unyilded morfology, for $J = 10^4$, $n = 0.5$, $G_0 = 10$ and (a) $U^* = 0.002$; (b) $U^* = 0.006$; (c) $U^* = 0.01$

Fig 3 shows the influence of the dimensionless flow rate in the morphology of the unyielded zones on a inertialess flow of elasto-viscoplastic fluid with $J = 10000$, $n = 0.5$ and $G_0 = 10$, whit U^* ranging from 0.002 to 0.01. Note that the unyielded zones decrease when U^* increases. Another observed effect is that this zones becomes more asymmetric with the increasing of U^*. Such kind of behavior may be explained by the decreasing of yield stress effects thanks to the increasing of U^*. This behavior can also be seen mathematically by Eq. (12), which relates the flow intensity with the classical Herschel-Bulkley number. Note that there is an inverse relationship between U^* and HB.

$$HB = \frac{\tau_0}{K(\dot{\gamma}_1 U^*)^n} = \frac{1}{(U^*)^n} \qquad (12)$$

The elastic deformation of the fluid for the same three cases, that is, keeping fixed the rheological parameters $J = 10000$, $n = 0.5$ and $G_0 = 10$ and ranging U^* from 0.002 to 0.01, are ilustrated in Fig. 4. It can be seen that elastic deformation is larger in a region near the surface of the flat plate as well as around the symmetry line downstream of the plate. Moreover, it is clear that increasing the flow intensity, the elastic deformations are carried downstream from the flat plate. This can be attributed to the upwind effect from velocity and elastic stress fields.

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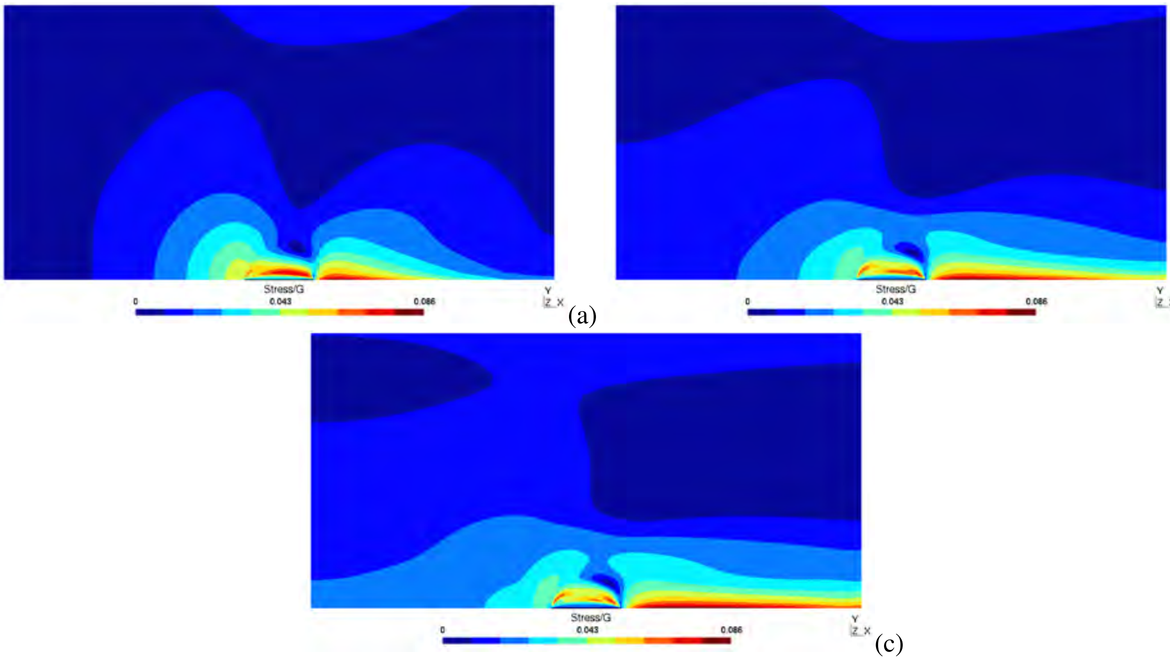

(a) (b) (c)

Figure 4. Elastic deformations, for $J = 10^4$, $n = 0.5$, $G_0 = 10$ and (a) $U^* = 0.002$; (b) $U^* = 0.006$; (c) $U^* = 0.01$

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