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COMPARISON OF ACTIVE VIBRATION CONTROL STRATEGIES FOR A BEAM MODELED USING SPECTRAL ELEMENT METHOD

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Abstract. *The objective of this work is to present a comparison of different control strategies applied to a cantilever beam structure. The mechanical system is modeled by means of spectral element method such that the equations of motion are not approximations as in finite element method, allowing accurate response of the system using a few elements to discretized the system. Numerical simulations are carried out comparing different control strategies, with feedforward control. Position of secondary control forces are studied in terms of cost function minimization. Numerical simulations where performed in order to indentify positions for the control force taking into account the system mode shapes.*

Keywords: *Vibration Control; Spectral Element Method; Feedforward; cantilever beam structure*

1. INTRODUCTION

With the growing need for mechanical design to achieve high efficiency, structural-dynamic systems are increasingly flexible and often possess low damping coefficient, which can cause severe vibration and instability compromising the system's performance. A method to eliminate this problem is by performing vibration control, which can be by passive or active methods, where the main objective is the reduction of the amplitudes of the displacements of a structure over a range of excitation frequency.

Passive control strategies are usually associated to structural modification by addition of mass. Usually by adding damping as viscoelastic materials, friction dampers, shock absorbers adjusted with base-isolation which have been. Its main function is to dissipate the internal energy of the system in the form of heat. An important feature of passive systems is their inability to destabilize the structure because no external energy is added to the system.

Another strategy to control vibration is to use active methods, which uses an external power source for applying secondary forces to control the structure. Generally these forces are applied through a control law with entries on measurements of displacements (velocities or accelerations) of the structure.

Usually in active control, the forces are inserted into the frame through "smart" materials, which can produce a mechanical stress with a specific input of an electrical signal. The piezoelectric materials come with great emphasis in the area of measuring sensors, the main innovation capacity, both measured (sensor) and, if of interest, function as actuator. The most common piezoelectric materials are the PZT's (Lead Zirconate Titanate) and PVDF (Polyvinylidene Flouride), which may function as sensors or actuators, because of their known direct and indirect effect, i.e. the sensor undergoing deformation it produces a potential difference can be measured and in the other case, if it receives a potential difference will respond with a deformation. See Figure 1 below. (Prazzo, Carlos Eduardo, 2011).

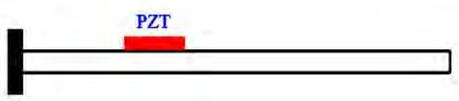

Figure 1 – Beam with piezoelectric

The most known strategies of active control is the Feedback and Feedforward. In this article we will explore the possibilities of a control of optimum feedforward control.

The concept of a feedforward control is based on the superposition principle of linear systems, i.e. waves entered into the system by an actuator to control and eliminate the cause interference wave disturbance. The block diagram illustrating this strategy is presented in figure 2:

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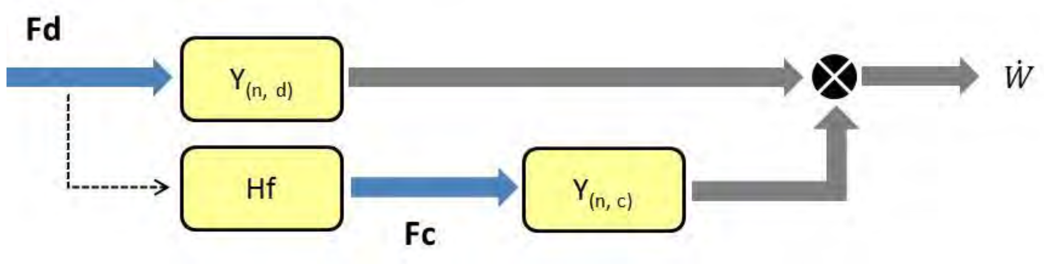

Figure 2 – Block diagram of feedforward control

Where:
Fd = Disturbance force;
Fc = Control force;
$Y_{(n,d)}$ = Mobility matrix of beam, related of Fd with de velocity in a point of beam.
$Y_{(n,c)}$ = Mobility matrix of beam, related of Fc with de velocity in a point of beam.
Hf = Feedforward controller
$\dot{W}$ = Velocity of interest point of beam.

2. NUMERICAL MODELING

The finite element method is the most used and accepted today in structural analysis. In this method, static solutions are used with great success for static problems, where the system is divided into several elements to represent the geometry of the body, with boundary conditions and loading certain.

As we evolving from static to the dynamic problem, the solution should converge to acceptable values which generally imply increasing the number of elements of the system in relation to the static problem, where the elements is increased in proportion to the increased frequency. With the larger number of elements simulations are longer and sometimes out of a viable time.

An alternative is the use of spectral elements that is based on exact solution of the beam equation, unlike the polynomial approximation of finite elements. The trigonometric and hyperbolic functions are incorporated in the frequency response resulting in accurate dynamic response element.

Several articles have explored the method in comparison with the traditional method of finite elements with satisfactory results (Thomas Black, 2005), (Lee, Kim, Leung, 2000).

Another known modeling method is impedance matrix. Although well documented (Rubin, 1967), this technique began to be used more recently. Its main advantage is that, unlike the method of transfer matrices, the matrices are expressed directly in terms of frequency response functions.

Consider an Euler-Bernoulli beam, which can be modeled by partial differential equation:

$$E.I.\frac{\partial^4\omega(x)}{\partial x^4} + \rho.S.\frac{\partial^2\omega(x,t)}{\partial t^2} = -f(x,t) \qquad (1)$$

Where E = Young´s Modulus; I = Second moment of area; x = relative position in the beam; ρ = density of material; S = cross section area; t = time.

The system showing in equation 1 can be represented in frequency domain by use of impedance matrices.

$$[V] = [Y].[F] \; or \; [F] = [Z].[V] \qquad (2)$$

Where:
[Y] = Mobility matrix;
[Z] = Impedance matrix;
[V] = Velocity vector;
[F] = Force and bending vector;

Applying the boundary conditions at the ends of the beam (free-free) in figure 2, we have the following solution in equation 2:

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Figure 2 – Free-Free beam

$$\begin{bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{bmatrix} = \frac{E.I.k_b^3}{j.\omega.N} \cdot \begin{bmatrix} -K_{11} & -P & K_{12} & V \\ -P & Q_{11} & -V & Q_{12} \\ K_{12} & -V & -K_{11} & P \\ V & Q_{12} & P & Q_{11} \end{bmatrix} \cdot \begin{bmatrix} \dot{W}_1 \\ \dot{\theta}_1 \\ \dot{W}_2 \\ \dot{\theta}_2 \end{bmatrix} \tag{3}$$

Where:
j = Complex number;
ρ = Density of material;
S = Cross section area;
E = Young modulus of material;
I = Momento de inércia de área da secção transversal da viga;
l = Length of beam;
ω = Frequency of excitation;
See appendix for more impedance matrix equations;

Considering a beam with boundary conditions at the ends of the clamped-free type, as shown in figure 3, the forces and velocities at point 1 are equal to zero due to the locking of the degrees of freedom. Therefore, the impedance matrix equation and the beam are described as:

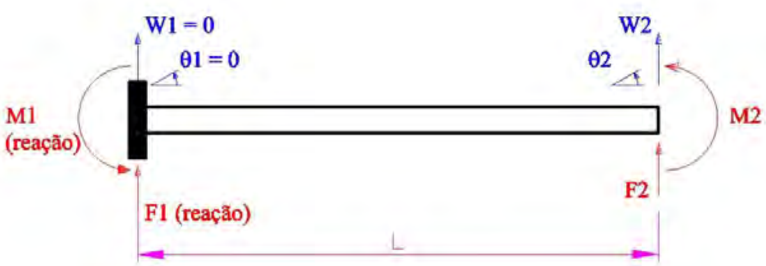

Figure 3 – Beam clamped-free type

$$\begin{bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{bmatrix} = \frac{E.I.k_b^3}{j.\omega.N} \cdot \begin{bmatrix} -K_{11} & -P & K_{12} & V \\ -P & Q_{11} & -V & Q_{12} \\ K_{12} & -V & -K_{11} & P \\ V & Q_{12} & P & Q_{11} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \dot{W}_2 \\ \dot{\theta}_2 \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} F_2 \\ M_2 \end{bmatrix} = \frac{E.I.k_b^3}{j.\omega.N} \cdot \begin{bmatrix} -K_{11} & -P \\ -P & Q_{11} \end{bmatrix} \cdot \begin{bmatrix} \dot{W}_2 \\ \dot{\theta}_2 \end{bmatrix} \tag{5}$$

However, we wish to introduce a control force (F_c) to reduce velocities and displacements resulting from the application of a disturbance force (F_d). As it is not always physically possible application of these forces at the same point, we have to split the beam into two elements as the figure 4:

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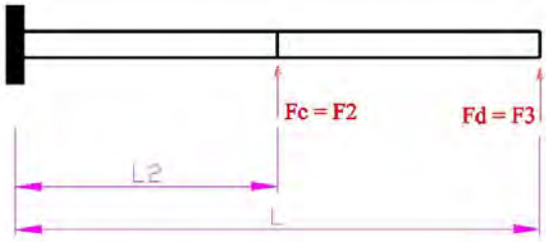

Figure 4 – Beam of two spectral elements

The variables of this new beam with two elements are illustrated in the figure 5:

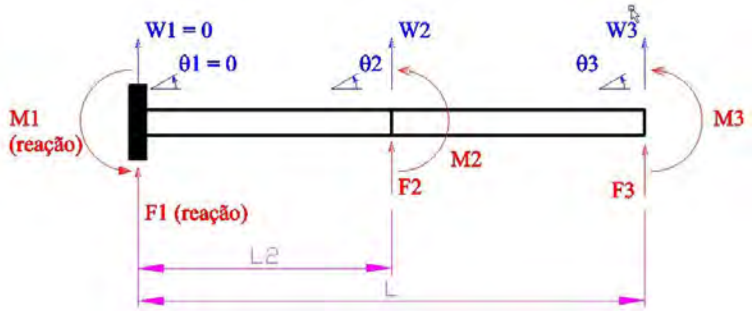

Figure 5 – Beam with two elements and their variables

And follow the equations:

⇨ Equation of impedance matrix of first element:

$$\begin{bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{bmatrix} = Ta. \begin{bmatrix} -K_{11\backslash A} & -P_A & K_{12\backslash A} & V_A \\ -P_A & Q_{11\backslash A} & -V_A & Q_{12\backslash A} \\ K_{12\backslash A} & -V_A & -K_{11\backslash A} & P_A \\ V_A & Q_{12\backslash A} & P_A & Q_{11\backslash A} \end{bmatrix} . \begin{bmatrix} \dot{W}_1 \\ \dot{\theta}_1 \\ \dot{W}_2 \\ \dot{\theta}_2 \end{bmatrix} \tag{6}$$

⇨ Equation of impedance matrix of second element:

$$\begin{bmatrix} F_2 \\ M_2 \\ F_3 \\ M_3 \end{bmatrix} = Tb. \begin{bmatrix} -K_{11\backslash B} & -P_B & K_{12\backslash B} & V_B \\ -P_B & Q_{11\backslash B} & -V_B & Q_{12\backslash B} \\ K_{12\backslash B} & -V_B & -K_{11\backslash B} & P_B \\ V_B & Q_{12\backslash B} & P_B & Q_{11\backslash B} \end{bmatrix} . \begin{bmatrix} \dot{W}_2 \\ \dot{\theta}_2 \\ \dot{W}_3 \\ \dot{\theta}_3 \end{bmatrix} \tag{7}$$

Where:

$$Ta = \frac{E.I.k_b^3}{j.\omega.N_A} \quad \text{and} \quad Tb = \frac{E.I.k_b^3}{j.\omega.N_B} \tag{8}$$

We must add the impedances of the two elements of the beam results in the equation:

$$\begin{bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \\ F_3 \\ M_3 \end{bmatrix} = \begin{bmatrix} Ta.(-K_{11\backslash A}) & Ta.(-P_A) & Ta.(K_{12\backslash A}) & Ta.(V_A) & 0 & 0 \\ Ta.(-P_A) & Ta.(Q_{11\backslash A}) & Ta.(-V_A) & Ta.(Q_{12\backslash A}) & 0 & 0 \\ Ta.(K_{12\backslash A}) & Ta.(-V_A) & \begin{pmatrix} Ta.(-K_{11\backslash A}) \\ +Tb.(-K_{11\backslash B}) \end{pmatrix} & \begin{pmatrix} Ta.(P_A) \\ +Tb.(-P_B) \end{pmatrix} & Tb.(K_{12\backslash B}) & Tb.(V_B) \\ Ta.(V_A) & Ta.(Q_{12\backslash A}) & \begin{pmatrix} Ta.(P_A) \\ +Tb.(-P_B) \end{pmatrix} & \begin{pmatrix} Ta.(Q_{11-A}) \\ +Tb.(Q_{11\backslash B}) \end{pmatrix} & Tb.(-V_B) & Tb.(Q_{12\backslash B}) \\ 0 & 0 & Tb.(K_{12\backslash B}) & Tb.(-V_B) & Tb.(-K_{11\backslash B}) & Tb.(P_B) \\ 0 & 0 & Tb.(V_B) & Tb.(Q_{12\backslash B}) & Tb.(P_B) & Tb.(Q_{11-B}) \end{bmatrix} . \begin{bmatrix} \dot{W}_1 \\ \dot{\theta}_1 \\ \dot{W}_2 \\ \dot{\theta}_2 \\ \dot{W}_3 \\ \dot{\theta}_3 \end{bmatrix} \tag{9}$$

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Eliminating the degrees of freedom related to the boundary condition, we have:

$$\begin{bmatrix} F_2 \\ M_2 \\ F_3 \\ M_3 \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} Ta.\left(-K_{11\backslash A}\right) \\ +Tb.\left(-K_{11\backslash B}\right) \end{pmatrix} & \begin{pmatrix} Ta.\left(P_A\right) \\ +Tb.\left(-P_B\right) \end{pmatrix} & Tb.\left(K_{12\backslash B}\right) & Tb.\left(V_B\right) \\ \begin{pmatrix} Ta.\left(P_A\right) \\ +Tb.\left(-P_B\right) \end{pmatrix} & \begin{pmatrix} Ta.\left(Q_{11-A}\right) \\ +Tb.\left(Q_{11\backslash B}\right) \end{pmatrix} & Tb.\left(-V_B\right) & Tb.\left(Q_{12\backslash B}\right) \\ Tb.\left(K_{12\backslash B}\right) & Tb.\left(-V_B\right) & Tb.\left(-K_{11\backslash B}\right) & Tb.\left(P_B\right) \\ Tb.\left(V_B\right) & Tb.\left(Q_{12\backslash B}\right) & Tb.\left(P_B\right) & Tb.\left(Q_{11-B}\right) \end{bmatrix} . \begin{bmatrix} \dot{W}_2 \\ \dot{\theta}_2 \\ \dot{W}_3 \\ \dot{\theta}_3 \end{bmatrix} \tag{10}$$

2.1 – Feedforward control

Considering two forces acting on the beam, that the speed at the desired point is the sum of the individual effects of forces on the beam as follows:

$$\mathbf{V}_{(n,\omega)} = \mathbf{Y}_{d(n,\omega)}.\mathbf{F}_d + \mathbf{Y}_{c(n,\omega)}.\mathbf{F}_c \tag{11}$$

Where:
n = Control point;
ω = Frequency of excitation;
$\mathbf{Y}_d$ = Mobility matrix related to $\mathbf{F}_d$;
$\mathbf{Y}_c$ = Mobility matrix related to $\mathbf{F}_c$;
d = Disturbance point;
c = Control point;

As the theory explains about Feedforward control, a controller with a certain gain is calculated to minimize any physical quantity, in case, we have velocity. The force has a gain and phase control, relating the disturbance force (F_d), we have the following equation:

$$\mathbf{V}_{(n,\omega)} = \left(\mathbf{Y}_{d(n,\omega)} + \mathbf{Y}_{c(n,\omega)}.H_{f(n,\omega)}\right).\mathbf{F}_d \tag{12}$$

Where:
H_f = Feedforward controller;

According to (Fuller, Elliot, Nelson, 1996), the optimum controller of the cost function can be solved as follow:

$$\mathbf{H}_{ff} = -\left(\mathbf{Y}^H_{c(n,\omega)}.\mathbf{Y}_{c(n,\omega)}\right)^{-1}.\mathbf{Y}^H_{c(n,\omega)}.\mathbf{Y}_{d(n,\omega)} \tag{13}$$

In this equation, the sensor is positioned exactly at the point of force control (c) and the controller H_{ff} will reduce the velocity at this point, without consider other points of the beam.

As is usually necessary to control primarily a point of the beam, but while observing others, the form of optimal controller can be written as follows:

$$\mathbf{H}_{ff} = -\left(\begin{bmatrix} \mathbf{Y}_{c(1,\omega)} \\ \mathbf{Y}_{c(2,\omega)} \end{bmatrix}^H . \begin{bmatrix} \mathbf{Y}_{c(1,\omega)} \\ \mathbf{Y}_{c(2,\omega)} \end{bmatrix}\right)^{-1} . \begin{bmatrix} \mathbf{Y}_{c(1,\omega)} \\ \mathbf{Y}_{c(2,\omega)} \end{bmatrix}^H . \begin{bmatrix} \mathbf{Y}_{d(1,\omega)} \\ \mathbf{Y}_{d(2,\omega)} \end{bmatrix} \tag{14}$$

Where the indices "1" and "2" are the points where the sensors, which in the case are the points in "F_d" and "F_c". In this equation, the controller H_{ff} try to achieve the best compromise of velocity reduction on these two points.

3. NUMERICAL SIMULATIONS AND RESULTS

The main objective is to control the velocity at certain points of the beam to the right of a range of excitation frequency. This control technique involve Feedforward positions and will be operated under control and the number of velocity sensors.

For the analyzes we set the parameters of a clamped-free beam with the following values in table 1:

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Table 1 – Assumptions for simulations

Lenght (L):	0,5 meters
Constant cross section:	Circular / ∅ = 0,0254 m
Material:	Aluminum ρ = 2710 kg/m³ E = 7e10 Pa
Disturbance force (Fd):	1 N
Frequency of excitation:	$0{,}8.\Psi_n(1)$ to $1{,}2.\Psi_n(5)$ Where: $\Psi_n(1)$ = 1st natural frequency $\Psi_n(5)$ = 5th natural frequency

First, it is necessary to know the natural frequencies and mode shapes of the beam in question and the results are presented in table 2.

Table 2 – Natural frequencies of the beam

Natural Frequencies and Natural Modes Equations	**Natural modes**	**K_{nb}** (Bending wavenumber)	**Results for ω_n**
$\omega_n = \sqrt{\frac{E.I}{\rho.S}}.K_{nb}^2$ $\Psi_n(x) = [cosh(K_{nb}.x) - cos(K_{nb}.x)] - \sigma_n.[sinh(K_{nb}.x) - sin(K_{nb}.x)]$ $\sigma_n = \frac{sinh(K_{nb}.L) - sin(K_{nb}.L)}{cosh(K_{nb}.L) - cos(K_{nb}.L)}$	n=1	$\frac{1{,}87510}{L}$	72,2 Hz
	n=2	$\frac{4{,}69409}{L}$	452,7 Hz
	n=3	$\frac{7{,}85476}{L}$	1267,6 Hz
	n=4	$\frac{10{,}9955}{L}$	2484 Hz
	n=5	$\frac{14{,}1372}{L}$	4106,3 Hz
	n=6, 7, ...	$\frac{(2.n-1).\pi}{2.L}$	---

The graphs of vibration modes, showing the relative displacement in relation of the length of the beam as show in figure 6:

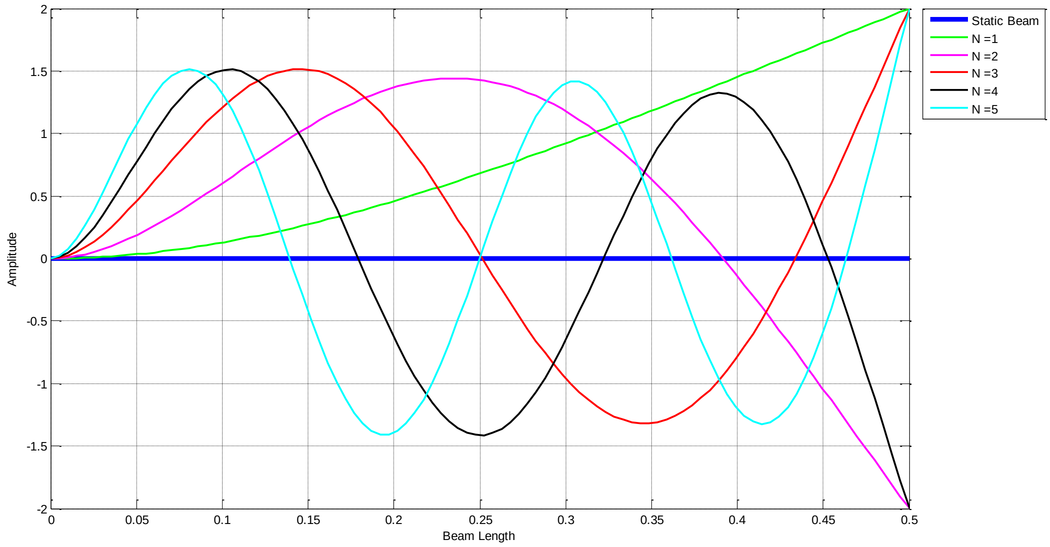

Figure 6 – Modes of vibration

Analyzing the graph of the relative displacements one can conclude that the greatest displacement are the free end and therefore this point is of the greatest interest.

Another observation is that the points of zero velocity (nodes) in the beam changes according to the natural frequency. This observation is important for the correct positioning of the sensor (s) and force of control to meet the goal of reducing the vibration of the beam.

Now, follow the velocity of the free end of the beam with a disturbance force positioned at the same end, according to figure 7 and 8.

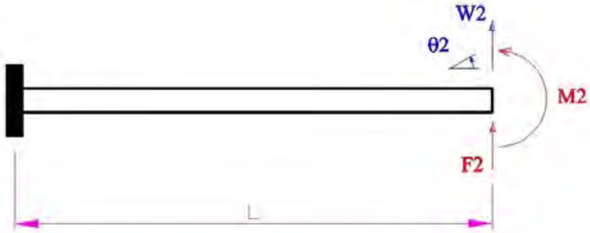

Figure 7 – Model of clamped-free beam with disturbance force

Development from equation 2:

$$\begin{bmatrix} \dot{W}_2 \\ \dot{\theta}_2 \end{bmatrix} = \mathbf{Y}.\begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{15}$$

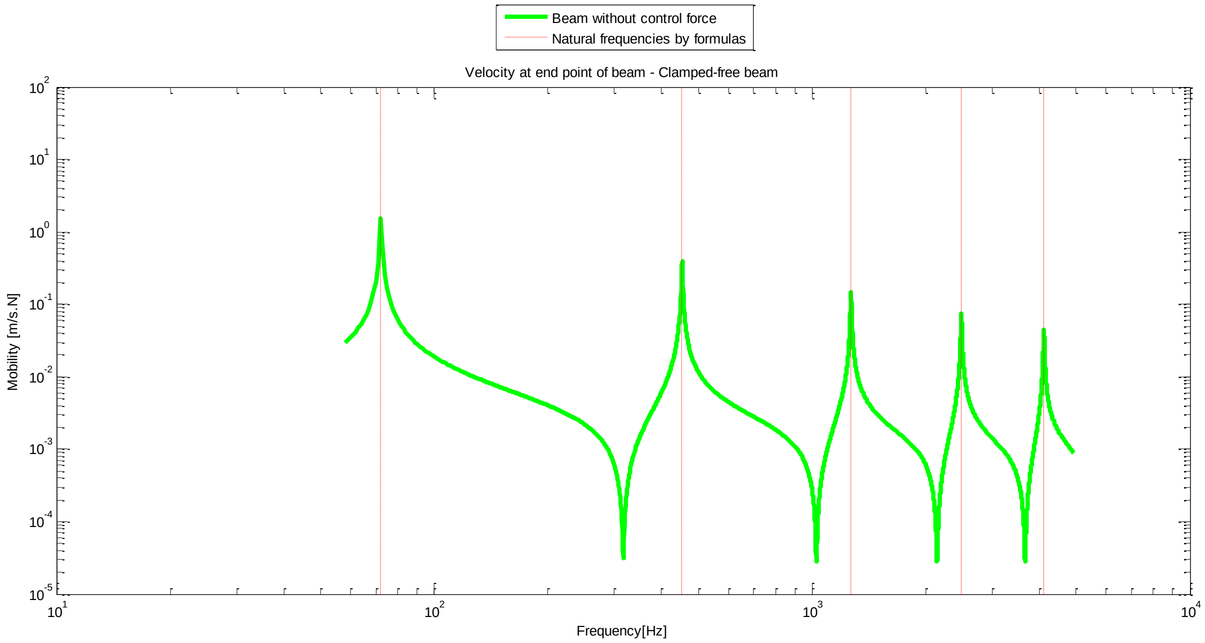

Figure 8 – Velocity of free end of the beam – 1 Element – Without control force.

The valleys are observed in the graph excitation frequencies on which the velocity and consequently the displacement of the beam are minimal.

Using feedforward technique, placing a force control in L2 = 0,2.L and the sensor and the same point according figure 8, we have:

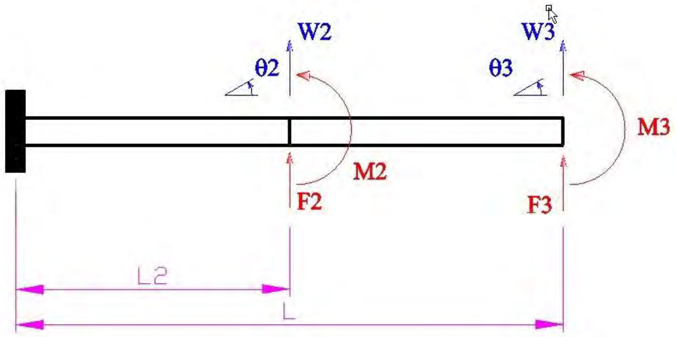

Figure 8 - Model of clamped-free beam with disturbance force and control force

Development from equation 10:

$$\begin{bmatrix} \dot{W}_2 \\ \dot{\theta}_2 \\ \dot{W}_3 \\ \dot{\theta}_3 \end{bmatrix} = \mathbf{Y}.\begin{bmatrix} 1.H_{ff} \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{16}$$

The figure 9 and 10 shows, respectively, the velocity in the beam end and velocity of the point sensor:

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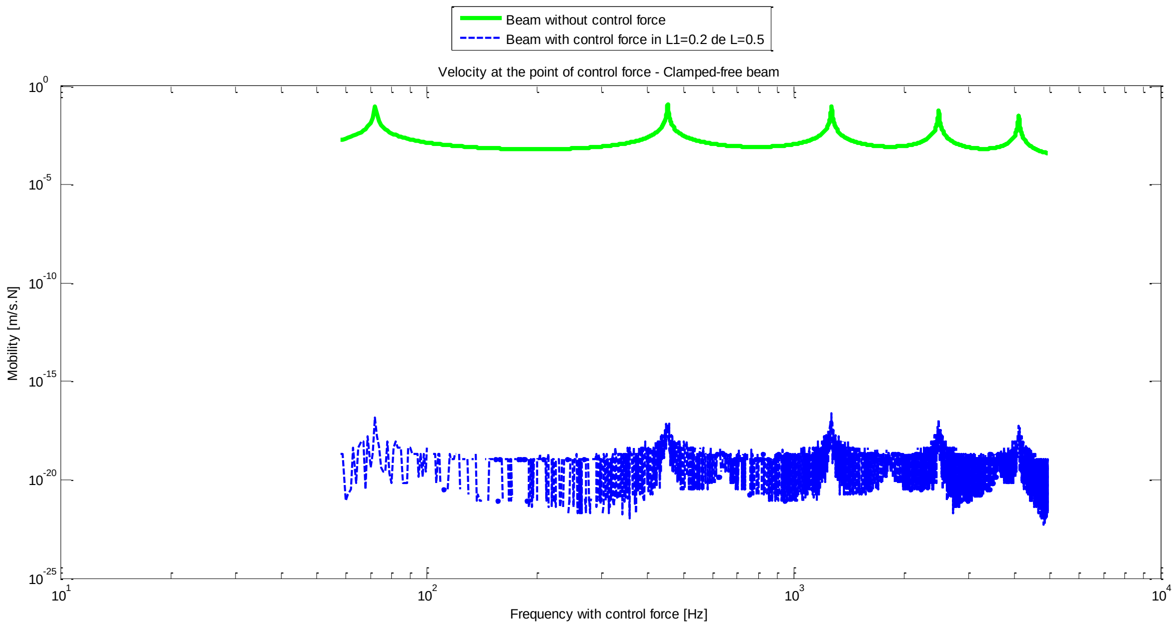

Figure 9 - Velocity of sensor point of the beam – 2 Element – With control force.

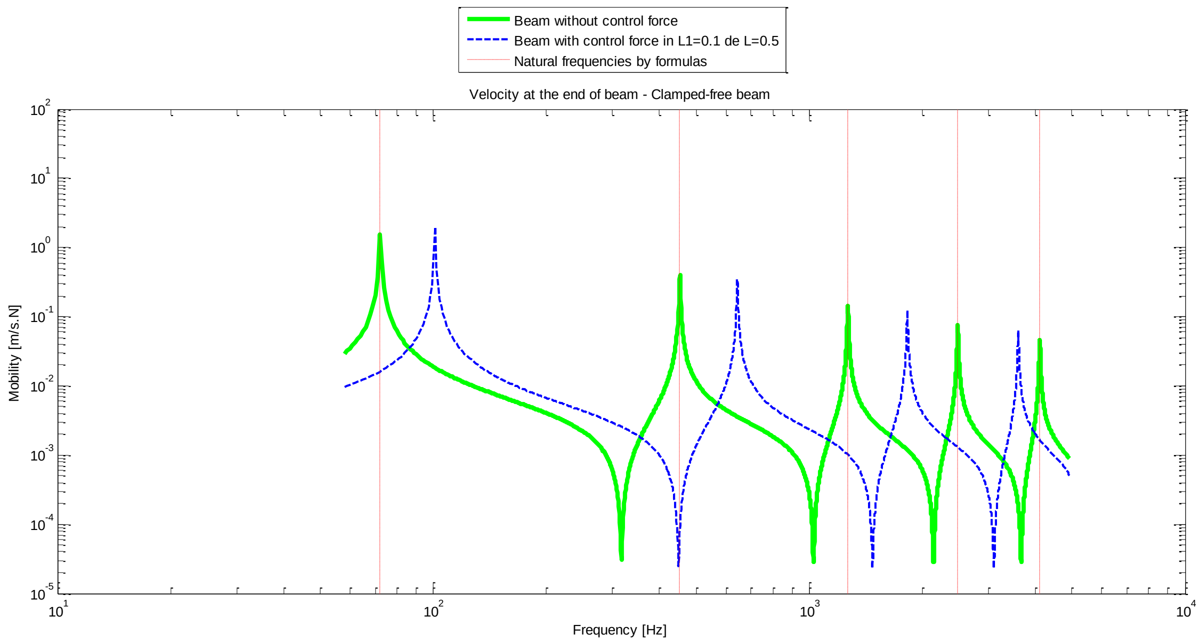

Figure 10 - Velocity of free end of the beam – 2 Element – With control force.

It is possible to see in figure 9, the controller H_{ff} is effective in controlling the velocity at the sensor point, at which the velocity is practically zero. At the free end of the beam there is a displacement of the natural frequency to higher frequencies. This fact is explained precisely from the fact that the control force to be behaving like a pin.

Positioning control force and the force sensor at values L2 40%, 60%, 80% of the length L, it is possible to note the displacement of the frequency in relation the position of the controller, as show in figure 11.

Important to note when the position of control force is 80% of L, i.e., a distance of 0.4 m from the clamped end, there isn't effective control in second mode frequency. This is explained by the fact that the control force is positioned exactly at the point of zero velocity of the second mode of vibration, as we see in figure 6.

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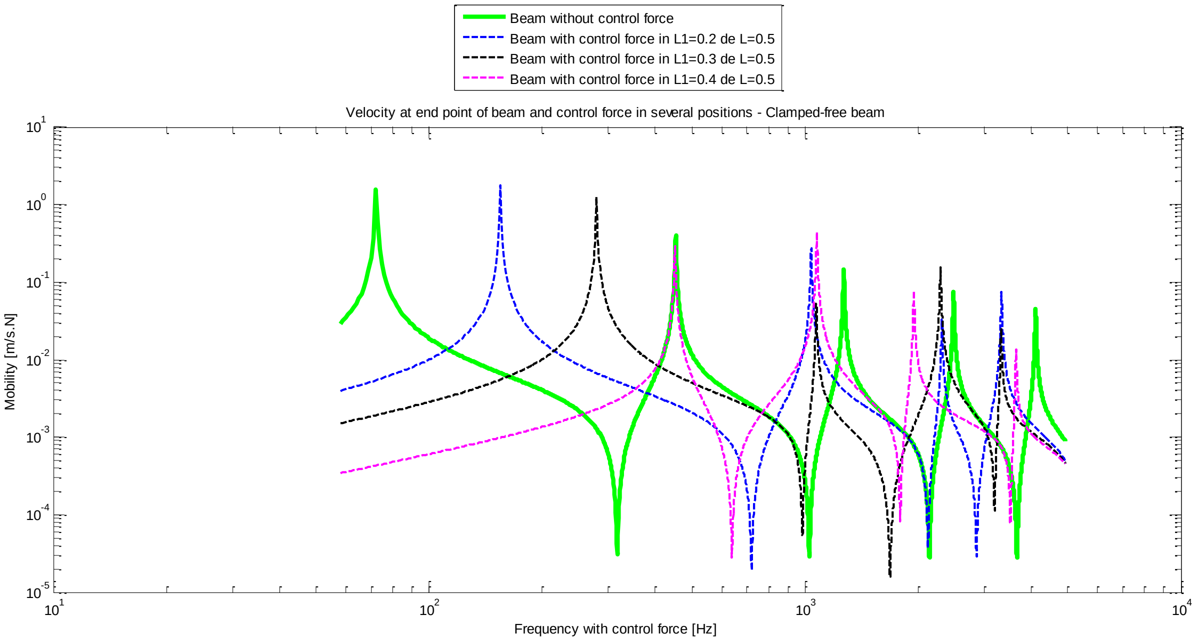

Figure 11 - Velocity of free end of the beam – 2 Element – With control force in several positions.

Modifying the strategy and including a second sensor velocity controller, positioned at the point of force control and at the end of the beam, in order to achieve the best compromise between the velocities at the free end and the point of force application control.

Applying the new controller we have the following results in figure 12 and 13:

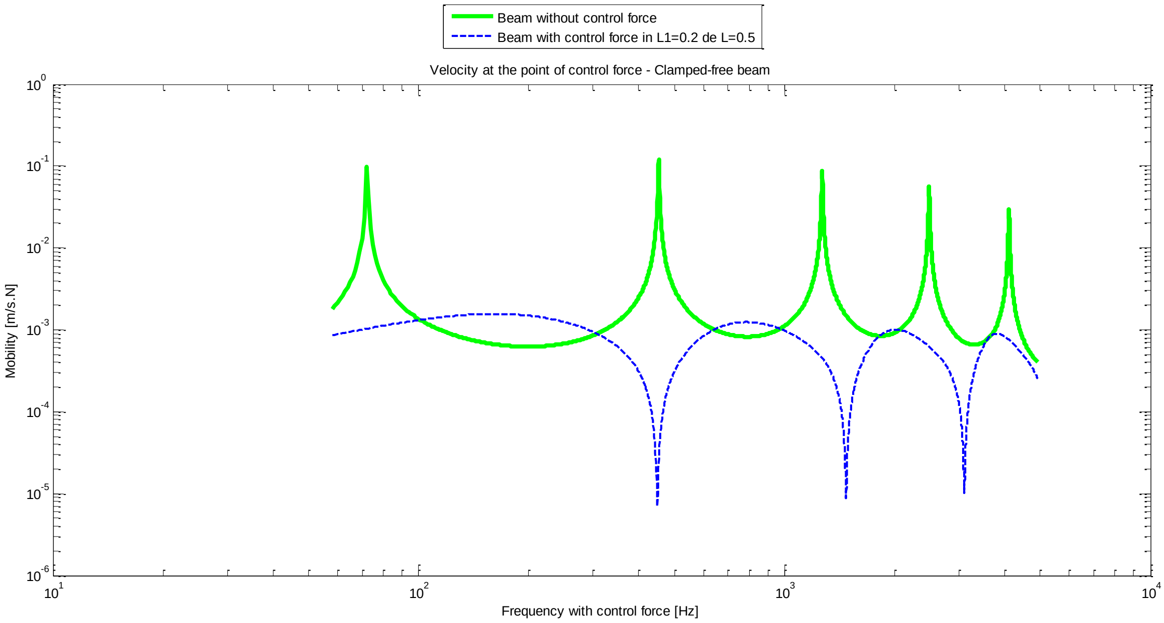

Figure 12 - Velocity of sensor point of the beam – 2 Sensors – With control force.

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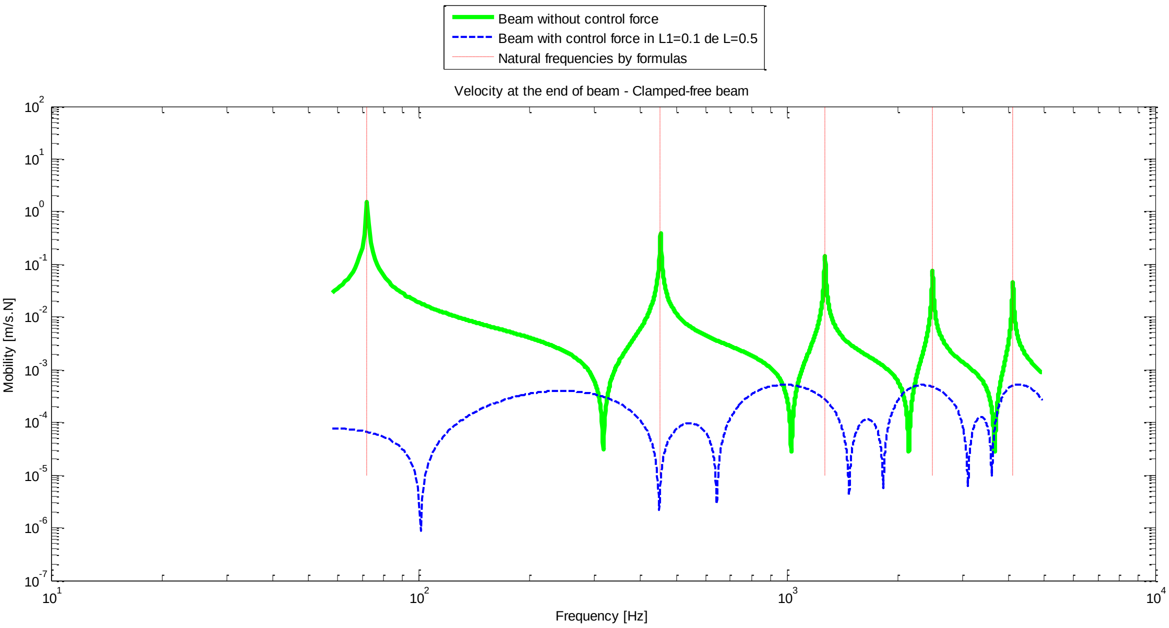

Figure 13 - Velocity of free end of the beam – 2 Element – With control force

We can note that the velocities were significantly reduced at the points of natural frequency in relation of the beam without control.

Positioning control force and the force sensor at values L2 40%, 60%, 80%, we noted even the position of control force is close to zero velocity point, in accordance with figure 6, the controller is able to reduce the velocities, as show in figure 14.

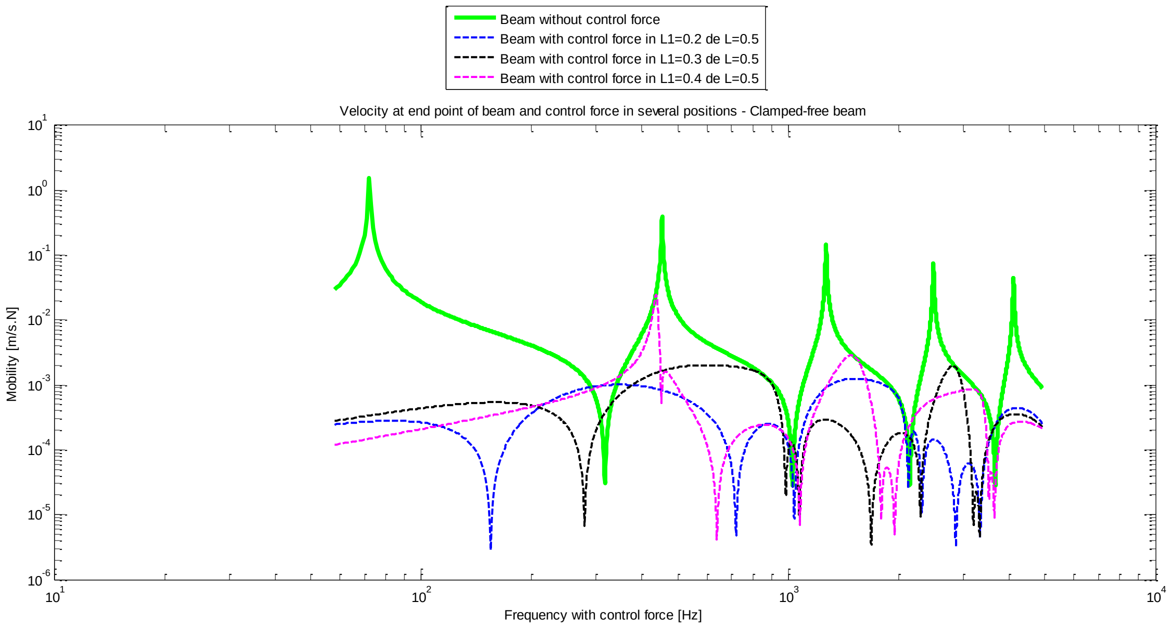

Figure 14 - Velocity of free end of the beam – 2 Sensors – With control force in several positions.

4. CONCLUSIONS

As shown in Figures 6 and 8, the dynamic model using mobility matrix with spectral elements was accurate when comparing the natural frequencies of the resultant model and the frequency calculated by the exact formulas.

For the classic model of a clamped beam as shown in figure 7, we confirmed that the feedforward control is effective to control the velocity at the point where the sensor is located. However, as shown in Figure 10, the end points of the beam, as well as other points are not adequately controlled. As shown in Figure 9, the control force with the sensor in a single point reduces the velocity at the desired point like a pin, modifying the dynamic model of the beam and modifying the natural frequencies and mode shapes of beam.

We can also conclude seeing de figure 11 that the controller with only one sensor position are not effective to reduce velocity in all ranges of excitation frequency, because of the fixed position of the sensor is coincident with a point of zero velocity at some vibrate mode.

Using the technique of feedforward control with two sensors was more effective in controlling all frequency bands, as shown in Figures 12, 13 and 14, without frequency no controls as seen in Figure 11.

5. APPENDIX

$$K_b = \sqrt[4]{\frac{\rho.S}{E.I}}.\sqrt{\omega}$$

$$K_{11} = cos(K_b.L).sinh(K_b.L) + sin(K_b.L).cosh(K_b.L)$$

$$K_{12} = sin(K_b.L) + sinh(K_b.L)$$

$$P = \frac{sin(K_b.L).sinh(K_b.L)}{K_b}$$

$$V = \frac{cos(K_b.L) - cosh(K_b.L)}{K_b}$$

$$Q_{11} = \frac{cos(K_b.L).sinh(K_b.L) - sin(K_b.L).cosh(K_b.L)}{{K_b}^2}$$

$$Q_{12} = \frac{sin(K_b.L) - sinh(K_b.L)}{{K_b}^2}$$

$$N = cos(K_b.L).cosh(K_b.L) - 1$$

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