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ECOLOGICAL OPTIMIZATION OF AN IRREVERSIBLE RANKINE CYCLE BASED IN AN ECOLOGICAL COEFFICIENT OF PERFORMANCE (ECOP)

André Luiz Salvat Moscato
Unesp/Bauru – Mechanical Engineering Department
andremoscato@yahoo.com.br

Santiago del Rio Oliveira
Unesp/Bauru – Mechanical Engineering Department
santiago@feb.unesp.br

Alcides Padilha
Unesp/Bauru – Mechanical Engineering Department
padilha@feb.unesp.br

Abstract. *In this paper, the thermodynamics of finite time will be applied to a Rankine cycle to optimize the power output. The cycle is design between two reservoirs where their rates of thermal capacitances are infinites and the inlet and outlet temperature are different both in the hot steam as in the cold steam. The irreversibility of the processes are originated mainly of the non-isentropic pumping and expansion in the turbine, the heat exchange processes occurring in finite time and heat leak from the hot source to the cold source. This type of optimization represents the best compromise between generating work of a heat engine and the environment that surrounds it. It will be present the values of the optimum and maximum power, optimum and maximum efficiency, besides optimum parameter value of operation. It will be presented the curves of efficiency and ecological power, power and ecological power. Another analysis that will be discussed and deserves featured is the ratio of power conditions and optimum power according maximum ecological conditions and the ratio of entropy generation in maximum condition and entropy generation according to ecological conditions. The results are a tool for qualitative analysis of the process.*

Keywords: Rankine cycle, power, optimization, ECOP.

1. INTRODUCTION

One of the most used ways for converting thermal energy into power scale, the Rankine cycle, has wide application in industry mainly due to the capacity of the boilers are fed by different types of fuels, such as coal, nuclear sources, petroleum and biomass. Besides these factors, the use of water as the working fluid is an advantage because is relatively cheap and abundant in much of the planet. Recently, the Rankine cycle has been gaining new applications through the use of organic fluids as reported Tchanche, *et al*. (2011), which enables the use of heat sources at low temperature. Due to the large number of plants operating in this cycle, it has been studied not only this cycle, but several other modified Rankine cycles in order to improve the performance of the cycle.

The great contribution to the advancement of these studies was performed by Curzon and Ahlborn (1975), have analyzed the efficiency of the Carnot cycle for the cases in which the output power is limited by the addition of heat in the working fluid and the heat rejection of the fluid. The cycle is modeled with a heat source for addition of heat to the system and a cold source for heat rejection (thermal reservoirs). It was found that for the cycle to operate at Carnot efficiency heat exchange should take place in an infinite time, thus having no temperature difference between the working fluid and the thermal energy sources for a finite net power. In this case, however, the work cycle would run at a speed so low that the power produced would be practically nil when compared to the long time heat exchange. Thus, the authors have proposed a more realistic model in which exists a temperature difference between the heat reservoir and the working fluid. In this case the authors showed that $\eta_{max} = 1 - \sqrt{T_2/T_1}$, where T_2 is the temperature at which the heat is rejected to a low temperature source and T_1 is the temperature at which heat is added from the high temperature source. It shows that this expression is more suitable for real situations than the Carnot cycle efficiency.

Bejan (1988) evaluated the degree of irreversibility existing in a steam power plant operating in steady state. It is considered that these irreversibilities are derived from the heat transfer that occurs in heat exchangers and heat leak occurs from the source of high temperature to the low temperature source. Lee and Kim (1991) optimized Rankine cycle through the concepts of finite time thermodynamics in order to analyze its efficiency in terms of power. And Ion Popescu (2011) also used the finite time thermodynamics to perform an ecological analysis of a Rankine cycle with irreversibilities resulting from the compression and expansion processes non isoentropics. Thus it was possible to find an expression for optimized power in terms of ecological function.

Ecological optimizations appeared from an ecological function proposed by Angulo-Brown (1991) and after applied to a endorreversível Carnot cycle. The ecological criterion appears as an option for analyzing power cycles, defined as the difference between the net power produced by a cycle and this cycle loss of power due to irreversibilities.

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Reviewing the work of Angulo-Brown, Yan (1991), it is showed that the power loss cited by Brown-Angulo as the criterion would be compatible if the environmental temperature of the low temperature heat reservoir was equal to room temperature, which represent the best compromise between power output and the power loss due to the interactions of the power cycle with the environment. Analyzing irreversible Dual cycle, Ust, et al. (2004) analyzed the performance of this cycle under a thermo-ecological criterion defining an ecological coefficient of performance (ECOP) to quantify this analysis. The ECOP was defined as the ratio of the net power available and the power loss due to irreversibilities. The authors showed that the cycle analysis by maximum ECOP has significant advantage over the cycle analysis by maximum ecological function, taking into other hand a decrease in power output.

This paper presents an analysis of an irreversible Rankine cycle in order to optimize the net power through the ECOP. The considered cycle operates between two heat reservoirs with infinite thermal capacity rates, it has internal irreversibilities originated from the non-isoentropic compression and expansion processes, irreversibilities stemming from the thermal resistance in heat exchangers and heat leaks from the high temperature to low temperature. It is presented an analytical expression for the optimum net power along with a graphical analysis, where some operating parameters of the cycle are analyzed for a better understanding of the effect of irreversibilities in optimized power.

2. MATHEMATICAL MODELING

Fig. 1 shows a schematic diagram of the Rankine cycle which will be analyzed, where $\dot{Q}_{H,C}$ is the rate heat effectively transferred from the reservoir of high temperature to the cycle, $\dot{Q}_{L,C}$ is the rate heat effectively transferred from the cycle to the reservoir of low temperature, while $\dot{Q}_I$ is the rate heat transferred directly from high temperature to the low temperature reservoir. This term is also known as "heat leakage". Finally, $\dot{W}$ is the net power of the cycle.

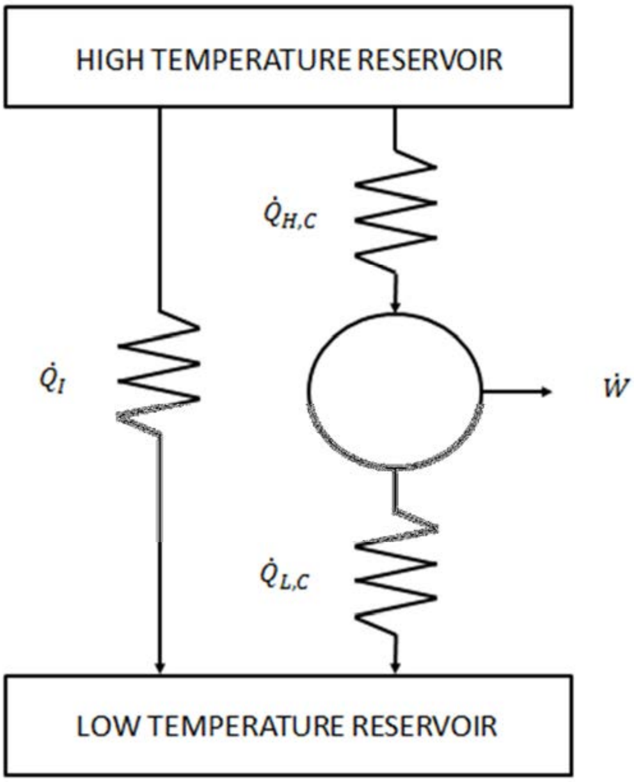

Figure 1 - Schematic diagram of the Rankine cycle.

The thermodynamic processes of the irreversible Rankine cycle can be represented in temperature-entropy diagram according to Fig 2:

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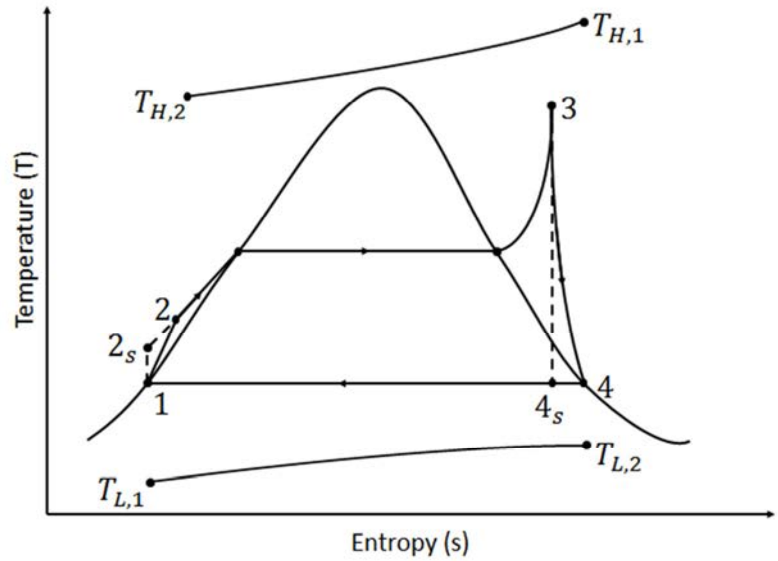

Figure 2 - Temperature-entropy diagram for the irreversible Rankine cycle.

The processes by which the working fluid is submitted to the cycle of Figure 2 are described as follows:

- Process 1-2: Irreversible compression in the pump.
- Process 2-3: Adding heat at constant pressure in the boiler.
- Process 3-4: Irreversible expansion in the turbine.
- Process 4-1: Rejection of heat at constant pressure in the condenser.

According Ion and Popescu (2011), in order to facilitate the mathematical modeling of the Rankine cycle, the cycle can be approximated to a Carnot cycle through the definition of the average entropy temperature. Thus, as shown in Fig 3, the Rankine cycle may be converted into a Carnot cycle through the use of the average entropy temperature defined as:

$$T_l = \frac{H_4 - H_1}{S_4 - S_1} \quad (1)$$

$$T_h = \frac{H_3 - H_2}{S_3 - S_2} \quad (2)$$

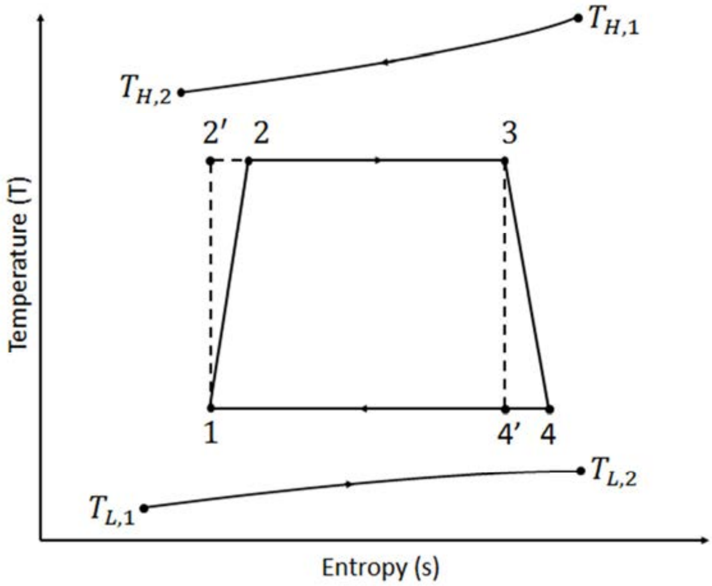

Figure 3 - Temperature-entropy diagram of a modified Rankine cycle modified to a Carnot cycle.

where H_i and S_i ($i = 1,2,3,4$) are, respectively, the enthalpy and entropy of the working fluid. It is assumed that the thermal conductance $U_H A_H$ and $U_L A_L$ and the heat capacity rates are variable and inlet temperatures of the hot and cold fluids, $T_{H,1}$ and $T_{L,1}$, respectively, are fixed. U_H and U_L are the overall heat transfer coefficients of the reservoirs of high and low temperature respectively, and A_H and A_L are the areas of heat transfer in thermal reservoirs of high and low temperature, respectively. Similarly, $\dot{m}_H$ and $\dot{m}_L$ are the mass flows of the stream at high and low temperature respectively, $c_{p,H}$ and $c_{p,L}$ are the specific heats at constant pressure of fluids at high and low temperature, respectively.

Heat transfer rates absorbed and rejected by the fluid are written as follows:

$$\dot{Q}_H = \dot{Q}_{H,C} + \dot{Q}_I \tag{3}$$

$$\dot{Q}_L = \dot{Q}_{L,C} + \dot{Q}_I \tag{4}$$

with:

$$\dot{Q}_{H,C} = U_H A_H \Delta T_{LMTD,H} = \dot{m}_H c_{p,H}\left(T_{H,1} - T_{H,2}\right) = \dot{C}_{W,H}\left(T_{H,1} - T_{H,2}\right) \tag{5}$$

$$\dot{Q}_{L,C} = U_L A_L \Delta T_{LMTD,L} = \dot{m}_L c_{p,L}\left(T_{L,2} - T_{L,1}\right) = \dot{C}_{W,L}\left(T_{L,2} - T_{L,1}\right) \tag{6}$$

where $T_{H,1}$ and $T_{H,2}$ are the inlet and outlet temperatures of the hot stream in the high temperature reservoir, and $T_{L,1}$ and $T_{L,2}$ are the inlet and outlet temperatures of the cold stream in the low temperature reservoir. The average logarithmic temperature differences are defined as:

$$\Delta T_{LMTD,H} = \frac{[(T_{H,1} - T_h) - (T_{H,2} - T_h)]}{\ln\left(\dfrac{T_{H,1} - T_h}{T_{H,2} - T_h}\right)} \tag{7}$$

$$\Delta T_{LMTD,L} = \frac{[(T_l - T_{L,1}) - (T_l - T_{L,2})]}{\ln\left(\dfrac{T_l - T_{L,1}}{T_l - T_{L,2}}\right)} \tag{8}$$

with $\dot{C}_{W,H}$ and $\dot{C}_{W,L}$ the heat capacity rates of the hot and cold fluids, respectively. The heat transfer rate between the reservoirs can be calculated as:

$$\dot{Q}_I = \dot{C}_I\left(T_{H,1} - T_{L,1}\right) \tag{9}$$

Combining Eqs. (5) and (7) and Eqs. (6) and (8), it can obtain the following expressions:

$$T_{H,2} = T_h + (T_{H,1} - T_h)e^{-NTU_H} \tag{10}$$

$$T_{L,2} = T_l + (T_{L,1} - T_l)e^{-NTU_L} \tag{11}$$

with:

$$NTU_H = \frac{U_H A_H}{\dot{m}_H c_{p,H}} \tag{12}$$

$$NTU_L = \frac{U_L A_L}{\dot{m}_L c_{p,L}} \tag{13}$$

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are the numbers of transfer units of the heat exchangers of high and low temperature reservoir, respectively. Net power cycle may be determined as follows:

$$W = \dot{Q}_H + \dot{Q}_L \tag{14}$$

Using Eqs. (5), (6), (10) e (11) into Eq. (14), it is obtained:

$$\dot{W} = c_{p,H}\left(T_{H,1} - T_h\right)\left(1 - e^{-NTU_H}\right) - c_{p,L}\left(T_l - T_{L,1}\right)\left(1 - e^{-NTU_L}\right) \tag{15}$$

Isolating T_l obtain:

$$T_l = \left[\frac{-\dot{W}ac_{p,H}\left(T_{H,1} - T_h\right)}{bc_{p,L}}\right] + T_{H,1} \tag{16}$$

where:

$$a = \left(1 - e^{-NTU_H}\right) \tag{17}$$

$$b = \left(1 - e^{-NTU_L}\right) \tag{18}$$

The addition of irreversibility is performed by a parameter proposed by Wu and Kiang (1992). This parameter quantifying the degree of internal irreversibilities of a Carnot cycle being written in the following form:

$$R = \frac{\Delta S_l}{\Delta S_h} = \frac{S_{4'} - S_1}{S_3 - S_{2'}} \tag{19}$$

where ΔS_h represents the variation of entropy in adding heat in an irreversible Carnot engine and ΔS_l represents the variation of entropy in heat rejection of the same engine. As can be seen in Figure 3, necessarily must be greater than one, it being understood that one has to be equal an endoreversible cycle. This parameter includes the efficiencies of irreversible processes of pumping and expansion. Applying the second law of thermodynamics to the irreversible Carnot cycle can be written the following inequality:

$$\frac{\dot{Q}_{L,C}}{T_l} - \frac{\dot{Q}_{H,C}}{T_h} > 0 \tag{20}$$

Substituting Eq. (19) in Eq. (20), it can rewrite the inequality of Eq. (20) by an equality of the form:

$$\frac{\dot{Q}_{L,C}}{T_l} - R\frac{\dot{Q}_{H,C}}{T_h} = 0 \tag{21}$$

Substituting Eq. (16) into Eq. (21) and solving to T_h is obtained:

$$T_h = \frac{-n_1\dot{W} - n_2 \pm \sqrt{n_1^2\dot{W}^2 + 2n_1n_2\dot{W} + n_2^2 - 4mp_1 - 4mp_2}}{2 \cdot m} \tag{22}$$

with:

$$n_1 = Rac_{p,H} + bc_{p,L} \tag{23}$$

$$n_2 = ac_{p,H}T_{H,1}bc_{p,L} - 2a^2{c_{p,H}}^2T_{H,1} - Rac_{p,H}T_{L,1}bc_{p,L} \tag{24}$$

$$p_1 = Rac_{p,H}T_{H,1} \tag{25}$$

$$p_2 = Ra^2{c_{p,H}}^2{T_{H,1}}^2 + Rac_{p,H}T_{H,1}bc_{p,L}T_{L,1} \tag{26}$$

$$m = ac_{p,H}bc_{p,L} + Ra^2c_{p,H}{}^2 \tag{27}$$

The ecological coefficient of performance (ECOP) can be calculated from its definition, that is:

$$ECOP = \frac{\dot{W}}{T_0\left[\left(\frac{\dot{Q}_L}{T_l}\right) - \left(\frac{\dot{Q}_H}{T_h}\right)\right]} \tag{28}$$

To maximize the ecological coefficient of performance in relation to the power cycle, it can be used the following mathematical operation:

$$\frac{\partial(ECOP)}{\partial \dot{W}} = 0 \tag{29}$$

After algebraic manipulation is obtained an algebraic equation for the optimized power cycle:

$$A_1\dot{W}_{opt}^2 + A_2\dot{W}_{opt} + A_3 = 0 \tag{30}$$

where:

$$A_1 = -b_2{}^2m^2p_1{}^2 + b_2{}^2n_1n_2p_1m - b_2n_2b_3mn_1{}^2 + b_3{}^2m^2n_1{}^2 \tag{31}$$

$$A_2 = -4b_2{}^2m^2p_1p_2 + 4b_2n_2b_3m^2p_1 - 4b_3{}^2m^3p_1 + 2b_2{}^2n_1n_2mp_2 + 2b_3{}^2n_1n_2m^2 - 2b_2n_2{}^2b_3n_1m \tag{32}$$

$$A_3 = b_3{}^2m^2n_2{}^2 + 4b_2n_2b_3m^2p_2 - 4b_2{}^2m^2p_2{}^2 + b_2{}^2mp_2n_2{}^2 - 4b_3{}^2m^3p_2 - b_2n_2{}^3b_3m \tag{33}$$

with:

$$b_1 = T_{H,1}T_{L,1} \tag{34}$$

$$b_2 = T_{H,1}ac_{p,H} - T_{L,1}ac_{p,H} \tag{35}$$

$$b_3 = -T_{H,1}{}^2c_I - T_{H,1}{}^2ac_{p,H} + 2T_{H,1}T_{L,1}c_I + T_{H,1}T_{L,1}ac_{p,H} - T_{L,1}{}^2c_I \tag{36}$$

The solution of Eq. (30) is given for:

$$\dot{W}_{opt} = \frac{-A_2 \pm \sqrt{A_2{}^2 - 4A_1A_3}}{2A_1} \tag{37}$$

Substituting the results obtained by Eq. (37) into Eq. (22) and (16) it makes possible the calculation of optimized average entropic temperatures ($T_{h,opt}$ and $T_{l,opt}$). With results obtained by Eqs. (22) and (16) it can be calculated the heat transfer rates and the optimized temperatures using Eqs. (3), (5) e (10) and Eqs. (4), (6) e (11). Finally, through an entropy balance for the irreversible Carnot cycle, it can determine the optimal entropy generation rate of the cycle based on the optimized power output, namely:

$$\dot{S}_{g,opt} = \left(\frac{\dot{Q}_{L,opt}}{T_{l,opt}} - \frac{\dot{Q}_{H,opt}}{T_{h,opt}}\right) \tag{38}$$

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3. RESULTS AND DISCUSSION

In Fig. 4 it can be seen the variation of ECOP with power output to an irreversible Rankine cycle to values of $T_{H,1}$ equals to 1000K, 1200K and 1400K. It is observed that there is only one maximum power point and a point of maximum ECOP for each curve. Values to the left of the ECOP maximum values of power output are smaller, and in the same way, to values below the point of maximum power output, the values of the ECOP are low. Thus, it is evident that in the region between the ECOP and maximum power output occurs with higher values of ECOP and power output. Note also that with the increase of $T_{H,1}$, there is an increase of both the maximum ECOP as the maximum power output.

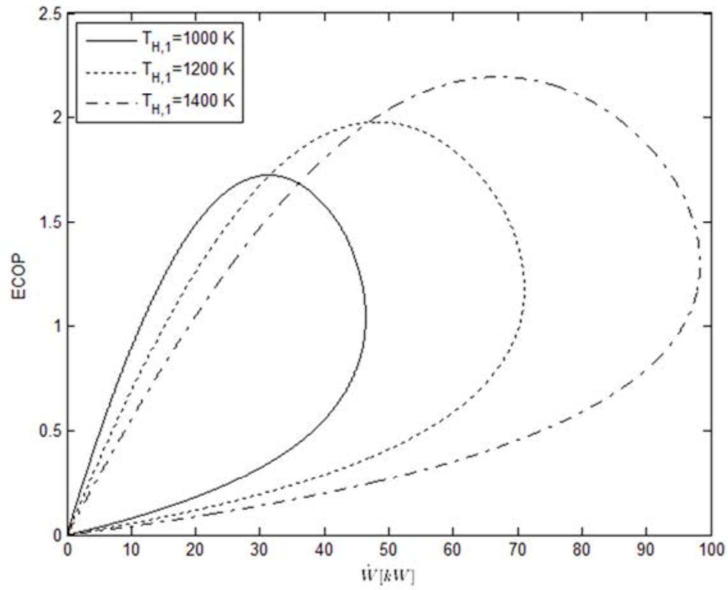

Figure 4. ECOP function of the power output, with $NTU_H = NTU_L = 1$; $R = 1.2$; $\dot{C}_I = 0.02$ kW/K; $T_0 = 298.15$ K.

In Fig.5 can be seen the variation of ECOP with the power output for various values of heat leakage capacity rates. It is observed that the increase of heat leakage heat capacity rates increases optimum power (the point where the maximum output corresponding to maximum ECOP) while the point at which the maximum ECOP is decreased. However, when the values of the heat leakage capacity rates are zero there is a maximum ECOP of cycle, as it can be seen in Fig 5.

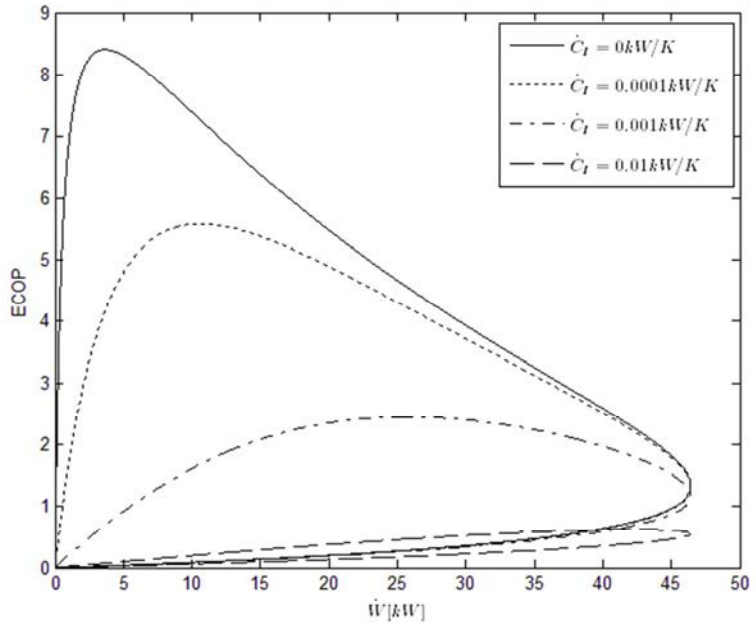

Figure 5. ECOP function of the power output $NTU_H = NTU_L = 1$; $R = 1.2$; $T_{H1} = 1000$ K; $T_0 = 298.15$ K.

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In Fig. 6 can be seen the variation of ECOP with the power output for different values of the degree of irreversibility R. It is observed that increasing the degree of irreversibility of the cycle causes a great reduction both power as the maximum ECOP.

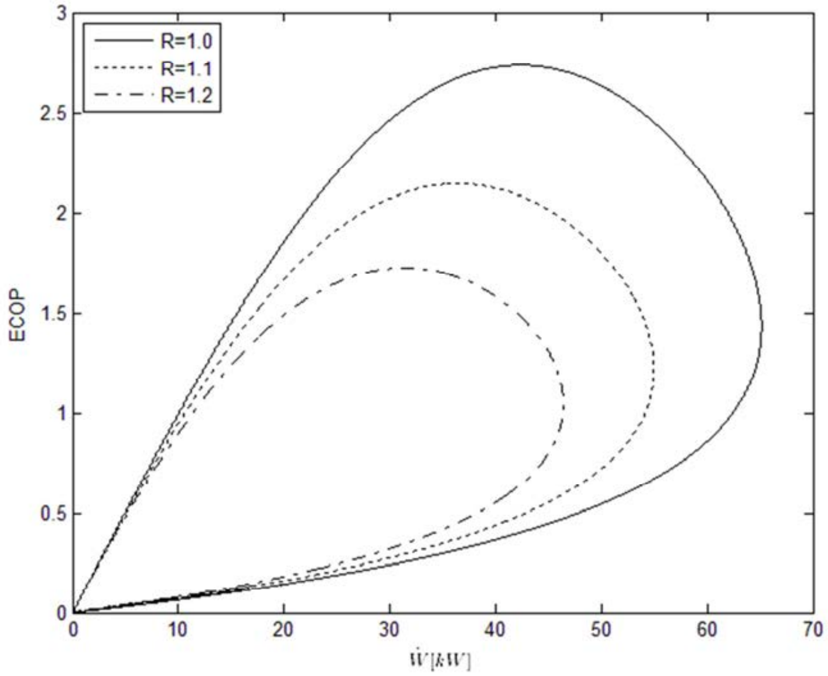

Figure 6. ECOP function of the power output, with
$NTU_H = NTU_L = 1;\ T_{H,1} = 1000\,\text{K};\ \dot{C}_I = 0.02\,\text{kW/K};\ T_0 = 298.15\,\text{K}.$

The variation of ECOP with thermal efficiency is shown in Fig. 7, it can be observed as the temperature of the hot source increase, ECOP and the thermal efficiency increases. The values of thermal efficiency increase to values close to Carnot efficiency where they are stabilized while the values of the ECOP tend to infinity. This is due to low values for entropy generation rate for these values of thermal efficiency.

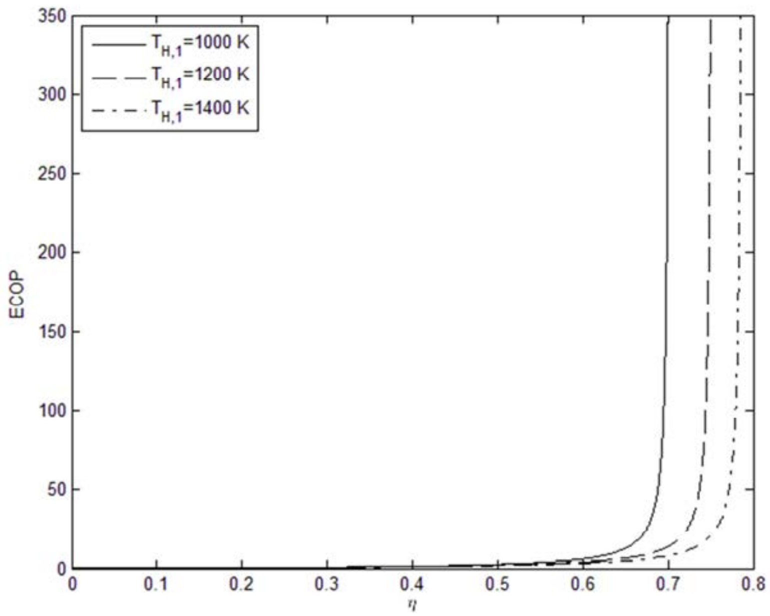

Figure 7. ECOP function of the thermal efficiency, with $NTU_H = NTU_L = 1;\ R = 1.2;\ \dot{C}_I = 0.02\ \text{kW/K};\ T_0 = 298.15\ \text{K}.$

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Figs. 8 and 9 represent the variation of ECOP with thermal efficiency. In Figure 8 is analyzed the influence of heat leakage capacity rates while in Fig. 9 is considered the influence of the degree of irreversibility in the cycle. In both figures it can be observed that the two parameters analyzed (heat leakage and the degree of irreversibility) does not change the behavior of the function, as observed by Ust, et al. (2005). According to the author, the behavior of the curve ECOP in function of thermal efficiency depends only on the extreme temperatures and the environment temperature.

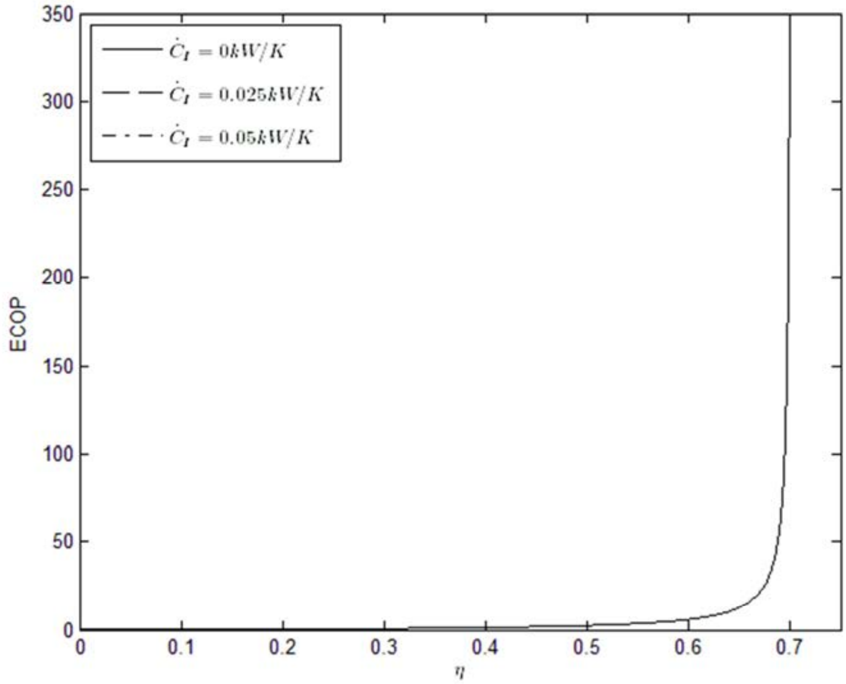

Figura 8. ECOP function of the thermal efficiency, with $NTU_H = NTU_L = 1$; $R = 1.2$; $T_{H,1} = 1000$ K; $T_0 = 298.15$ K.

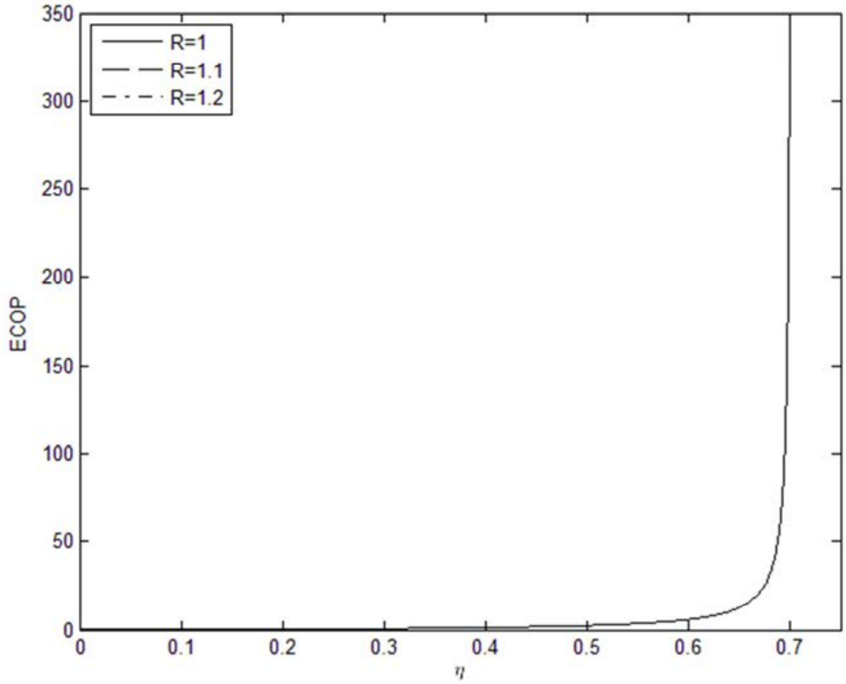

Figura 9. ECOP function of the thermal efficiency, with
$NTU_H = NTU_L = 1$; $T_{H,1} = 1000$K; $\dot{C}_I = 0.02$ kW/K; $T_0 = 298.15$K.

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In Fig.10 can be seen the relationship between the optimized power by the ECOP criteria and the optimized power by the maximum power criteria as a function of NTU_L. It is observed that increasing the temperature of the heat source causes an increase of this ratio, as the NTU_L increases the ratio of optimized power by ECOP criteria and the optimized power by the maximum power criteria decreases to a certain value, becoming stable. For the conditions adopted in this cycle, optimized power represents about 64.5% of the maximum power cycle.

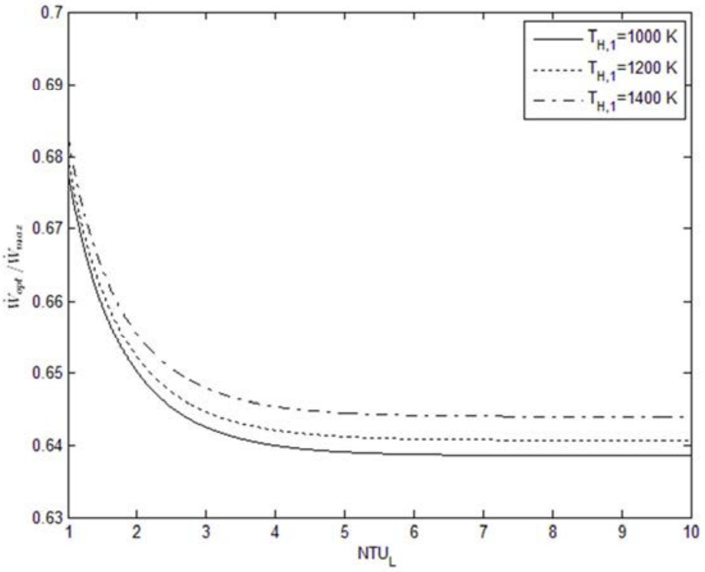

Figure 10. Ratio between the power optimized for ECOP and maximum power in function of NTU_L, with

$NTU_H = NTU_L = 1;\ R = 1.2;\ \dot{C}_I = 0.02$ kW/K; $T_0 = 298.15$ K.

Fig. 11 shows the ratio of the entropy generation rate due to the optimization by the ECOP criteria to the entropy generation rate due to the optimization by the maximum power criteria as a function of NTU_L. It is observed that an increase in temperature of the heat source causes a reduction in this ratio, as the NTU_L increases this ratio decreases to a certain value, stabilizing at a value close to 36%. This figure shows the capacity of this optimization to reduce the entropy generation rate.

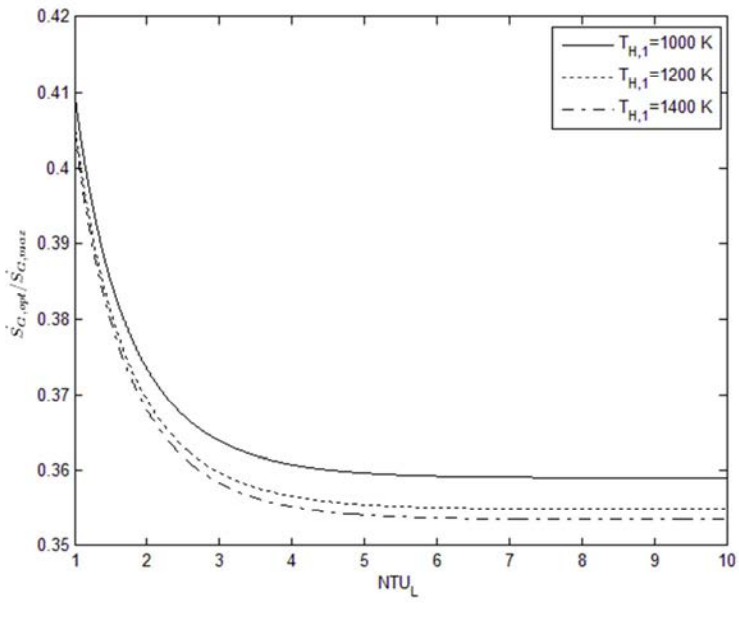

Figura 11- Ratio between the rate of generation of entropy due to optimized power by ECOP and the generation rate of entropy due to maximum power as a function of NTU_L, with

$NTU_H = NTU_L = 1;\ R = 1.2;\ \dot{C}_I = 0.02$ kW/K; $T_0 = 298.15$ K.

4. CONCLUSIONS

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In this work, it was performed an ecological optimization of an irreversible Rankine cycle based on ecological coefficient of performance (ECOP), where the irreversibility of processes are originated mainly of non-isentropic pumping and expansion in the turbine, the heat exchange processes that occur in finite time and the heat leak of hot source to the cold source.

The ECOP is an alternative function to the ecological function, defined by the ratio between the power and the rate of entropy generation. It was demonstrated that for the irreversible Rankine cycle optimized by the criterion of maximum ECOP: an increase of heat leakage capacity rates causes an increase of optimum power while the ECOP decreases; an increase of irreversibilities decreases the optimum power and the ECOP, the thermal efficiency of the cycle is not affected by the variation of heat leakage capacity rates and the irreversibilities.

Finally, it can be seen that the result of this optimization was satisfactory because, despite a little loss of power, the gains in the minimization of the entropy generation rate were higher when compared to the maximum power operating condition. Thus, these results can be used to determine optimum conditions for operation and design of new machines based on irreversible Rankine cycle.

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