

ATTITUDE AND POSITION TRACKING SYSTEM FOR A 6-6 STEWART PLATFORM

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Abstract. Application of multivariable control techniques with the aid of computing tools has been largely utilized in the design of complex control systems based on the concept of state variables. This paper presents the development and implementation of a multivariable control tracking system with state feedback. It also presents the modeling of a Stewart platform with six degrees of freedom, actuated by electromechanical actuators, obtained through the virtual prototyping environment system ADAMS®. It utilizes multi body systems modeling technique to obtain the equations of motion. The platform was developed for studies of control systems for flight simulators in the Laboratory of Airspace Control of the Engineering School of São Carlos of the University of São Paulo. The controller design is executed in a linear model obtained by ADAMS®. Design of the control system is done using the entire eigenstructure assignment technique. Inverse kinematic was applied to determine the length values of actuators that satisfy a known position and orientation of the platform. The modeling package ADAMS® provides flexibility and agility in obtaining the dynamic model. Simulation results are used to illustrate the robustness and performance of the designed feedback controller to cause the controller output to track a desired command input.

Keywords: Stewart Platform, Tracking System, Attitude Controller, Position Controller.

1. INTRODUCTION

The development of didactic systems that make possible the study of control techniques can result in great benefits for the presentation of current themes in an introductory control course (Campo, 2007).

The modern tendency of the engineering systems is to increase the complexity mainly due to the need of accomplishing high precision tasks. In order to support crescent and rigorous exigency of control systems performance, increasing systems complexity and the easy access to computers, the theory of modern control, which is an algebraic approach for the analysis and project of complicate control systems, is based on the state space concept.

Mechanical systems which allow a rigid body (here called effector) to move relatively to a fixed base, play a very important role in many applications, especially in flight simulators. A rigid body in space can move in various ways, in translation or rotation movements. These are called degrees of freedom. The total number of degrees of freedom of a rigid body in space may not exceed six (for example, three translational movements along mutually perpendicular axes and three rotational movements around these axes), (Montezuma, 2010).

So, the main purpose of this paper is to present the platform that was developed for studies of control systems for flight simulators in the Laboratory of Airspace Control of the Engineering School of São Carlos of the University of São Paulo.

2. THE STEWART PLATFORM

Usually, six legs are spaced around the top plate and share the load on the top plate (Stewart, 1965). This differs from serial designs, such as robot arms, where the load is supported over a long moment arm. The position and attitude of the movable platform varies depending on the lengths to which the six legs are adjusted (Rosario, 2007).

The tracking system strategy developed in this paper addresses a plant of a Stewart Platform, controlling six degrees of freedom. This system is basically composed of a fixed platform connected by six electromechanical actuators, on a movable platform.

Each actuator consists of an electric motor with gear transmission for a ball screw. The motor is actuated by a voltage signal with amplitude of up to 12V, and changes its direction of rotation when the polarization is reversed. The Actuators are connected to the fixed and movable platforms by means of universal joints.

The mathematical model is obtained using the virtual prototyping environment system ADAMS[®], shown in Fig. 1 that uses modeling techniques of multi-body systems to obtain the dynamic motion equations. This software has a graphic interface to aid the creation of the mathematical model and for the simulation results visualization where the wanted parameters are supplied in a simple and fast way. Afterwards the model is exported to the simulations environment MatLab[®].



Figure 1 - Stewart Platform modeled in ADAMS®

2.1 Inverse Kinematics

In the design of a position and attitude control system of the movable platform of a Stewart platform, it is necessary to know the inverse kinematics of this mechanism (Travi, 2009). The inverse kinematics uses the position and attitude of the movable platform with respect to the fixed platform to obtain the lengths of the actuators and can be addressed using tensor modeling (Zipfel, 2000) or modeling based on linear algebra (Nguyen et al., 1993). The modeling using linear algebra is presented in this paper.

The positions of the joints connecting the platforms to the actuators are defined in two coordinate systems (Gonzalez Acuña, 2009). A system with origin in the center of the fixed platform F and axis xf pointing between joints 1 and 6 of the fixed platform, axis zf perpendicular to the plane of the fixed platform pointing up and axis yf completing the right-hand rule. The other system has the origin in the center of the movable platform M and axis xm pointing between joints 1 and 6 of the movable platform, axis zm perpendicular to the plane of the movable platform M and axis xm pointing upward and axis ym completing the right-hand rule. Figure 2 shows the definitions of the two coordinate systems.

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Figure 2 - Coordinate systems for fixed and movable platform

The positions of the joints of the fixed and movable platforms coordinate systems centered at F_i and M_i respectively are expressed by Eqs. (1), (2), (3) and (4) as follows:

$$\{F_i\}^F = \{rf\cos(\Lambda f_i) \quad rf\sin(\Lambda f_i) \quad 0\}^T = \{F_{i1} \quad F_{i2} \quad 0\}^T, \ i = 1, 2, \dots, 6$$
(1)

 $\{M_i\}^M = \{rm\cos(\Lambda m_i) \quad rmsin(\Lambda m_i) \quad 0\}^T = \{M_{i1} \quad M_{i2} \quad 0\}^T$ (2)

$$\Lambda f_i = 60^{\circ}i - \lambda f, \ \Lambda m_i = 60^{\circ}(i-1) + \lambda m, \quad i = 1,3,5$$
(3)

$$\Lambda f_i = 60^{\circ}(i-1) + \lambda f, \quad \Lambda m_i = 60^{\circ}i - \lambda m, \quad i = 2,4,6$$
(4)

Where *rf* and *rm* are the radii of the circles centered at the center of the platform and contain the positions of the joints of the fixed and movable bases, respectively, and λf and λm are angles that help to define the positions of the joints of the fixed and movable platforms, respectively.

The vector representing the actuator in the fixed platform coordinate system $\{V_i\}^F$ is obtained using the equation (5).

$$\{V_i\}^F = \{M_i\}^F - \{F_i\}^F \tag{5}$$

and the vector representing the position of the joints of the movable platform in the fixed coordinate system is defined in Eq. (6)

$$\{M_i\}^F = \{M\}^F + [T^{MF}] \{M_i\}^M = \begin{cases} x \\ y \\ z \end{cases} + \begin{cases} u_i \\ v_i \\ w_i \end{cases}$$
(6)

Where $\{M\}^F$ is the vector that represents the position of the center of the movable platform in the coordinate system of the fixed platform and $[T^{MF}]$ is the transformation matrix of the movable coordinate system to the fixed coordinate system.

Using a sequence of three rotations, it is possible to obtain the transformation matrix $[T^{MF}]$. First, a rotation φ is applied around the axis *xm* until axis *ym* becomes parallel to the plane formed by *xf* and *yf*, and the rotation angle φ is called roll angle. Then, a rotation θ is applied around *ym* until *xm* is parallel to the plane formed by *xf* and *yf*, being θ the pitch angle. Finally, a rotation around *zm* is applied until *xm* is parallel to *xf*, and this angle of rotation is the yaw angle ψ . The resulting matrix of the three rotations is shown in Equation (7).

$$[T^{MF}] = \begin{bmatrix} c\varphi c\theta & c\varphi s\theta s\psi - s\varphi c\psi & c\varphi s\theta c\psi + s\varphi s\psi \\ s\varphi c\theta & s\varphi s\theta s\psi + c\varphi c\psi & s\varphi s\theta c\psi - c\varphi s\psi \\ -s\theta & c\theta s\psi & c\theta c\psi \end{bmatrix}$$
(7)

where *c* is the cosine of the angle and *s* the sine.

The vector representing the *i* th-actuator $\{V_i\}$ is obtained using information about the geometry of the Stewart platform and the position and attitude set of the movable platform. The module of this vector $|V_i|$ is equal to the length of the actuator it represents.

2.2 Control System

A controllable open-loop system is represented by the *n* th-order state and *p* th-order output equations of the form

$$\dot{x} = Ax + Bu \tag{8}$$

$$y = Cx = \begin{bmatrix} E \\ F \end{bmatrix} x \tag{9}$$

where y is an $p \times 1$ vector and w = Ex is a $m \times 1$ vector representing the outputs which are required to follow a $m \times 1$ input vector r.

A feedback controller is required to cause the output vector w to track the command input r in the sense that the steady state response is:

$$\lim_{t \to \infty} w(t) = r(t) \tag{10}$$

where *r* consists of piecewise-constant command input.

The design method consists of the addition of a vector comparator and an integrator which satisfies the equation:

$$\dot{z} = r - w = r - Ex \tag{11}$$

The composed open-loop system is therefore governed by the augmented state and output equations formed from Eqs. (8) to (11):

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -E & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} r = \bar{A}x' + \bar{B}u + \bar{B}'r$$
(12)

$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} = \bar{C}x'$$
(13)

where

$$\bar{A} = \begin{bmatrix} A & 0\\ -E & 0 \end{bmatrix} ; \ \bar{B} = \begin{bmatrix} B\\ 0 \end{bmatrix} ; \ \bar{B}' = \begin{bmatrix} 0\\ I \end{bmatrix} ; \ \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}$$
(14)

D'Azzo and Houpis, (1995) present the state feedback control law to be used here which is:

$$u = K_1 x + K_2 z = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix}$$
(15)

$$\overline{K} = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \tag{16}$$

A block diagram representing the feedback control system, consisting of the plant state and output equations given by equations (8) and (9) and the control law given by equations (15) and (16), is shown in Fig. 3. This control law assigns the desired closed loop eigenvalues spectrum if and only if the matrices pair $(\overline{A}, \overline{B})$ is controllable. It has been shown that this condition is satisfied if (A, B) is a controllable pair and

$$Rank\begin{bmatrix} B & A\\ 0 & -E \end{bmatrix} = n + m \tag{17}$$

This condition is much easier to use than the determination of the rank of the controllability matrix for $(\overline{A}, \overline{B})$. Equation (17) can be satisfied only if the number of outputs p which are required to track the input r is less than or equal to the number of control inputs m. Satisfaction of the condition of Eq. (17) guarantees that a control law of the form of Eq. (15) can be synthesized such that the closed-loop output tracks the command input. In that case the closed-loop state equation is:

$$\dot{x}' = \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A + BK_1 & BK_2 \\ -E & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} r = A'_{cl}x' + B'r$$
(18)

The feedback matrix of Eq. (16) can be selected so that the eigenvalues are in the left-half plane for the closed-loop plant matrix of Eq. (18).

The purpose in applying state feedback is to assign a closed loop eigenvalue spectrum

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$$\sigma(\bar{A} + \bar{B}\bar{K}) = \{\lambda_1, \lambda_2, \dots, \lambda_{n+m}\}$$
(19)

and an associated set of eigenvectors

$$v(A + BK) = \{v_1, v_2, \dots, v_{n+m}\}$$
(20)

which are selected in order to achieve the desired time-response characteristics. The closed-loop eigenvalues and eigenvectors are related by the equation

$$(\bar{A} + \bar{B}\bar{K})v_i = \lambda_i v_i \tag{21}$$

This equation can be put in the form

$$\left[\bar{A} - \lambda_i I \bar{B}\right] \begin{bmatrix} \nu_i \\ g_i \end{bmatrix} = 0 \qquad for \ i = 1, 2, \dots n + m \tag{22}$$

where v_i is an eigenvector and

$$g_i = K v_i \tag{23}$$

In order to satisfy Eq. (22), the vector $\begin{bmatrix} v_i^T & g_i^T \end{bmatrix}$ must lie in the kernel or null space of the matrix

$$\bar{S}(\lambda_i) = [\bar{A} - \lambda_i I \bar{B}] \quad for \ i = 1, 2, \dots n + m \tag{24}$$

The notation $ker\bar{S}(\lambda_i)$ is used to define the null space which contains all vectors $[v_i^T \quad g_i^T]$ for which Eq. (22) is satisfied. Equation (23) can be used to form the matrix equality

$$[g_1 \ g_2 \ \dots \ g_{n+m}] = [\bar{K}v_1 \ \bar{K}v_2 \ \dots \ \bar{K}v_{n+m}]$$
(25)

Since \overline{K} can be factored from the right-hand matrix of Eq. (25)

$$\overline{K} = [g_1 \quad g_2 \dots g_{n+m}] [v_1 \quad v_2 \dots \quad v_{n+m}]^{-1} = QV^{-1}$$
(26)

Therefore, if the desired eigenvalue spectrum of Eq. (19) is specified and the associated eigenvectors are selected to satisfy Eq. (22), then Eq. (26) specifies the required states-feedback matrix \overline{K} .

The $ker\bar{S}(\lambda_i)$ imposes constraints on the eigenvector v_i that may be associated with the assigned eigenvalue- λ_i . The $ker\bar{S}(\lambda_i)$ identifies a specific subspace, and the selected eigenvectors v_i must be located within this subspace. In addition, the selected eigenvectors must be linearly independent so that the inverse matrix V^{-1} in Eq. (26) exists.



Figure 3 - The Simulink Tracking System Block Diagram

3. RESULTS

C =

The linearized model in terms of state-variables, according to Fig. 3, is used to accomplish the control around an operation position. It is represented by four matrices: *A*, *B*, *C* and *D*. The matrix *A* has dimensions $n \times n$ where *n* is the number of states which is twice the number of degrees of freedom of the dynamic system. Since the Stewart Platform system has six degrees of freedom, *n* is equal to twelve. The matrix *B* has dimensions $n \times m$ where *m* is equal to six, the number of inputs of the system. The matrix *C* is $p \times n$ where *p* is the number of outputs of the system and the matrix *D* is $p \times m$ which has all their elements null in the case. Also the linear model is used to calculate the gains of the control system.

The matrices generated by: ADAMS ® are:

	$\begin{bmatrix} 0\\1 \end{bmatrix}$	1.58	$\begin{array}{ccc} 3 & 0 & 0 \\ 0 & 0 \end{array}$.187 0	0.006	0 -0	.039	0 0	0.005	0 -0.1	³⁰]	
		0 350	0 -0	083 0	-0 157	0 0	129	0-0	0 065	0 00	74	
	0	0	1	0 0	0	0	0	0	0	0 0		
	0	-0.01	3 0 0	.094 0	1.693	0 -0	.051	0 -0	0.018	0 0.02	28	
	$A = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	0	0	0 1	0	0	0	0	0	0 0		
		0.01	1 0 0	.143 0	0.204	0 -0	0.148	0 0	0.290	0 0.1:	54	
		0.01'	7 0 -0	004 0	-0.017	1	033	0 1	0 680			
		0.01	/ 0 -0 0	0 0	-0.017	0 -0	0	1	0			
	0	-0.31	2 0 0.	.064 0	-0.179	0 0	.146	0 -0	0.078	0 -0.1	00	
	L ₀	0	0	0 0	0	0	0	0	0	1 0]	
		Г	-4.150	3.815	0.468	-0.5	35	-3.526	5 4.0	ר 280		
			0	0	0	0)	0		0		
			15.731	-69.731	63.869	-6.5	11	22.153	3 -4.	535		
			0 8 908	0 10 844	-1 155	-5.1	10	-0.264	5 2'	732		
		л	0.900	0	0	-5.1	10	-0.20.	, 2.	0		
		B =	63.406	5.893	-22.125	-4.7	78	-15.50	7 68.	.862		
			$0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\$	0	0		12	0		0		
			2.820	-0.286	-5.227	-1.2	13	10.730) 8.9	0		
			-22.081	5.045	15.748	69.3	392	-63.41	7 -6.	173		
		l	- 0	0	0	0		0		0		
гО	0.197	0	-0.283	0	-0.199	0	-	0.925	0	0.093	3 0	0.212 ₁
0	0.011	0	-0.983	0	0.173	0	-	0.000	0	-0.00	2 0	-0.001
0	-0.168	0	-0.949	0	-0.228	0	(0.239	0	0.060) 0	-0.290
	0.113	0	0.018	0	0.044	0	-	0.005	0	-0.13		-1.009
	-0.023	0	-0.011	0	-0.005	0	-	0.301	0	-0.12	2 0	-0.898
0.197	0	-0.283	3 0	-0.199	0.0.12	-0.92	25	0	0.093	3 0	0.212	0
0.011	Ō	-0.983	3 0	0.173	0	-0.00	00	0	-0.00	2 0	-0.001	0
-0.168	0	-0.949) 0	-0.228	0	0.23	9	0	0.060) 0	-0.290	0
0.113	0	0.018		0.044	0	-0.00)5	0	-0.13	0 0	-1.009	0
-0.023	0	0.219		-0.005	0	-0.30)]	0	0.282		-0.898	0
L-0 .111	0	-0.01	1 0	0.042	0	-0.95	0	0	-0.12	9 0	0.005	0 1
				D =	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0					

In order to analyze if the tracking system controller is able to follow the input signal of the plant Stewart Platform, some simulations were conducted.

The first experiment used the following values: dx = 0 mm, dy = 0 mm, dz = 300 mm and the angles φ , θ and ψ equal to zero. This value will serve as the initial condition for the study of the movement of the Stewart platform, considering that the real actuators, which served as a basis for data modeling, have 600 mm of length maximum in dz.



Figure 4 - Variation length of 300 mm in dz of Stewart Platform

It can be seen in Fig. 4 that the values calculated in inverse kinematics for the six actuator lengths converged to the desired input. Also in Fig. 5 one can see that the error of the length of all the actuators tended to zero, showing the effectiveness of the control system.



The second experiment used the following values: dx = 0 mm, dy = 0 mm, dz = 300 mm and the angles $\varphi = 0, \theta = 0$ 30° and $\psi = 0$. Figure 6 shows that in this condition the tracking system controller also tracks the desired inputs.



In Fig. 7, it turns out that the error of length of all actuators has tended to zero.



Figure 8 shows how the actuators lengths changed for variations in the position and attitude using the values dx = 20 mm, dz = 300 mm, $\theta = 20^{\circ}$, $\varphi = 10^{\circ}$ e $\psi = 30^{\circ}$.



Figure 8 – Length for a variation of attitude and position



The errors in the length of each actuator to variations in the attitude and position are shown in Fig. 9.

4. CONCLUSION

One can conclude that the model suggested in this work with a multivariable control tracking system with state feedback, responded with speed and efficiency to the desired command in the controller input. Also one can say that in all simulations the steady state error tended to zero. The modeling of the Stewart Platform using software *ADAMS*® is extremely important to provide flexibility and speed in obtaining the dynamic model.

5. ACKNOWLEDGEMENTS

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