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STUDY OF NONLINEAR DYNAMIC BEHAVIOR OF A MICROELECTROMECHANICAL ACTUATOR SYSTEM

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Abstract. *MicroElectroMechanical Systems (MEMS) are systems what are composite by electrical and mechanical devices which interact by means of transducers elements, so that forming one single and whole system. It's an invasive microsystem, intermediator and interactive what develops its activities very efficiently and versatility. MEMS dispositive have been considered a future technology, used in too many areas, like, for example, mobiles, audio equipments and hearing aids. MEMS are a combination of microstructures, microsensors and microactuators. The actuators electro-mechanics are transducers that convert electrical energy into mechanical energy, been composed by two basic functions: the first one, one univocal function, what sets the relationship between the input signal and the output signal; the second function is of driver, what has de function of monitoring and force's generation. The main electro-mechanic actuator used in MEMS' devices is the Electrostatic one. The goal in this work is studying the non-linear properties of a micro-actuator electrostatic, kind beam, into many numerical simulations (Time History, phase portrait, Spectrum of Frequency, Lyapunov's exponent) are done to illustrate the micro-actuator's dynamic behavior, what is shown.*

Keywords: MicroElectroMechanical System, Non-Linear Dynamics, Actuator.

1. INTRODUCTION

Micro electro mechanical System (MEMS) technology are linking devices in the micrometer range, i.e. less than *1mm* and greater than *1μm*. These devices collect, as its name suggests, properties of microelectronics, micromechanics and others. These devices have been used as temperature sensors, flow, speed, sound, linear and angular motion actuators, and as components of complex systems, such as robots, micro pumps heat etc. (Gad-El-Hak, 2012; Rugar *et al*, 1992).

MEMS are a technology which enables the development of smart products, increasing the computational ability of microelectronics with the control of micro sensors and micro actuators, thereby expanding the space of possible designs and applications. (Illic *et al*, 2000; Kenny, 2001)

Nowadays, manufacturing processes can produce very small devices, producing electrostatic, magnetic, electromagnetic, thermal and pneumatic actuators, motors, valves, gears etc. all with less than *100μm* in size. By the fact the devices are fabricated using MEMS manufacturing techniques similar to those used in the manufacture of integrated circuits, allowing unprecedented levels of functionality, reliability and sophistication can be placed on a small chip and a relatively low cost (Gad-El-Hak, 2012).

MEMS are found in diverse applications with accelerometers for cars. In MEMS accelerometers, measure vehicle acceleration through the oscillation of a small device and send an electrical impulse to a microprocessor. In the fraction of a second prior to the accident, there is a sharp drop in vehicle velocity, acceleration, what changes the frequency of the devices oscillation and the response of the microprocessor is to activate the air bags. In Brazil, CONTRAN (Conselho Nacional de Trânsito) determined that, from the year 2014, cars must have air-bags regardless of model (http://veja.abril.com.br). Thus, MEMS use will probably increase next years, making important its study.

In this work, is studied the behavior of a MEMS device, beam-type micro electrostatic actuator. This MEMS have been proposed by Takamatsu and Sugiura (2005) and the non-linear properties are simulated numerically demonstrating the dynamic behavior: stable, unstable and chaotic. Besides, an analytical study of stability is done in function of the system's eigenvalues, what is achieved with the linearization of the system.

The paper is organized as follows: in Section 2, we demonstrated the mathematical model for the micro electrostatic actuator in study and we discussed its linear stability. In Section 3, we discuss and include the parametric resonance causing chaotic behavior in the system. In Section 4, we make the concluding remarks of this paper. In Section 5, we make some acknowledgements. Finally, we list out the bibliographic references

2. MODEL

The beam-type micro electrostatic actuator which is object of our study is presented for the model of figure 1. The model consists of a mass-spring system for constant k ($1,31.10^{-8}$ kg/s) and mass m ($1,31.10^{-10}$ kg) of a degree of freedom that is a beam that is also the capacitor C ($1,07.10^{-13}$ F). Between the plates (electrodes) there is an applied voltage V (0) and the distance between them is denoted by D ($2,02.10^{-6}$ m). As we've said, one of the plates is movable and is connected to the spring constant k and the other is the stationary electrode, as shown in figure 1. The damping coefficient, characteristic of the geometry of the object and of the environment proprieties is denoted by c ($1,0.10^{-15}$ kg/s) (Takamatsu, Sugiura, 2005).

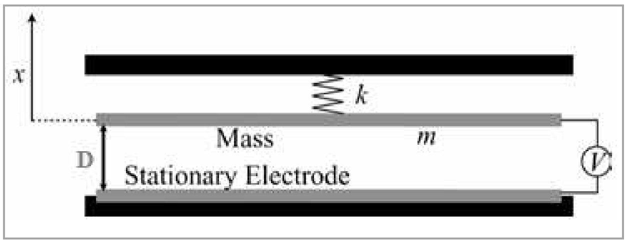

Figure 1: Modeling of a beam by a spring-mass system (Takamatsu, Sugiura, 2005).

As we can see at the figure 1, e x-axis is taken along the vertical direction with positive direction upwards. Its origin is defined as the position of the mass for $V=0$. We denote the charge on the electrode for q. We will adopt the system parameters constant, so our system is called autonomous.

2.1 Equation of motion and Stability

The equation of motion of the system shown in figure 1 can be achieved by the application of Newton's second law to the model and is written by:

$$m\ddot{x} + c\dot{x} + kx + \frac{q^2}{2CD} = 0 \tag{1}$$

where $\dot{x}$ represents the first derivative of x with respect to time, namely:

$$\dot{x} \equiv \frac{dx}{dt} \Rightarrow \ddot{x} = \frac{d^2x}{dt^2} \tag{2}$$

In equation (1), the term $m\ddot{x}$ represents the variation of the amount of linear movement provided by the Newton's Second Law, $c\dot{x}$ the damping force due to ambience. This force is opposite to the movement being proportional to the velocity of the mass; kx comes from Hooke's Law, and corresponds to the force applied by the spring on the mass, and as the damping force due to resistance offered by ambience, is contrary to the movement.

The electric Field ($\vec{E}$), is

$$\vec{E} = \frac{\vec{F}_{ele}}{q} \tag{3}$$

$\vec{F}_{ele}$ is the electric force vector acting on the charge q relative to the electric field $\vec{E}$, thus we have

$$E = \frac{F_{ele}}{q} \tag{4}$$

By the definition of capacitance,

$$C = \frac{q}{V} \tag{5}$$

From equations (4) and (5) we have:

$$F_{ele} = ECV \tag{6}$$

For a uniform electric field with equipotential surfaces of distant, the potential difference *V*:

$$E = -\frac{V}{D} \tag{7}$$

Of (6) in (7), has:

$$F_{ele} = -\frac{V^2 C}{D} \tag{8}$$

Of (5) and (8) is considering we want only the electric force acting on the movable plate, we obtain (Halliday *et al*, 2008):

$$F_{ele} = -\frac{q^2 C}{C^2 D} \Rightarrow F_{ele} = -\frac{q^2}{2CD} \tag{9}$$

To demonstrate the equation (1), consider first the second Newton's law of motion, namely:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \tag{10}$$

In our case mass *m* is constant, so we obtain

$$\sum \vec{F} = m\frac{d^2\vec{x}}{dt^2} \tag{11}$$

In the direction of the coordinate axis *x*, see figure 1, we can write:

$$\sum F = m\frac{d^2 x}{dt^2} \tag{12}$$

Replacing forces, we have

$$\sum F = F_{ele} + F_k + F_C \tag{13}$$

as described, we have

$$F_{ele} = -\frac{q^2}{2CD} \tag{14}$$

is the electric force on the movable plate;

$$F_k = -kx \tag{15}$$

corresponds to the spring force on the mass *m*, and lastly

$$F_C = -c\dot{x} \qquad (16)$$

is the drag force on the medium provides the body movement. Thus, by replacing (14-16) into (13) we obtain the equation, which substituted in (11) gives us:

$$m\ddot{x} + c\dot{x} + kx + \frac{q^2}{2CD} = 0 \qquad (16)$$

Whereas the applied voltage *V* is given by the combination of a DC voltage and the AC voltage given by:

$$V_{\sin} = A\sin(Nt) \qquad (17)$$

where *A* is the amplitude of the applied AC voltage and *N* the frequency.

The equation of motion (1) was obtained by considering the constant voltage *V*. If this tension is a composition of a DC voltage with an AC signal and variables *q*, *x*, *C*, and *D*, the relationship between the applied voltage and the load can be obtained (Takamatsu, Sugiura, 2005):

$$\frac{q(D + x)}{cD} = V_0 + A\sin(Nt) \qquad (18)$$

then

$$m\ddot{x} + c\dot{x} + kx + \frac{q^2(x + D)^2}{2CD^2} = 0 \qquad (19)$$

We can investigate the stability of a nonlinear system through the equilibrium points according to Lyapunov (Lyapunov, 1966) linearizing the equation of motion. The procedure consisted in transforming the second order differential equation (19) into a system of two first order equations by means of the following change of variables

$$x_2 \equiv \dot{x}_1 \qquad (20)$$

such that

$$\ddot{x}_1 \equiv \dot{x}_2 \qquad (21)$$

where $x_1 \equiv x$.

Resulting

$$\dot{x}_1 = f(x_1, x_2) \qquad (22)$$

$$\dot{x}_2 = -\frac{k}{m}x_1 - \frac{c}{m}x_2 - \frac{q^2(x_1 + D)^2}{2mCD^2} = g(x_1, x_2) \qquad (23)$$

In the system of differential equations, the stationary solution is obtained, by definition, on equilibrium points. Thus, the equilibrium points of the system of equations (22) and (23) can be obtained.

Therefore, the equilibrium points $P^*{}_1 = (x_1{}^*{}_1, x_2{}^*{}_1)$ and $P^*{}_2 = (x_1{}^*{}_2, x_2{}^*{}_2)$ are

$$x_1{}^*{}_1 = \frac{-\left(\frac{k}{m} + \frac{q^2}{mCD}\right) + \sqrt{\left(\frac{k}{m} + \frac{q^2}{mCD}\right)^2 - \left(\frac{q^2}{mCD}\right)^2}}{\frac{q^2}{mCD^2}} \qquad (24)$$

$$x_2{}^*{}_1 = 0 \qquad (25)$$

and

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$$x_1{}^*{}_2 = \frac{-\left(\frac{k}{m}+\frac{q^2}{mCD}\right)-\sqrt{\left(\frac{k}{m}+\frac{q^2}{mCD}\right)^2-\left(\frac{q^2}{mCD}\right)^2}}{\frac{q^2}{mCD^2}} \tag{26}$$

$$x_2{}^*{}_2 = 0 \tag{27}$$

To linearize the system around the equilibrium points, must apply Taylor's series truncated to the first order terms around the equilibrium points (Fielder-Ferrara, 1995). With the resulting systems, then linear, we obtain respective Jacobian matrices:

$$\vec{J}_1 = \begin{bmatrix} 0 & 1 \\ \sqrt{\left(\frac{k}{m}+\frac{q^2}{mCD}\right)^2-\left(\frac{q^2}{mCD}\right)^2} & -\frac{c}{m} \end{bmatrix} \tag{28}$$

$$\vec{J}_2 = \begin{bmatrix} 0 & 1 \\ -\sqrt{\left(\frac{k}{m}+\frac{q^2}{mCD}\right)^2-\left(\frac{q^2}{mCD}\right)^2} & -\frac{c}{m} \end{bmatrix} \tag{29}$$

The eigenvalues of the Jacobian matrices given in (28) and (29) can be obtained by the equation:

$$\det(\vec{J} - \lambda\vec{I}) = 0 \tag{30}$$

where λ is eigenvalues and $\vec{I}$ is identity matrix (Monteiro, 2002).

Have, for matrices in (28) and (29) respectively:

$$\lambda_{1,2_1} = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2+\sqrt{4\left(\frac{k}{m}+\frac{q^2}{mCD}\right)^2-\left(\frac{q^2}{mCD}\right)^2}}}{2} \tag{31}$$

$$\lambda_{1,2_2} = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2-\sqrt{4\left(\frac{k}{m}+\frac{q^2}{mCD}\right)^2-\left(\frac{q^2}{mCD}\right)^2}}}{2} \tag{32}$$

appointing in equation (31),

$$\frac{c}{m} = \phi \tag{33}$$

$$\sqrt{\left(\frac{c}{m}\right)^2+\sqrt{4\left(\frac{k}{m}+\frac{q^2}{mCD}\right)^2-\left(\frac{q^2}{mCD}\right)^2}} = \psi \tag{34}$$

and in equation (32),

$$\frac{c}{m} = \phi \tag{35}$$

$$\sqrt{\left(\frac{c}{m}\right)^2+\sqrt{4\left(\frac{k}{m}+\frac{q^2}{mCD}\right)^2-\left(\frac{q^2}{mCD}\right)^2}} = \theta \tag{36}$$

So that the equations (31) and (32) can be represented in terms of these new variables defined.

$$\lambda_{1,2_1} = \frac{-\phi \pm \psi}{2} \tag{37}$$

$$\lambda_{1,2_2} = \frac{-\phi \pm \theta}{2} \tag{38}$$

Table 1 shows the analysis of stable equilibrium point P^*_1, therefore, made on the basis of $\lambda_{1,2_1}$. The stability of P^*_2 according to $\lambda_{1,2_2}$ is similar.

Table 1: Stability conditions in terms of $\lambda_{1,2_1}$.

Stability	Type Fixed Point	Condition
Asymptotically stable	Node (hyperbolic)	$\phi > 0$ and $0 < \psi < \phi^2$
Unstable	Node (hyperbolic)	$\phi < 0$ and $0 < \psi < \phi^2$
Unstable	Saddle (hyperbolic)	$\phi > 0$ and $\psi > \phi^2$
Unstable	Focus (hyperbolic)	$\phi < 0$ and $\psi < 0$
Asymptotically stable	Focus (hyperbolic)	$\phi > 0$ and $\psi < 0$
Stable	Center (hyperbolic)	$\phi = 0$ and $\psi < 0$
Unstable	Hyperbolic	$\phi < 0$ and $\psi = 0$
Asymptotically stable	Hyperbolic	$\phi > 0$ and $\psi = 0$

2.2 Nondimensional Equation

Using this scale transformation for the variables:

$$t = \sqrt{\frac{m}{t}t^*},\ x = dx^* \tag{39}$$

the nondimensionalized governing equation can be obtained,

$$\ddot{x} + 2\gamma\dot{x} + x + \frac{4\beta^2(1+\alpha\sin \delta t)^2}{27(1+x)^2} = 0 \tag{40}$$

where the notation * is omitted. Nondimensional parameters are:

$$\gamma = \frac{c}{2\sqrt{mk}},\ \beta = \sqrt{\frac{27CV_0^2}{8d^2k}} = \frac{V_0}{V_b},\ \alpha = \frac{A}{V_0},\ \delta = \frac{N}{\sqrt{m/k}}. \tag{41}$$

γ is the damping coefficient, β is the DC voltage (the ratio to V_b that the equilibrium point disappears according to the bifurcation), a is the amplitude of AC voltage (the ratio to DC voltage), and δ is the frequency of AC voltage.

3. SIMULATIONS

The simulations were performed using the non-dimensional motion equation (Eq. (40)). Figure 2 shows our simulations for the parameters $a = 0.33$, $\beta = 0.78$, $\delta = 0.86$ and $\gamma = 5.5$, the same used by Takamatsu and Sugiura (2005).

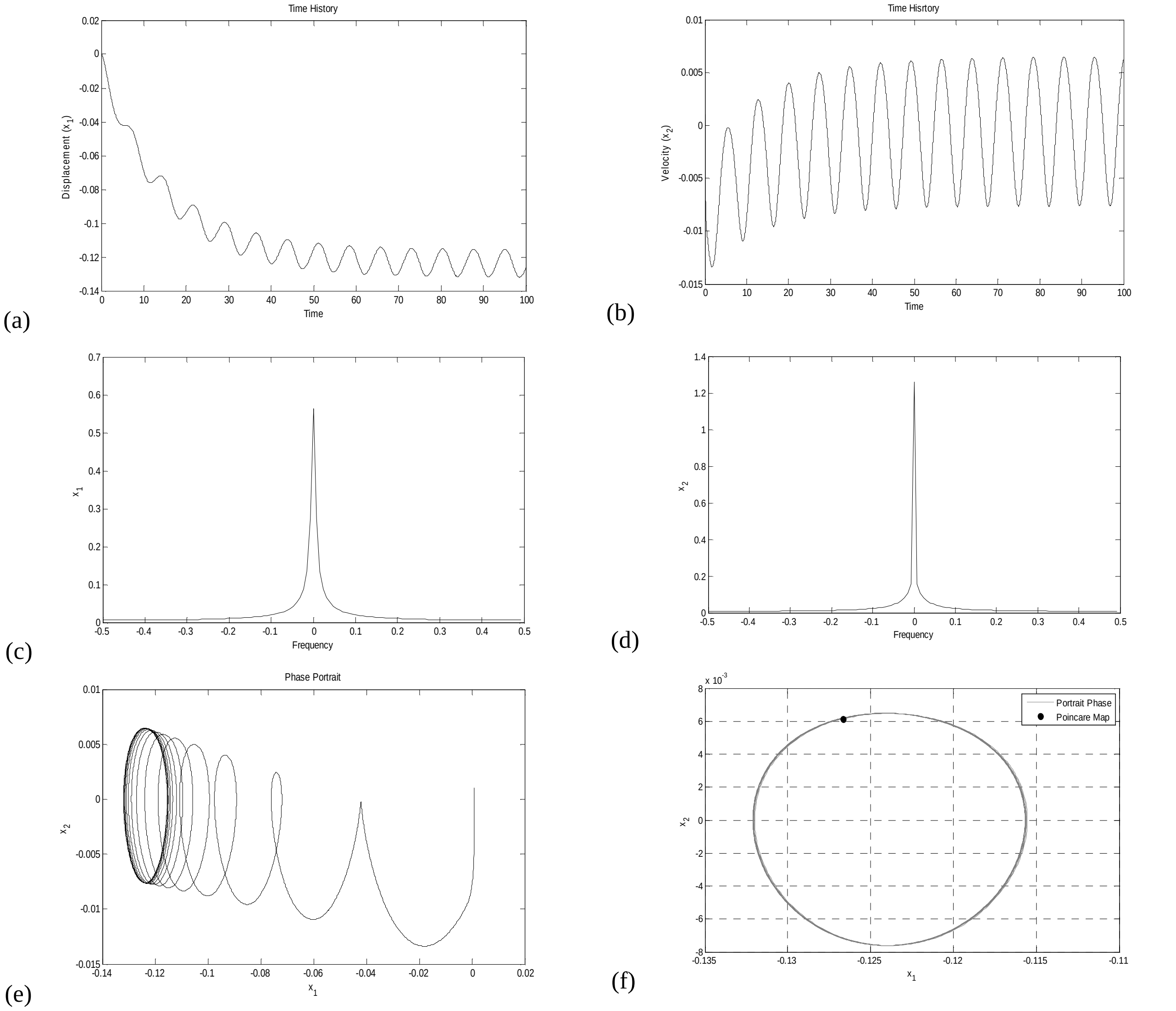

Figure 2: MEMS system. (a) Time History (x_1), (b) Time History (x_2), (c) Frequency Spectrum (x_1), (d) Frequency Spectrum (x_2), (e) Portrait Phase (x_1,x_2) and, (f) Poincare Map (x_1,x_2).

The coordinate of the equilibrium point of the system are $x_1=0.27451$, $x_2=0.0$, and the eigenvalues are $\lambda_1= -0.0836;$ $\lambda_2= -10.9164$. The eigenvalues $\lambda_{1,2}$ indicates that the micro electrostatic actuator is stable and the Figure 3 shows the dynamics of Lyapunov exponents (λ_1=-0.0688), (Wolf *et al*,1985) without the presence of chaotic behavior.

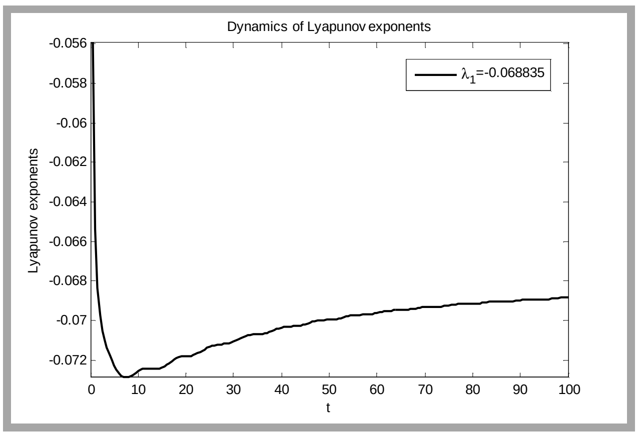

Figure 3: Lyapunov Exponents.

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3.1 Stability diagram

Chosen numerical values for the parameters are: a =0.33 and δ =0.86, where as β and γ are varied. The Jacobian matrix J is:

$$J = \begin{bmatrix} 0 & 1 \\ -1 + \dfrac{0.3113\ \beta^2}{1.0303} & -2\gamma \end{bmatrix}. \tag{41}$$

The eigenvalues of J, will provide the conditions for stability or instability of the equilibrium points: an equilibrium point is asymptotically stable if all the eigenvalues have negative real part and unstable if at least one of them has positive real part (Guckenheimer, J. and Holmes, 1983). The stability diagram for Eq. (4) is showed in Figure 4.

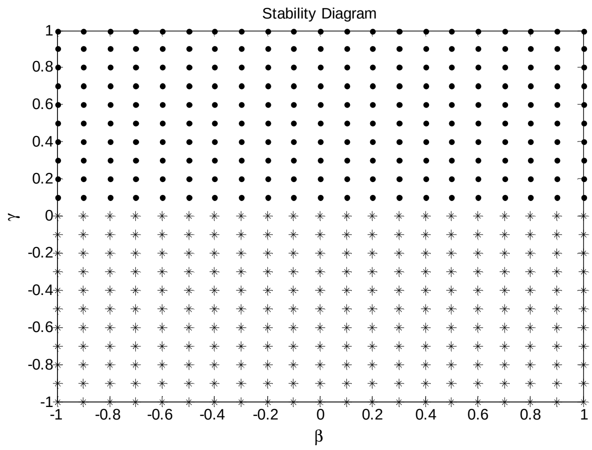

Figure 4: Stability Diagram (•) Stable points and (*) Unstable points.

The stability diagram is a graphical representation of a suggestive way of points, depending on two parameters β (damping coefficient) and γ (DC voltage), in which the system is stable or not. Figure 4 is a special case of discrete points and the choice of the parameter γ is because of the dampening effect on the system. The points (•) represent the stable points of the system and the points (*) represent unstable points. Noted in the Figure 4 that for damping zero or less than zero the system becomes unstable, otherwise there is the stability.

3.2 Stability as a function of β

In the Figure 5, are shown the points as compared to its stability parameter, that is, it is possible to see that the regions where the equilibrium point are stable and unstable in function of this parameter.

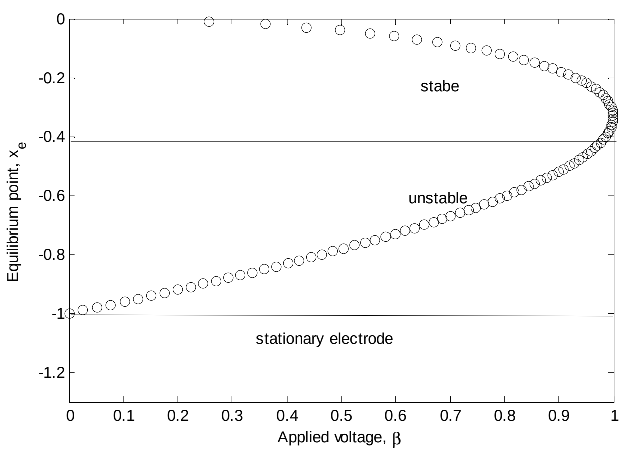

Figure 5: Stability diagram as a function of β.

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4. CHAOS

When a system has a parametric excitation, some parameter of the system is a modified variant with time. In this case, this vibration is said parametric vibration causing very high amplitudes.

In the forced vibration, power is simply added to the system, in accordance with the resonance conditions, there is the system response. In such cases there is no variation in the parameters, they are constant throughout the oscillation process.

In the system adopted, the forced excitation term including (*sinδt*) is primary resonance can occur when frequency of AC voltage is close to the natural frequency. Another possibility mechanism is parametric resonance caused by the term including *(acos2δt)* when 2δ is twice the natural frequency. Figure 6 shows the dynamic behavior of the model (3) for the parameters $a = 0.33$, $\beta = 0.78$, $\delta = 0.86$ and $\gamma = 5.5$.

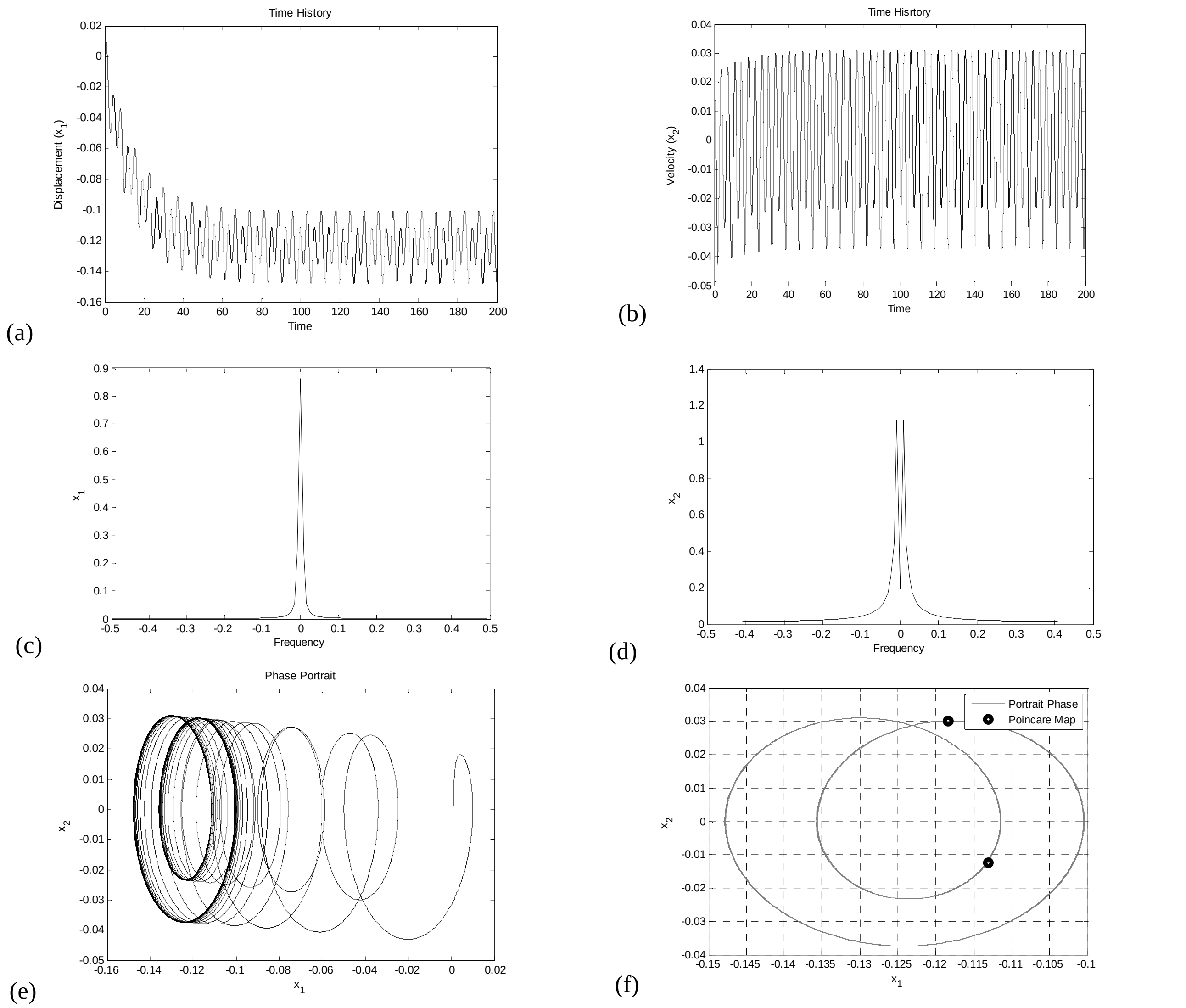

Figure 6: MEMS system. (a) Time History (x_1), (b) Time History (x_2), (c) Frequency Spectrum (x_1), (d) Frequency Spectrum (x_2), (e) Portrait Phase (x_1,x_2) and, (f) Poincare Map (x_1,x_2).

Figure 7 shows the dynamics of unstable chaotic behavior of Lyapunov exponents (λ_1=+2.6239; λ_2=+0.3666), (Wolf *et al*,1985) caused by parametric resonance.

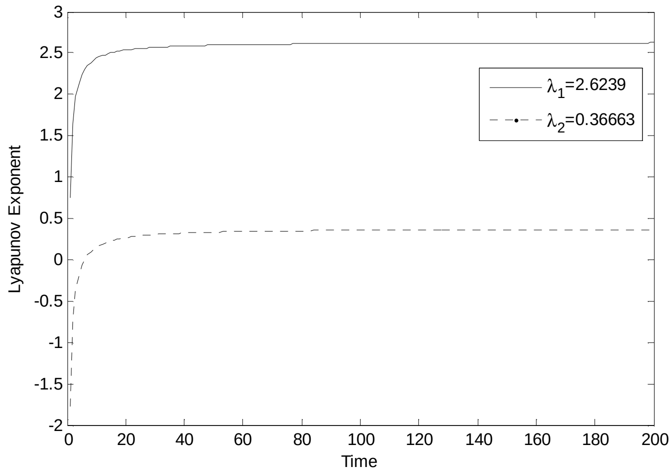

Figure 7: Lyapunov Exponents.

5. CONCLUSION

In this work, a dynamics of the micro electrostatic actuator proposed (Takamatsu, Sugiura, 2005) is investigated through numerical simulations using the software Matlab 6.5®. The model of the micro electrostatic actuator is shown in Figure 1 and in section 2, the equation of motion and stability are demonstrated and analyzed in accordance with the work (Takamatsu, Sugiura, 2005).

Figure 2 shows stable behavior of the proposed system, the Figure 3 and 4 shows the stability studies carried out aiming to find a chaotic behavior in the proposed system, we include parametric excitation, as shown in section 4. Figure 7 illustrates the lyapunov exponent for the model with parametric excitation, indicating that this excitement was efficient for this purpose.

Our next objective of our work, numerous design methodologies exist for the control design of nonlinear system. These include any of a huge number of linear design techniques (Dahlel, Pearson, 1987; Horowitz, 1991), used in conjunction with gain scheduling (Rugh, 1991; Apkarian, Gabinet 1995); nonlinear design methodologies include Jacobian linearization, recursive backstepping, feedback linearization, gain scheduling, sliding mode control, and adaptative control (Khalil, 1992). Chaotic behavior shown in Figures 6 and 7, motivates us to propose a method of stabilization control for the purpose of canceling this chaotic behavior.

6. ACKNOWLEDGEMENTS

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