



DEVELOPMENT OF A METHODOLOGY FOR DETERMINATION AND ANALYSIS OF THERMAL DISPLACEMENTS OF MACHINE TOOLS USING ELEMENTS METHOD FINITE AND ARTIFICIAL NEURAL NETWORKS

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Abstract. In manufacturing processes, machine tools play an important role in the manufacture of workpieces with a complex geometric and dimensional accuracy and high. Much of the errors of a machine tool are those that are thermally induced, caused by heat sources internal and external operating the machine. This work presents a methodology to determine and analyze thermal displacement machine tool using the Finite Element Method (FEM) and Artificial Neural Networks (ANN). The machine is modeled using FEM, defined the location of the heat source, it is possible to obtain the temperature gradient and the corresponding thermal displacement at predetermined periods. The results obtained from simulations using the software NX7.5 shown that this method is an effective tool to determine the thermal displacement of the machine correlating the temperature reading at strategic points to the volumetric displacement at the tip of the tool. Therefore, the thermal analysis of the errors in the pair tool part can be established. After the process of training and validation, the network will be able to make the prediction of thermal errors only indicate the temperature values of specific points of each heat source, providing a form of compensation of thermally induced errors.

Keywords: Thermal displacement, Machine tool, Finite element method, Artificial neural network.

1. INTRODUCTION

The influence of temperature variation on Machine Tools (MT) is a physical phenomenon that can not be completely eliminated. However, it should be monitored and included in the development process of the machine. The undesirable effects of heat and especially heat flow is represented by geometrical changes and/or structural volume of the machine. These changes cause an unwanted relative motion between the tool and the workpiece, which can negatively influence the specified tolerances during machining or accuracy (Vyroubal, 2012). Thus, this paper seeks to provide a contribution towards increasing the accuracy in machine tools, as compensation mechanisms are thoroughly studied to try to reduce thermal effects.

The main reason for the geometric and dimensional errors in production of workpieces in MF includes static laws of the rigid structure of the machines, the performance of the power law dynamics drives used and thermal deformations or displacements in the tool and the workpiece (Week et al. 1995).

In order to increase the accuracy of machined parts for machine tool CNC high-speed (HSM), by compensating the deformations caused by thermal variations during the process, which transforms the energy into thermal energy machining and propagates in the conduction, convection and radiation, the latter being discarded and the concentration of studies usually done only by conduction and convection, or just taking into account the deformations arising under heat sources involved in the machine. Importantly, it is essential to develop methodologies able to analyze and evaluate the performance of the machine tools high accuracy and high speed cutting, with regard to thermal effects.

In this paper a methodology is established to decrease, even if it is simulated the effects thermals in machine tools, but that can later be used in real cases. Being used the tools of Finite Element Method (FEM) and Artificial Neural Networks (ANN), using the software NX and MatLab 7.5 respectively.

2. MACHINE TOOL ANALYZED

Romualdo F., Francisco A. V., João Bosco de A. S., José Carlos L. J.

Development of a Methodology for Determination and Analysis of Thermal Displacements of Machine Tools Using FEM and ANN.

Considering the machine tool type column three axes, Fig. 1, the methodology that was employed for the analysis of thermal errors displayed on this machine, through the FEM and ANNs, has been adapted into a virtual machine to similar from Fig.1, this simplified model of MF was redesigned, adapted Heui (1997), the simulation software in finite element NX-7.5, simplified form, whose considerations are discussed in following sections.

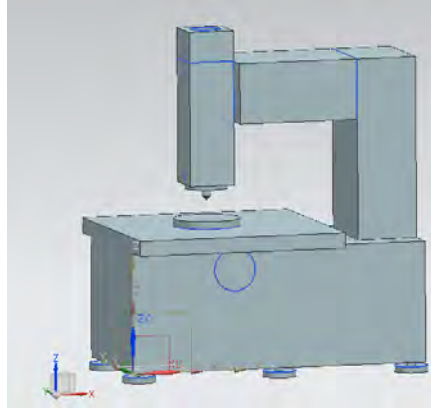


Figure 1: Machining center type column redesigned in the NX environment.
(Adapted from Heui 1997)

3. METODOLOGIA USED

Shortly after executing the design of the machine, set the criteria for the creation of the finite element mesh, creating a mesh compatible with both the structural solution as thermal, then preceded with the following steps:

- a) Create a mapping file (file ".bun" in the NX environment) which initially contains the values of temperatures for the conditions specified in the contours of a permanent state of temperature over a period of 8 hours.
- b) Export the conditions of thermal contours for structural solution and then simulate the displacements occurring at the tool tip for the three axes of analysis. This operation being repeated 80 times for predetermined time interval of 8 h or 28800s using a time increment between each simulation 360s over the initial state at time zero, to complete the cycle.
- c) Collect the solutions for each increment of time considered and plot the data on the time x displacement, as well as temperature versus displacement in the MatLab environment.
- d) Creates an RNA through the environment MatLab function having as input the temperature values of each node represent the thermocouples on the machine tool and as "target" or desired value for each respective displacements of the shaft.e) Training the ANN by means of programs developed in MatLab using the functions contained in this software.
- f) Run the validation process to verify that the Network could actually learn and predict the thermal displacements only making use of the temperature readings on the specified nodes.
- g) To develop this method for two different cases, namely to a permanent state of temperature and temperature transient state.
- h) Simulate offsets in each axis, by means of the MATLAB programs in two states analyzed to verify the efficiency of the technique developed.

4. THERMAL ANALYSIS OF SHIFTS IN PERMANENT STATE TEMPERATURE

In the machine tool of Fig. 1 analyzed the thermal behavior and structural to find a relationship between the thermal volumetric displacements occurring at the tool tip with tapered temperature reading strategic points on the machine. That is, the thermal behavior of the machine by reading eight nodes symbolizing virtual thermocouples (T1, T2, T3 ... T8.) as illustrated in fig. 2 and 3. Near any heat sources, considering the amount of work, in the simulations, one eight-hour shift.

The thermocouples were distributed for the simulations as follows:

- T1: Base part (node 1935)
- T2: Mesa (node 1605)
- T3: Base tool holder (node 3416)
- T4: upper bearing spindle (node 3146)
- T5: Main Engine (node 2455)
- T6: Base table motor (node 2878)
- T7: coolant pump (node 1795)
- T8: Oil pump (node 2564)

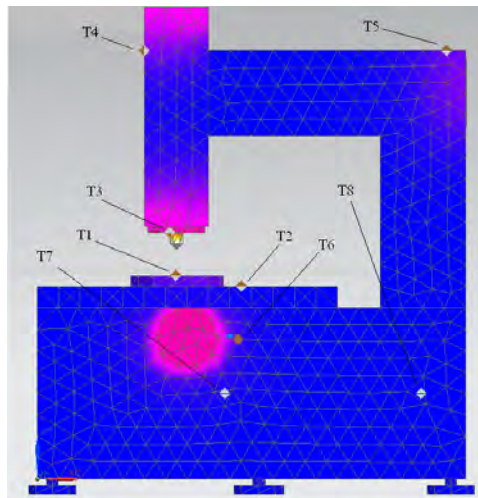


Figure 2: Distribution of thermocouples in the structure of the virtual machine.

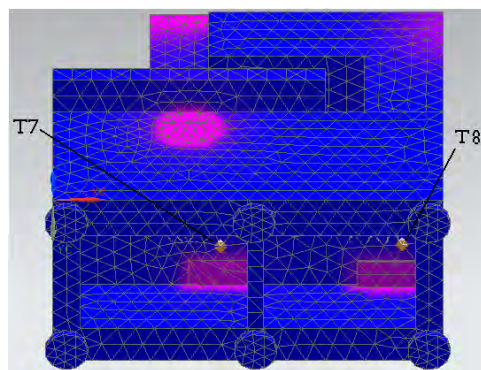


Figure 3: Thermocouple virtual oil pumps and refrigerant.

The considerations for analysis in this section are as follows:

- Material Machine-Tool: steel density $7.829e-6\text{kg/mm}^3$.
- Heat sources: Some internal heat sources exist in machine tool variations in time.
- Hazard analysis: 0 to 28800s.
- Thermal mapping performed for the times: Every 360s.
- Temperature range: 20-2000°C.
- Structural Boundary Conditions: Feet Machine tool set.
- Thermal Boundary Conditions: Temperature set at each side of the machine tool in contact with the engine, with the bearings and the pair play tool, through a fixed temperature, as figs 4 and 5.
- Element size: 50mm.
- Type of the element: tetrahedron with four nodes.

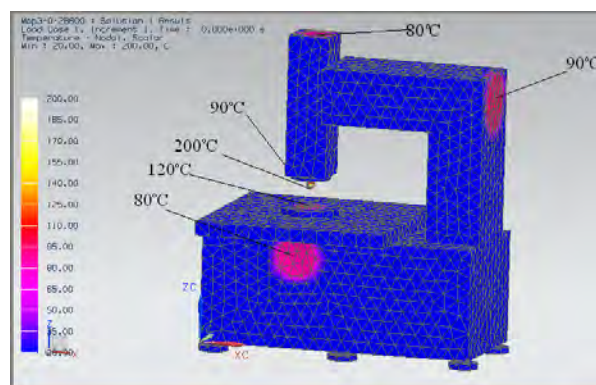


Figure 4: Conditions of thermal contours.

Romualdo F., Francisco A. V., João Bosco de A. S., José Carlos L. J.
 Development of a Methodology for Determination and Analysis of Thermal Displacements of Machine Tools Using FEM and ANN.

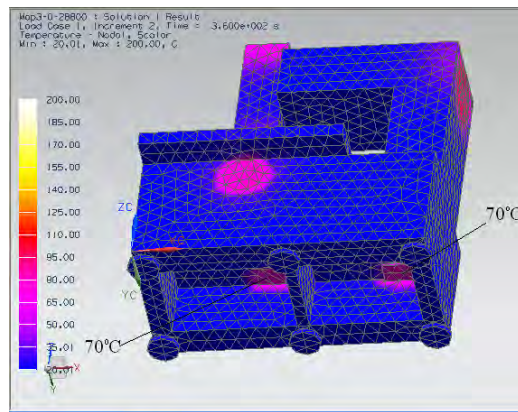


Figure 5: Conditions of contours thermal oil pumps and cooling.

The conditions of thermal contours are shown in figures 4 and 5, wherein the indications of temperatures from the heat sources in each internal face contact with the machine motors, bearings etc. and the pair tool part, are illustrated. The reading of each thermocouple virtual over time, for the case considered in this section is shown in the graph of fig. 6.

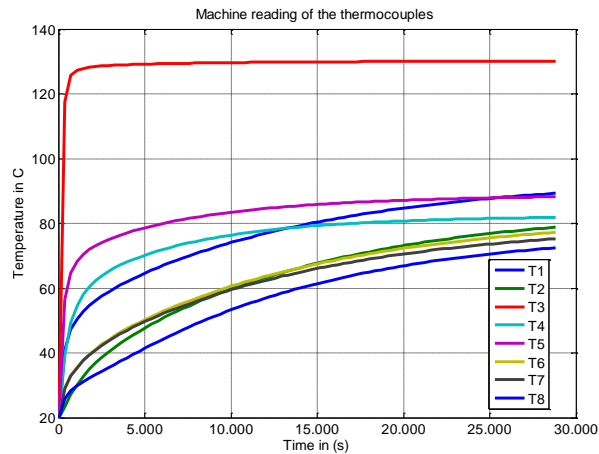


Figure 6: Curves of the eight thermocouples in permanent state

It is observed mainly in the thermocouple located close to the tool as the temperature rapidly enters the steady state, and as others take longer, it happens mainly because of the heat generated at the tool end, assuming machining and proximity for each thermocouple.

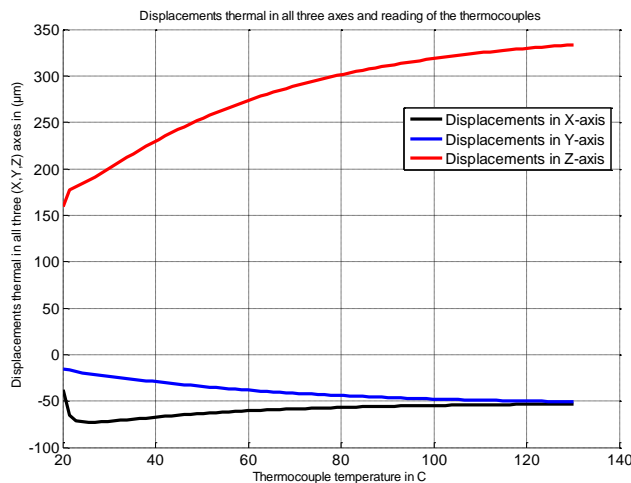


Figure 7: Behavior of the three axes simultaneously-permanent state.

When were performed the simulations, taking into consideration a lathe eight hours of machining, the curves of the displacements thermal insulation in function of time respective axes are shown in fig. 7, where if observe behavior smoother, due the fact of having been collected a number sufficient of points, as also the period analyzed time in the simulations is coherent with the performed in the machining process of an environment manufactures. Observes-if still that although the simulations commences in the zero instant, the displacements thermal on each axis already exhibit a significant error thermal, this happens due the fact the temperature gradient be elevated in the permanent state temperature provoking great displacements thermal already in beginning of the process for this case.

4.1 Development of an rna for learning axes simultaneously on the basis of the readings of the thermocouples in the steady state temperature.

Artificial neural networks was formulated one of 16 neurons in the first layer and the second layer 1 neurons with feed-forward backpropagation and feedback from output to input, with activation functions "tansig" and "purelin" respectively supervised learning by correcting errors standard mode, using eqs 1 and 2, whose topology is shown in Fig. 8.

$$\text{tansig}_j = \frac{2}{(1 + \exp(-2w_i t_j))} - b_i \quad (1)$$

$$a_j = w_i t_j + b_i \quad (2)$$

Where: w_i : The weight for each iteration i
 t_j : Mother mains input for each element j .
 b_i : Bias or (threshold) of each neuron for n iterations.

Created after the network passed the learning process of the displacements of the three axes simultaneously depending on the readings of the temperatures of the nodes that represent the thermocouples on the machine.

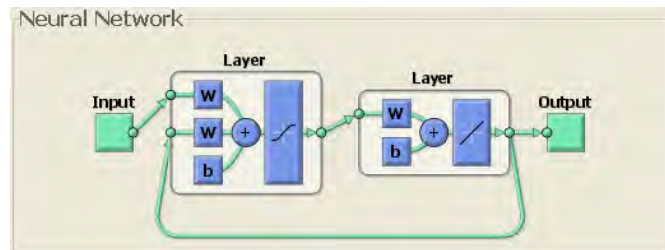


Figure 8: ANN for three axis-permanent state.

The entry "input" was introduced arrays temperature of each thermocouple, eq. 3 over time and considered as "target" or desired output array of thermal displacement of the axes X, Y and Z eq. 4.

$$T = \{ [T_j] \} \quad (3)$$

$$A = \{ [D_x] [D_y] [D_z] \} \quad (4)$$

Where:

T = Matrix of readings of all thermocouples in the analyzed period.

A = The Matrix "target" or desired output of the thermal displacements in the period analyzed.

$[T_j] = \{ T_j(1, i) \}$ Matrix line of each thermocouple j for each increment i .

$[D_x] = \{ D_x(1, i) \}$ Matrix line of each matrix row in the X axis displacement for each increment i .

$[D_y] = \{ D_y(1, i) \}$ Matrix line every shift on the Y axis for each increment i .

$[D_z] = \{ D_z(1, i) \}$ Matrix line every shift on the Z axis for each increment i .

The result of the learning process to the three axes using the ANN carried in this section is shown in fig. 9, which shows the learning simultaneously to the respective deflections along the three axes as a function of the temperatures developed in strategic points of the machine tool analyzed for 500 iterations. It is seen in the same figure, convergence occurred for all axes.

Romualdo F., Francisco A. V., João Bosco de A. S., José Carlos L. J.
Development of a Methodology for Determination and Analysis of Thermal Displacements of Machine Tools Using FEM and ANN.

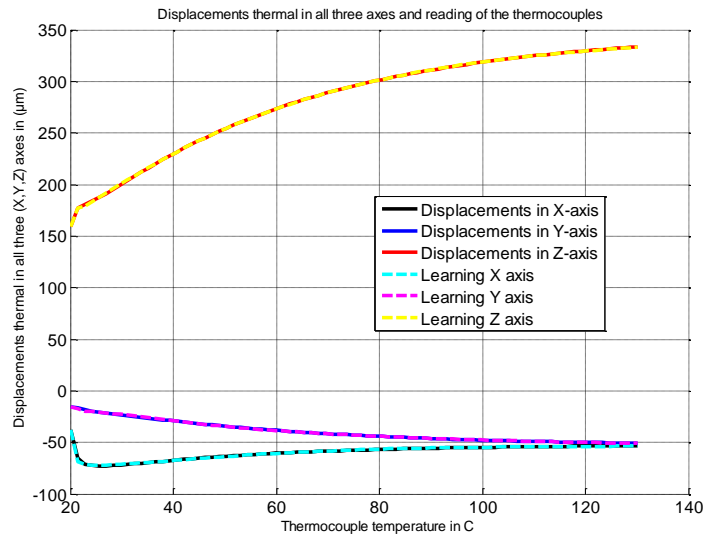


Figure 9: Displacements as a function of volumetric temperature-permanent state.

500 iterations for the aforementioned performance curve, extracted from the environment Matlab is shown in fig. 10 which is observed that the network could learn and that the error reached $0.01\mu\text{m}$ compared to the simulated results.

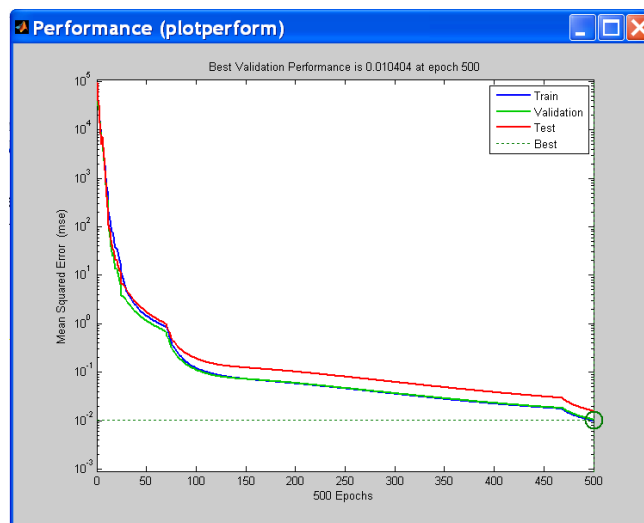


Figure 10: Graph of the performance of ANN learning

4.2 Validation of ANN for the three axes in a permanent state

Network After going through the process of learning, it is important to submit the network to another process called validation. The validation process of an ANN is to provide the network after trained, different values of the input data which she was trained and then compare them with the simulated results of these new input data to verify that the network could actually learn satisfactorily and whether it is possible to predict the deformations providing only the values of the temperatures measured by the thermocouples. For the validation process steady state temperature. The procedure was as follows:

- Again performed simulations, being the time interval for each reading 180s. Of the 162 new temperature readings of the thermocouples, disregarding that coincided with the previous analysis.
- Next to the new temperature readings of the eight thermocouples, fed to the network only at the new readings without the network was retrained.

After performing the above procedure is obtained the validation curves as shown in fig. 11. Thus, ANN, made the prediction of deformations, even entering different values (of temperature) of those she was trained. Being able to reach the goal of validation and this section to the permanent state of temperature.

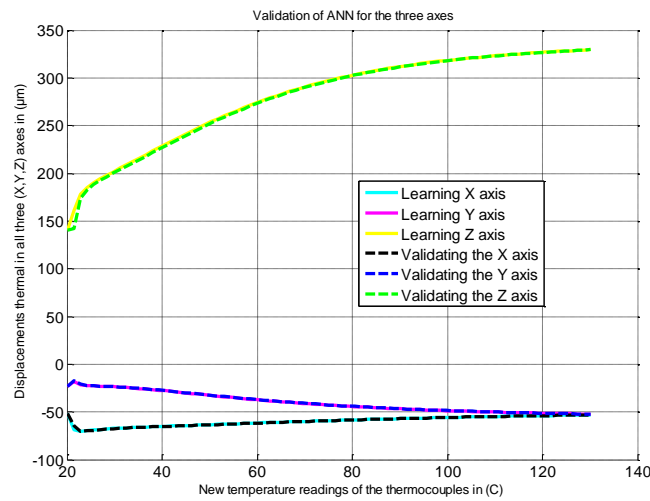


Figure 11: Validation of ANN for the three axes, heat sources contained

4.3 Analysis of results and compensations for the steady state temperature.

Using the curve that the ANN learned the thermal behavior of the X axis to predict the thermal deformation of this axis machine tool analyzed and make possible corrections or compensations. Is the curve of the deformation corrected or compensated in fig. 12. Where one can observe that with the correct coordinate axis X, using the technique proposed virtually zero thermal error on this axis. Using the same technique for the Y axis, can be the result obtained in fig. 13th. Ditto for the Z axis Fig. 14. What one finds the efficiency of the method.

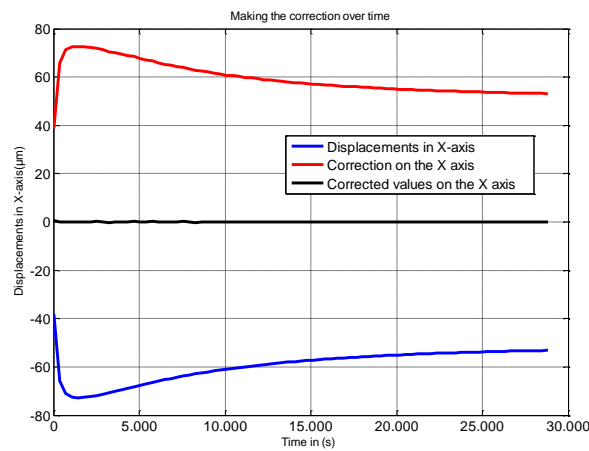


Figure 12 Corrections in the X axis along time

5. THERMAL ANALYSIS OF SHIFTS IN TRANSIENT STATE TEMPERATURE

In this section, the same procedure will be adopted in the previous section, in other words, the same configuration of the machine tool, the thermocouples located on the same nodes and the same conditions of structural contours, but just changing the conditions of thermal contours, which will be replaced by heating equations for each heat source.

The considerations for the simulation results in this section are the same as in the previous section, but changing only the transient thermal boundary conditions, in other words, temperature set at each side of the machine tool in contact with the engine, with the bearings and the pair play tool, by heating the equation of Newton, eq.5. The graph of Figure 13 was plotted from the law of heating / cooling Newton, according to eq. 5, each heat source. As seen in its initial temperature (T_m), which coincides with the temperature at 20°C if the instant "0s" and a final temperature T_∞ (∞), which is the maximum power that can reach through the simulation conditions preset to a final state of thermal equilibrium.

$$\frac{dT}{dt} = k(T - T_m) \tag{5}$$

Romualdo F., Francisco A. V., João Bosco de A. S., José Carlos L. J.

Development of a Methodology for Determination and Analysis of Thermal Displacements of Machine Tools Using FEM and ANN.

Where:

T_m = room temperature;

T = temperature of the body at time t ;

K = proportionality constant

Solving the eq. 5, by separation of variables and replacement of boundary conditions, according to Bronson (2008) is eq. 6:

$$Tl(t_i) = T_{(\infty)} + C_j \times \exp(-k_j \times t_i) \quad (6)$$

Where $Tl(t_i)$ is the local temperature of each heat source at time t_i ; $T_{(\infty)}$ is the steady state temperature; C_j constant is found from the condition of initial and final contours; K_j is time constant (Incropera et al, 2008) found for each heat source in unit time (s). It represents the time that the source spends to reach 63.2% of the final value of the temperature rise corresponding to its operation in a state of thermal equilibrium or steady state. And t_i is the time to consider every moment according to the temperature transient.

The value of the time constant for all sources, were found from the initial boundary condition $T(0) = 20^\circ\text{C}$ and intermediate condition $T(1800) = 0.632 \times T_{(\infty)}$. That is the temperature at time 0s and the temperature at which the source can reach the elapsed time when 1800s or 30min.

As showing only two are assigned numerically as an example, the eight equations as conditions found in simulations contours of heat sources close to the thermocouples installed in the structure, as seen in figs. 4 and 5, the two equations are the following:

- Temperature in the base part (close to T1) via the associated eq. 7:

$$Tb(t_i) = 120 - 100 \times \exp(-0,000451072620305507 \times t_i) \quad (7)$$

- Temperature in the toolholder (next to the tool, T3), linked by eq. 8:

$$Tf(t_i) = 200 - 180 \times \exp(-0,000493828754270023 \times t_i) \quad (8)$$

In the graph of fig. 13, is shown at eight curves from data collection of virtual thermocouples throughout the simulation period considered and the thermal displacement on the Y axis. It is observed eight different curves relating to data collected from each heat source, whether transients exist. Three equations equal, this happens because the position or location of each thermocouple being different in the structure of the machine. What is more coherent because all heat sources begin at 20°C which is exactly considered the initial temperature of the entire structure of the machine and the thermal shifts starting $0 \mu\text{m}$. Although the graph of fig.13 shows only the displacements in the Y axis can also be achieved for the other axes respectively at the end of the tool achieved by the FEM analysis.

There has been an irregular deformation in the first 1800s (fig. 13), although the heat sources have a rather regular behavior. Such irregularities are usually due to temperature gradients which are greater at the beginning of machining machine tool of any or due to other structural parameters which provide voltages at certain points in the machine frame. It was also observed that the initial deformation on the X axis is zero for the initial time, the initial temperature of 20°C , in other words it is more consistent with the reality machining than in the situation considered in the section where the heat source had a temperature constant, in other words, steady. Besides the fact of observing the deformation at the tool tip on the X axis and the temperature of each heat source with the reading of the respective thermocouples on the same graph.

The fact show deformations in the negative Y axis is due, fig. 13, the tool tip to deform in the direction opposite the direction of the axis of the machine and no shrinkage or the influence of other axes, as well as heat sources.

The deformations also occur in the other two axes having different behavior due to the fact that they have different stiffness and thermal diffusivity on each axis as well as the extent of deformation over time considered. In this paper it is shown only one axis due to synthesis of the work, but the analysis may be performed either individually for each axis or in a volumetric showing the thermal behavior of all the axes simultaneously, because the methodology allows.

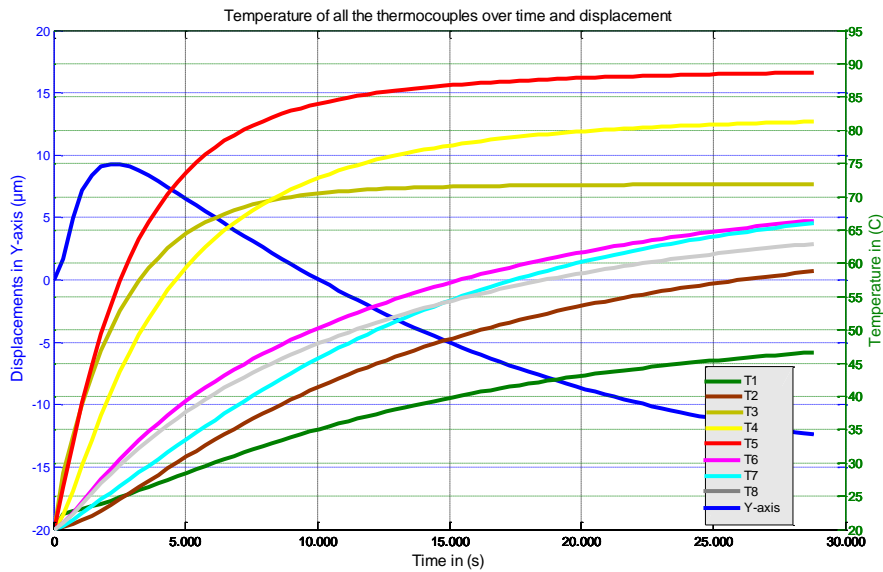


Figure 13: Reading of the thermocouples and displacement in the Y axis over time.

5.1 ANN learning for state transient temperature.

For learning the volumetric deformations, that is, the thermal deformation occurring simultaneously in each axis, whereas the same machine fermentation heat sources and variations in time (transient event). Was formulated an ANN with the same characteristics of the section steady state temperature, changing only the values of the input array and the "target", using the same topology of the previous case for the three axes, but with data from transient state temperature. In fig. 14, has been learning for Artificial Neural Network for 170 iterations, as shown:

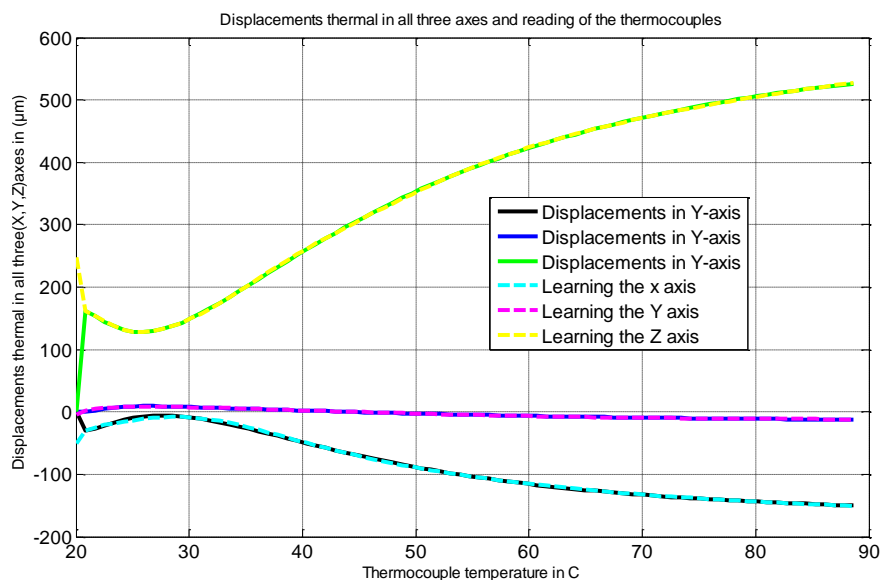


Figure 14: Learning the neural network model for 170 iterations.

It can be seen from the graphs that the network successfully learned the thermal volumetric machine for the transient event, through the convergence curves, that is, the thermal displacement on each axis simultaneously. But that's not enough; the network will have to go through the validation process.

In Table 1, there is an example of learning of ANN for the 10 initial training data network, having as input the temperatures of the thermocouples, and one of the "target" thermal displacement of the tool tip relative to the Z axis, with their errors of learning in relation to the displacement occurred on the same axis, collected as example after 239 iterations, a maximum of 500 iterations.

Romualdo F., Francisco A. V., João Bosco de A. S., José Carlos L. J.
Development of a Methodology for Determination and Analysis of Thermal Displacements of Machine Tools Using FEM and ANN.

Temperature of thermocouples (°C)-ANN Input								Thermal displacement in Z-axis (μm)	Learning ANN (μm)	Error(μm)
T1	T2	T3	T4	T5	T6	T7	T8			
20,0	20,0	20,0	20,0	20,0	20,0	20,0	20,0	0,00	239,72	-239,72
21,9	20,2	25,5	20,8	22,9	20,4	20,4	20,2	161,40	161,06	0,34
22,4	20,4	28,5	22,1	26,3	21,0	20,6	21,0	154,10	155,84	-1,74
22,8	20,8	34,0	25,7	33,0	22,5	21,5	22,4	144,80	149,09	-4,29
23,1	21,4	38,8	29,6	39,1	24,2	22,5	23,9	137,20	139,81	-2,61
23,5	22,1	43,1	33,4	44,6	25,9	23,5	25,4	131,80	132,53	-0,73
23,9	22,8	46,8	37,1	49,4	27,5	24,5	26,9	128,80	128,57	0,23
24,3	23,7	50,0	40,5	53,7	29,1	25,5	28,3	128,10	127,81	0,29
24,8	24,6	52,9	43,7	57,4	30,6	26,5	29,7	129,50	129,45	0,05
25,3	25,5	55,3	46,6	60,7	32,1	27,6	31,0	132,70	132,68	0,02

In the table below, again as the network converged, or learned by reducing the error. You can clearly see this statement, besides the graphs shown in fig. 14. Or by means of the graph of fig. 15, which shows the behavior of the mean square error relative to the axis Y. Dropping to less than $0.1 \mu\text{m}$ approximately one hour when it was simulated machining.

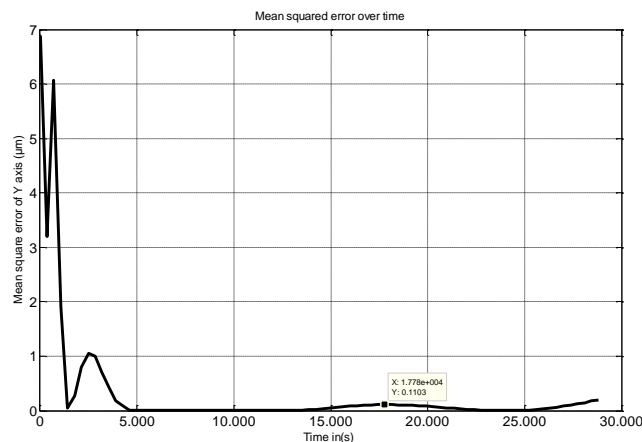


Figure 15: Mean square error for the Y-axis transient state

5.2 Validation of ANN for the three axes in the transient state

In the process of validation of the ANN to the transient event, the procedure was the same as the section to steady state temperature, or consist of the supply network after trained different values of input data with which it was trained, to verify that the network actually managed to learn the thermal behavior satisfactorily and make the prediction of thermal displacements of the tool only with the entry of the temperature readings of the thermocouples transiently.

For these simulations validation process proceeded as follows:

- Executed again the simulations, being in a time interval for each reading 180s. 162 of the new temperature readings of the thermocouples, disregarding that coincided with the increase of time of 360s.
- Then with the new temperature readings of the eight thermocouples fed into the network only at the new readings without the network to be retrained, in other words without a "target." Curves were obtained for validation as shown in fig. 16. In other words, ANN made the prediction of deformations, even entering different values (temperature) of those she was trained also to transient state. It is observed that there is little discrepancy between the curve simulated deformation and validation for each axis only at the start of the curves where the network begins the process of learning and the temperature gradient is increased, there is a greater difference between the curves and values. Soon the validation process was satisfactory.

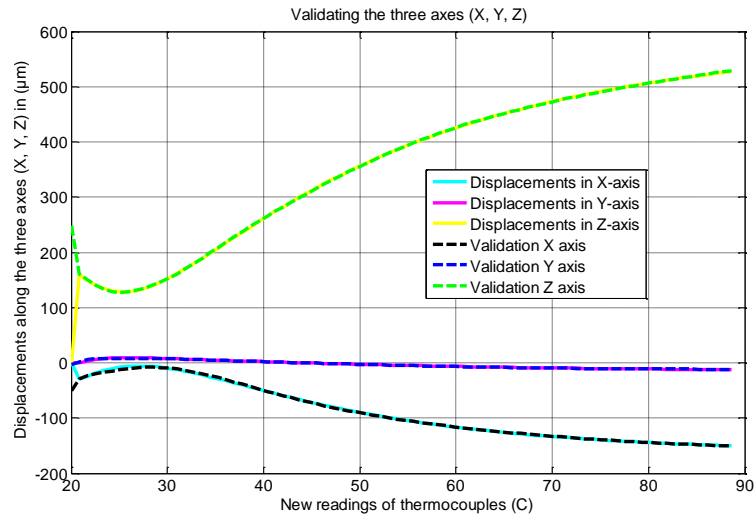


Figure 16: Validation of ANN-case transient.

5.3 Corrections to the transient case

Using the curve that the ANN learned, considered as an example in this section, only the thermal behavior in the X axis, to predict the thermal deformation of this axis machine tool analyzed, and make possible corrections. Is the corrected strain curve of the X-axis in fig. 17. Where one can observe that with the correct coordinate axis X, using the technique proposed virtually zero thermal error on this axis, immediately when the machine starts the heating process. The same procedure for corrections or offsets Y and Z axes can be performed.

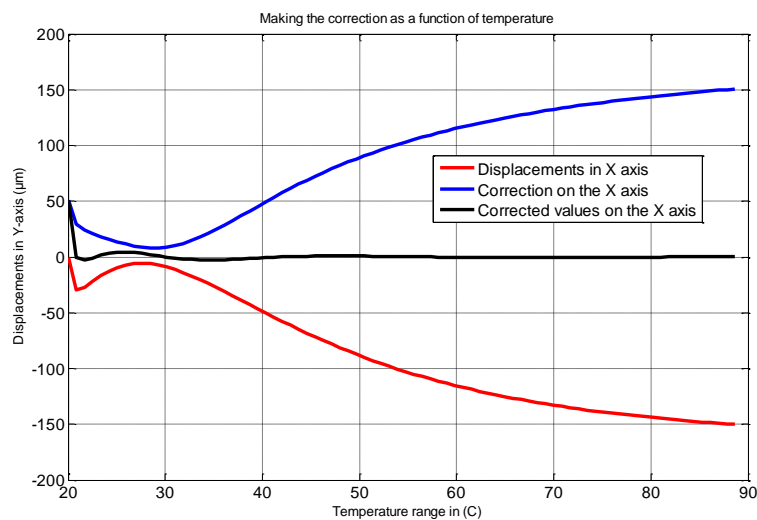


Figure 17: Correction of the X axis, if transient.

Performing the corrections in the X-axis, Y and Z to become volumetric error is, as shown in Fig. 18, almost constant after 25.09°C, and less significant than in the situation without correction. In percentage terms, considering the last time increment, where the thermal displacements were increased to 150.75µm and 0.65µm (reduction of 99.57% in the X axis), 11.83µm to 0.56µm (reduction 95.1% on the Y axis), and 528.38µm to 1.98µm (reduction of 99.62% in the Z-axis). In other words, the joint work of the FEM, ANN and methodology developed in this work provide significant results in the case of correction of thermal errors in machine tools even in a simulated, but the methodology can be used in real cases of compensation thermal errors even in the most critical case is that the transient case.

Romualdo F., Francisco A. V., João Bosco de A. S., José Carlos L. J.

Development of a Methodology for Determination and Analysis of Thermal Displacements of Machine Tools Using FEM and ANN.

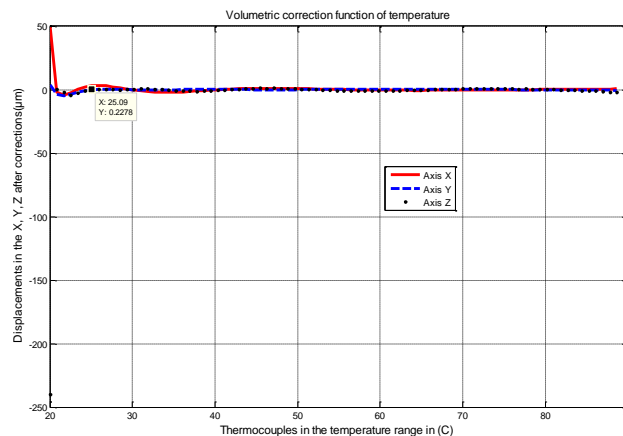


Figure 18: Volumetric Correction for axis.

6. CONCLUSIONS

The data collected from the thermal displacement of machine tool model analysis, can also be collected in any model of machine tool, since it applies the same methodology or that already has the design of the machine using the technology and export to CAD software working with the FEM and then develop a ANNs.

The thermal behavior of the machine is very different from that which was used at a constant temperature from start to finish the simulations, but it is closer to the reality of a machining process is therefore more reliable and can show that thermal errors consistent with the reality of machine design. The fact that there are more irregular thermal errors in the first 30min proves the two thermal conditions studied contour, what is said in the literature, in other words, the machine tool studied is consistent with a real model, considering only the conditions studied. It is observed that in both cases the heat source is variable or not the network has difficulty in learning at the beginning of the deformation. What one can optimize this by refining the network and increasing the number of data in the beginning of deformations is increased or the number of points collected between 0 and 360s. It is also observed that, although the heat sources variations in time, that is, the temperature rise is smoother, has more considerable deformations mainly in the Y and Z axes in relation to the heat sources constant. Due to the fact that the temperature gradient is due to greater thermal stability and be slower. Regarding the neural networks, the temperature, the coordinates of each point analyzed and the thermal displacements corresponding to each axis of MF derived from the thermal expansion are the input and output of the artificial neural network respectively. That through these data, the network topology suggested, "learned" significantly offsets in each axis simultaneously being able to predict the movement only with the entry of the matrix temperature, as verified in the validation section of ANN. Therefore, this work contributes significantly, through the methodology suggested for the reduction and/or reduction of thermal displacement in machine tool. This optional section must be placed before the list of references.

7. REFERENCES

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8. RESPONSIBILITY NOTICE

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