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## MODELING OF A CASCADE CRYOGENIC REFRIGERATOR

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**Abstract.** *This paper presents a study on modeling and analysis of a cascade two-stage cryogenic refrigerator. The system is composed by two refrigeration cycles working with R404A and R23 in the high and low pressure cycles, respectively. The system contains tubing, reciprocating compressors, capillary tubes, intermediary heat exchanger, condenser, evaporator and the cabinet. The complete model is composed by a group of sub-models for each component, adopting one dimensional and steady state condition approaches. Energy balance, heat transfer and pressure drop processes in each component are modeled. Mathematical models for each component are described. A brief analysis of the simulated results is performed.*

**Keywords:** *Cryogenic refrigerator, Cascade, Simulation, Steady-State*

### 1 INTRODUCTION

Cryogenic refrigerators are widely used in research and in laboratories, for example for conservation of embryos. According to ASHRAE (2006), cryogenics refers to equipments that operate or processes that occur at low temperatures. On contrary to household refrigerators that comprises a single vapor-compression cycle, cryogenic refrigerators work with two or more cascade refrigeration cycles.

Up to the present date, according to the authors' knowledge, studies focusing on the modeling of a refrigeration system operating with cascade cycles are not found in the literature. On the other hand, studies concerning household refrigerators are frequently published, e.g. Hermes *et al.* (2009) presented a study focused in modeling and analysis of a domestic refrigerator, operating under steady state condition, and in another study, Hermes *et al.* (2008) have investigated a household refrigerator operating under transient condition in order to estimate its energy consumption.

Based on these aspects, this paper presents a study focused on the modeling and analysis of a cryogenic refrigeration system for laboratory application. The system operates with two cycles. The high pressure cycle operates with the refrigerant R404A and the low pressure cycle with the refrigerant R23.

The presented model is based on commercial equipment manufactured by ColdLab Company, and is being developed in order to help the design and to diminish the number of physical prototypes.

The complete model of the refrigerator is composed of a group of sub-models, divided in terms of heat exchangers, compressors and capillary tubes. Figure 1 presents schematically the cryogenic circuits, including the subsystems and the states. In the subsequent sections, each sub-model of the complete system is described. It was considered a steady state condition for the refrigerator, including the compressor.

The model was implemented in EES (Engineering Equation Solver, Professional V9.303-3D), and each sub model was implemented as procedures or functions. Condenser geometric aspects and fan operational curves were inserted as tables.

### 2 MODEL DESCRIPTION

The heat exchangers were modeled based on the discretization of their geometry. This approach was selected in order to avoid the necessity of adopting effectiveness factors for the analysis. The heat exchangers were modeled assuming all tubes positioned horizontally. Additionally, the frictional pressure drop in the bends was assumed as equal to 4 times the pressure drop of a straight tube with similar length. Figure 2 illustrates the schematics of the heat exchanger discretization, for an element in coordinate  $l$  and length  $\Delta l$ , operating under condition of counter-current flow.

In Figure 2, the internal fluid is denoted by the sub index  $i$ , and has the following inlet properties: pressure  $p$ , Temperature  $T$ , vapor quality  $x$ , enthalpy  $j$ , and mass flow  $\dot{m}$ . Figure 2 also gives the outlet properties indicated by the sub index  $out$ . The heat transfer coefficient  $h_{i,x}$  and the pressure gradient  $dp/dl_{i,l}$  are evaluated for each element  $l$ . Similarly, the same properties are presented for the external flow, that can be the high pressure fluid in the intermediary heat exchanger, or the air in the condenser and cabinet.

From continuity, the inlet mass flow is equal to the outlet mass flow. The other outlet properties are evaluated as follows:

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$$p_{i,out,l} = p_{i,in,l} - \frac{dp_{i,l}}{dl} \Delta l = p_{i,in,l+\Delta l} \tag{1}$$

$$j_{i,out,l} = j_{i,in,l} - \frac{q_l}{\dot{m}_{i,in,l}} = j_{i,in,l+\Delta l} \tag{2}$$

where  $q_l$  is the heat transferred from the inner flow to the external flow. Temperature and vapor quality are evaluated based on pressure and enthalpy. The same approach is adopted for the external flow, however different directions are considered for counter current flow.

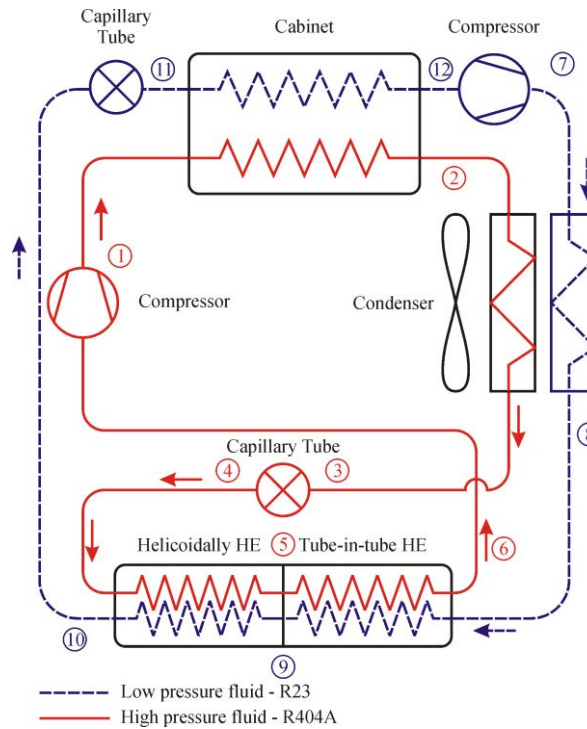


Figure 1. Schematic diagram of the refrigeration system.

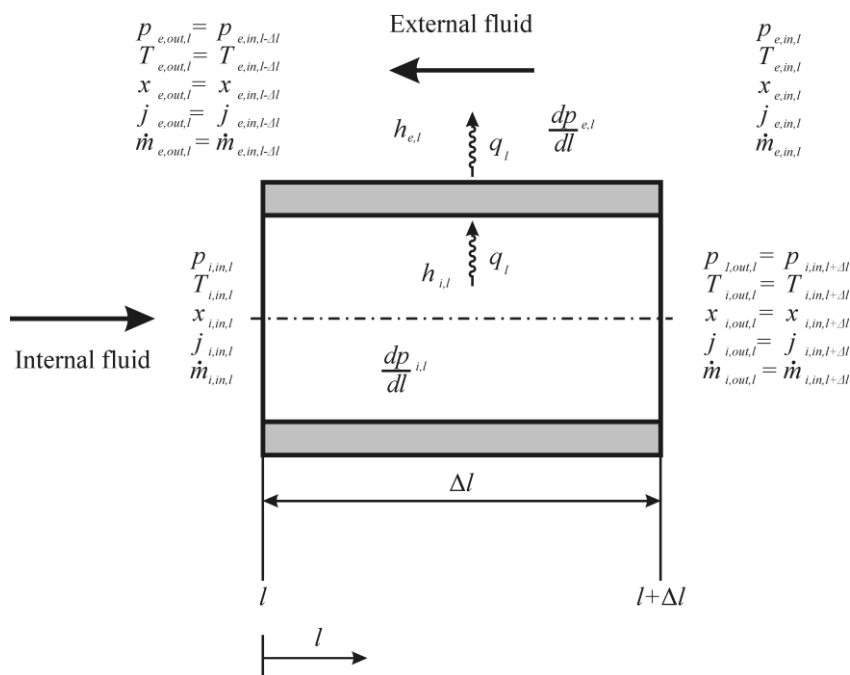


Figure 2. Schematics of discretization of heat exchangers.

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Heat exchange  $q_l$  is evaluated based on the overall heat transfer coefficient that includes internal convection heat transfer coefficient, wall conduction resistance, and external conduction and/or convection. For models that require the heat flux, as Cavallini *et al.* (2003), loops were performed in order to obtain convergence of the heat flux with the overall heat transfer coefficient.

For single-phase flow, Gnielinski (1976) correlation was adopted for determination of heat transfer coefficient in straight tube, and the friction factor given by the Churchill (1977) correlation was adopted for the estimative of frictional pressure drop. For curved tubes in heat exchangers with predominantly straight tubes, Gnielinski (1976) was adopted for the heat transfer coefficient, and an increment of four was adopted for the pressure drop. In the case of spirally curved tubes, the single-phase heat transfer coefficient is given by Gnielinski (2010) correlation, and the pressure drop is estimated with Churchill (1977) friction factor, with an increment of four times.

The internal flow boiling heat transfer coefficient was estimated according to the correlation of Liu and Winterton (1991). The correlation proposed by Cavallini *et al.* (2003) was used in order to estimate the heat transfer coefficient during in-tube condensation. The frictional pressure drop was estimated according to Friedel (1979) correlation in both cases (condensation and evaporation).

## 2.1 Cabinet

In the cabinet, heat exchanges among the low pressure fluid, the air inside the cabinet, the external environment, and the high pressure fluid were modeled. Figure 3 presents schematically the modeled cabinet.

The evaporator inside the cabinet consists of a copper tube attached to the internal surface of the cabinet. The tube presents multiple passes on the top, followed by multiple passes that covers both internal laterals, and the back surface. The low pressure fluid enters the evaporator at the top region of the cabinet, and follows to the laterals and back region.

Additionally to the low pressure refrigerant fluid, the system counts with a single pass of the high pressure refrigerant fluid, in the frontal surface of the cabinet, between consecutives sealings. The adoption of this pass is performed in order to avoid heat conduction from the environment to the inner region through this surface that causes condensation of air moisture in the external surface.

For the modeling of the heat transfer between the refrigerant fluids and the cabinet surfaces, it was considered the intube convection heat transfer, tube wall conduction, conduction between the tube and the plane surface considering a form factor, and the convective resistance between the plane surfaces and the air. The conduction between the internal and external surfaces was modeled as one-dimensional conduction through the insulation, and the heat transfer between the external surface and the environment was modeled as free convection.

The free convection heat transfer coefficients were modeled according to Kast and Klan (2010) correlation for free convection that distinguishes among surface orientations and heat transfer direction. The insulation consists of expanded polyurethane, 120 mm thick.

A loop of the entire cabinet, including the refrigerants circuits and the internal and external air sides, was executed until no variation was observed in the refrigerants outlet conditions.

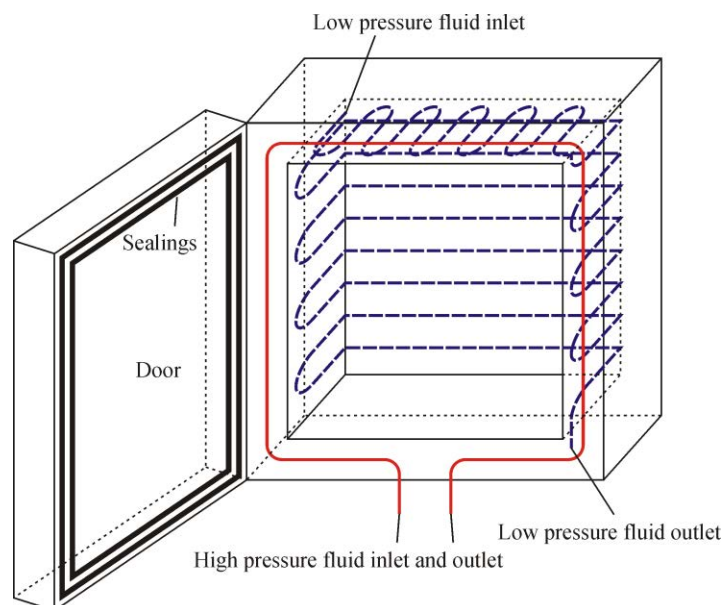


Figure 3. Schematics of the cabinet.

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## 2.2 Condenser

The condenser consists of finned heat exchangers, with the air propelled by a pair of fans. The aluminum fins have louver profile, and the copper tubes presents internal diameter of 9.525 mm (3/8 inches) of. The condenser presents a total of 40 passes, with 28 of the upper region for the high pressure fluid and the rest for the low pressure fluid. Figure 4 presents the schematics of the condenser. Filled lines represent curves in the front face, and dotted lines represent curves in the back face.

As can be observed the condenser counts with two circuits, one for the high pressure fluid and another for the low pressure fluid. The tube distribution presents triangular normal configuration, with the fluids inlets in the bottom region of each respective circuit.

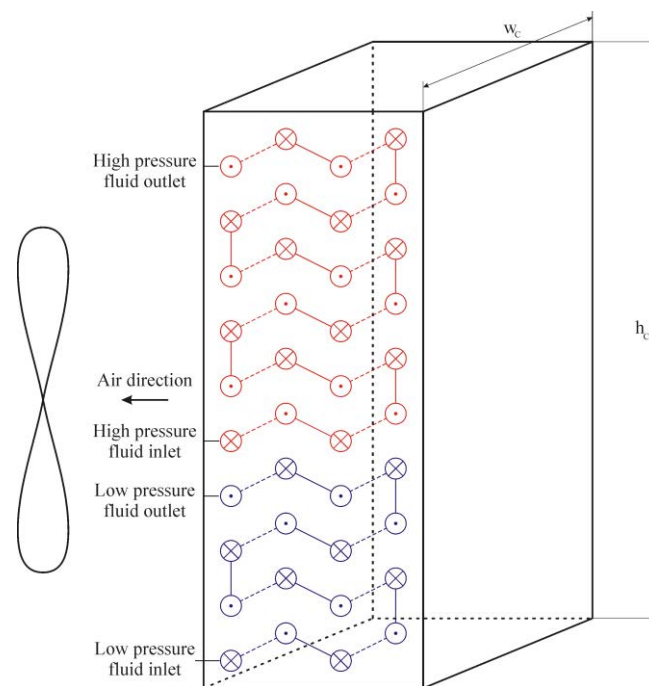


Figure 4. Schematics of condenser model.

The model was implemented similar to the methodology described by Zoghbi (2004), according to which the circuits are entered in the model as tables containing the following information: the position of the tube pass (column and row), flow direction, next pass, and pass function (first pass of the circuit, last pass of the circuit and regular pass). Each pass was discretized, and based on the external and internal convection heat transfer coefficient and the conduction through the wall, the heat flux, and consequently the energy balance was solved for each element.

A loop for the entire condenser, including the air and refrigerants sides, was executed until no variations of the outlet conditions were observed for the refrigerants.

Air flow rate was estimated from the fans curve provided by the manufacturer and the pressure drop of the air-side of the condenser. It is considered that this model has two similar fans, each one with 62.8 W. The adopted methodology for estimating the air pressure drop is given by Wang *et al.* (1999), considering the average temperature of the air between the inlet and outlet. Based on the air velocity, the air side heat transfer coefficient was estimated based on the Wang *et al.* (1999) correlation.

## 2.3 Intermediary heat exchanger

The intermediary heat exchanger is the evaporator of the high pressure cycle and the condenser of the low pressure cycle. It is composed of a helicoidally and tube-in-tube heat exchangers, with the higher pressure refrigerant flowing in the shell side and the low pressure fluid in the tube side. Figure 5 presents the schematics of the intermediary heat exchanger.

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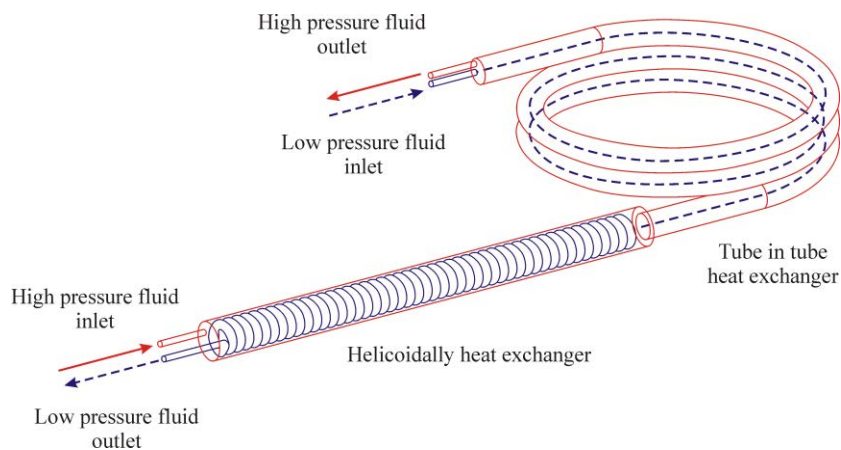


Figure 5. Schematics of intermediary heat exchanger.

As can be observed by Fig. 5, the heat exchanger works as a counter current device, in order to enhance the heat transfer between both fluids.

The model of the intermediary heat exchanger neglects heat transfer with the environment. For the helical region of the heat exchanger, the discretization element correspond to each coil of the inner tube, and for the tube-in-tube heat exchanger, the discretization considered the total length divided into small equal parts.

The thermo hydraulic model for the high pressure fluid in the helical region was divided into two regions, one for the inner region of the coil and another for the annulus region between the coil and the shell. The mass flow rate in each region was determined through the estimative of pressure drop in both regions that must be equal, and the sum of individual mass flow rate must be the total mass flow rate. Single-phase heat transfer was estimated adopting the correlation from Gnielinski (1976) for the inner region and Gnielinski (2010a) for the annular region. During two-phase flow, the heat transfer was evaluated based on the Liu and Winterton (1991) model, considering the inner diameter and hydraulic diameter as characteristics lengths. Frictional pressure drop was estimated based on the correlation proposed by Friedel (1979).

For the flow inside the tube in the helical region, during single-phase flow the heat transfer coefficient was estimated with the correlation proposed by Gnielinski (2010b), and the pressure drop was estimated considering the friction factor given by the Churchill (1977) correlation, multiplied by a factor equal four. For two-phase flow, it was considered the correlation of Liu and Winterton (1991) for flow boiling and Cavallini *et al.* (2003) for condensation.

For the tube-in-tube region, the shell side flow was considered as an annulus region and heat exchanges with environment were neglected.

## 2.4 Capillary tube

The expansion devices of the system modeled in the present study are capillary tubes. This expansion method is the cheapest, although it requires significant number of trial and errors tests in order to obtain an appropriate length for the operational condition desired. This system also presents disadvantage due to the fact that it does not compensate variations in thermal charge of the system.

In the present study, the capillary tubes were modeled as suggested by Hermes *et al.* (2010). This method, developed for adiabatic capillary tube, provides the mass flow rate through the capillary tube based on fluid properties at capillary tube inlet.

It was found in the literature several methods that model the capillary tube as discretized elements, however these methods are computationally expensive, and according to Hermes *et al.* (2010) does not necessarily imply in better prediction of the capillary tube behavior.

In the method of Hermes *et al.* (2010), the mass flow rate is given according to the following relationship:

$$\dot{m} = \left\{ \frac{\pi^{2-C_4} 2^{2C_4-3}}{C_3} \frac{d^{5-C_4}}{\mu_f^{C_4} L_{cap}} \left[ \frac{p_{cap,in} - p_f}{v_f} + \frac{p_f - p_{cap,out}}{C_1} + \frac{C_2}{C_1^2} \ln \left( \frac{C_1 p_{cap,out} + C_2}{C_1 p_f + C_2} \right) \right] \right\}^{1/(2-C_4)} \quad (3)$$

where the sub index  $f$  corresponds to the liquid properties at the flashing point,  $v_f$  is the liquid specific volume at the flashing condition,  $\mu_f$  is the liquid viscosity,  $L_{cap}$  is the capillary length,  $p_{cap,in}$  and  $p_{cap,out}$  are the inlet and outlet pressure. Flashing point corresponds to the pressure in which saturated liquid presents the same enthalpy as the inlet condition. Outlet pressure,  $p_{cap,out}$ , can be defined as the evaporation pressure. The terms  $C_1$  and  $C_2$  are given by Yilmaz and Ünal (1996), as follows:

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$$C_1 = v_f(1 - k_f) \quad (4)$$

$$C_2 = v_f p_f k_f \quad (5)$$

$$k_f = 1.63 \times 10^5 p_f^{-0.72} \quad (6)$$

and the constants  $C_3$  and  $C_4$  were obtained by Hermes *et al.* (2010) through regression of experimental results and are given as follows:

$$C_3 = 0.14 \quad (7)$$

$$C_4 = 0.15 \quad (8)$$

From Eq. (3), the mass flow through the capillary tube is obtained without iterative processes.

## 2.5 Compressor

The refrigeration system comprises two reciprocating compressors model NJ 2212GK produced by Embraco. The compressors were modeled as steady state devices, operating between the evaporator and the condenser region of each cycle.

The mathematical models of the compressors were based on the methodology presented by Saiz-Jabardo *et al.* (2002) that take into account the volumetric efficiency of the compressor, pressure ratio, and inlet fluid properties. The mass flow through the compressor is given according to the following equation:

$$\dot{m}_{comp} = V_{comp} \rho_{evap} \eta_V f \quad (9)$$

where the volumetric efficiency is estimated according to the following equation:

$$\eta_V = f_{n,DV} \left\{ 1 - \varepsilon_{cv} \left[ \left( \frac{P_{cond}}{P_{evap}} \right)^{1/np} - 1 \right] \right\} \quad (10)$$

The unknown parameters in Eqs. (9) and (10) were estimated based on the data provided by the catalogue of the manufacturer, Embraco (2004), considering an electrical frequency of 60 Hz. From regression through minimum square errors and considering the data from the catalogue, the volumetric efficiency  $f_{n,DV}$  and the clearance volume fraction  $\varepsilon_{cv}$  were found equal to 0.535 and 0.052, respectively. The cylinder volume  $V_{comp}$  was estimated based on the geometry data of the compressor and was found equal to  $34.38 \cdot 10^{-6} \text{ m}^3$ . Since the term in the exponent of pressure ratio,  $np$ , was obtained for R404A based on the data from the catalogue, its value was corrected for the refrigerants R23 in the low pressure cycle. The procedure adopted for this correction was the following: i) a value of  $np$  was obtained for R404A; ii) this value was divided by the specific heat ratio of R404A that correspond to the exponent for an adiabatic polytropic process and a value of 0.976 was found; iii) then, the  $np$  for R23 is calculated as the product between 0.976 and the specific heat ratio of R23.

The command group that defines the compressor was solved iteratively, in order to obtain the same mass flow rate given by the capillary tube, consequently the outlet condition was obtained from this loop.

## 2.6 Solution methodology

This section presents the solution methodology for the entire model. As above mentioned, the model was implemented in EES software, with the sub-models as functions or procedures.

Each sub-model was executed sequentially, with the output conditions being applied as input conditions of the next sub-model in the cycle. Figure 6 presents the diagram block adopted for the solution of the complete model, with the outlet states indicated below each block, numbered according to Fig. 1.

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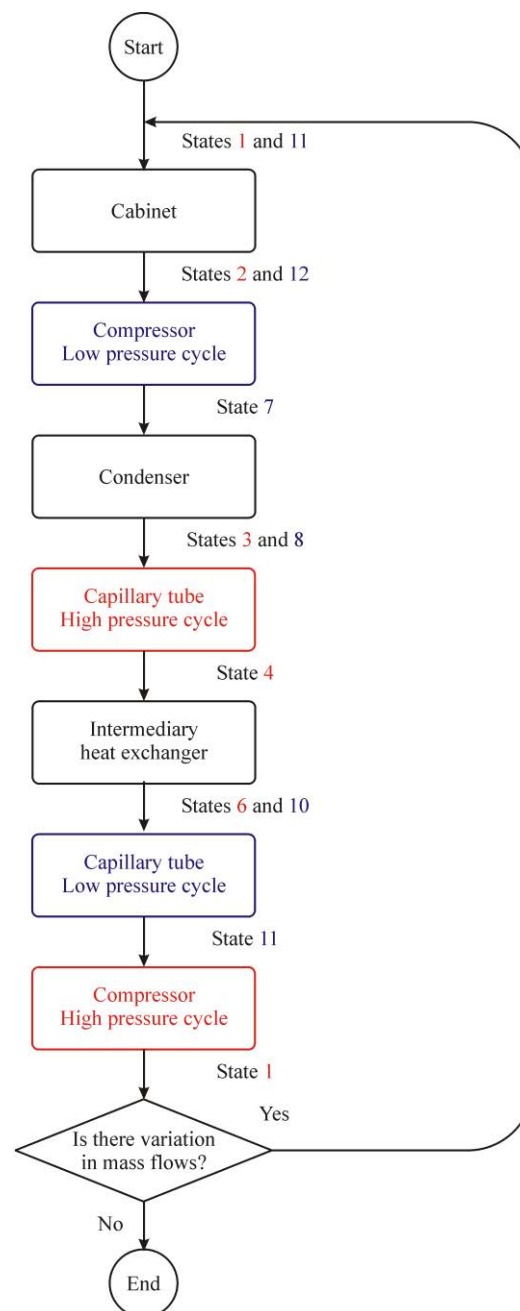


Figure 6. Diagram block of numerical solution.

As the starting point, it was assumed guess values for the states conditions at the cabinet inlet, numbered as 1 and 11 in Fig. 1. Guess values for mass flow rates were also assumed, in order to solve the first loop of the system. In subsequent loops, the cabinet inlet conditions and mass flow rates were estimated with the entire system.

An evaporation pressure corresponding to saturation temperature of  $-70\text{ }^{\circ}\text{C}$  was assumed for the state 11, low pressure fluid at the cabinet inlet, due to the fact that the desired temperature inside the cabinet is  $-50\text{ }^{\circ}\text{C}$ .

The compressor outlet conditions were estimated through an iterative method, with the mass flow obtained from the capillary tube formulation. With this procedure, the pressure ratio was obtained, and the outlet conditions could be estimated.

It was assumed convergence of the complete model when small variations of the mass flows were observed. The limit value for the sum of the absolute variations of both fluxes adopted was equal to  $1.10^{-6}\text{ kg/s}$ .

### 3 ANALYSIS RESULTS AND DISCUSSION

This section presents results of the analysis performed with the above described model. An analysis was performed in order to obtain the working temperature of the cabinet, refrigerants inventories, mass flow rate and all the thermodynamic states.

The following geometric parameters were adopted for the simulation:

- Cabinet:
  - Evaporator tube inner diameter: 9.525 mm;
  - High pressure pass inner diameter tube: 7.938 mm;
  - Evaporator tube length: 24.9 m;
  - High pressure tube length: 4 m;
  - Insulation thickness: 130 mm;
- Condenser:
  - Tube inner diameter: 9.525 mm;
  - Fin pitch: 2 mm;
  - Tube transversal pitch: 25 mm;
  - Tube longitudinal pitch: 25 mm;
- Helicoidally heat exchanger:
  - Inner tube internal diameter: 6.35 mm;
  - Shell inner diameter: 28 mm;
  - Coil diameter: 14.65 mm;
- Tube in tube heat exchanger:
  - Inner tube internal diameter: 6.35 mm;
  - Shell inner diameter: 15.88 mm;
  - Two inner tubes;
- Capillary tube of the low pressure stage:
  - Inner diameter: 0.64 mm;
  - Length: 5.9 m;
- Capillary tube of the high pressure stage:
  - Inner diameter: 0.64 mm;
  - Length: 7 m;

An analysis was performed considering the geometrical aspects above described, and Tab. 1 presents the results for this configuration of the system, for the states indicated in Fig. 2.

Table 1. Thermodynamic states obtained from the analysis.

State	p [kPa]	T [C]	x [-]
1	2521	92.2	100
2	2517	75.6	100
3	2513	30.0	0.01
4	672.4	3.0	0.24
5	672.4	3.0	0.26
6	671.3	2.9	0.27
7	2521	133.3	100
8	2520	20.0	100
9	2519	4.9	0.52
10	2514	3.5	0.11
11	194	-70.0	0.47
12	182	-71.2	0.98

The mass flow rates obtained from the simulation are equal to 0.024 and 0.007 kg/s for the high and low pressure fluids, respectively. A refrigerant charge of 0.814 and 0.842 kg was obtained for the high and low pressure stages, respectively. The air temperature inside the cabinet is equal to -52.6 °C.

As can be observed based on the results presented on Tab. 1, the model shows reasonable results, indicating that it is a good starting point for the development of a design tool. Experimental results, as pressure ratio in the compressors, fluid temperatures in each heat exchanger inlet and outlet, would be required in order to validate the computational code, and also to improve the models.



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#### 4 CONCLUDING REMARKS

This paper presents a study focused on the modeling and analysis of a two-stages cascade refrigeration equipment for cryogenic application. The main purpose of the refrigerator is laboratorial applications, for cabinet internal temperature close to -50 °C.

A model was developed, in order to evaluate thermodynamic states, refrigerant inventories and working temperature inside the cabinet, under steady state condition.

The model is representative of a real system, although experimental data were not available up to the present date to be used to validate and adjust the models.

It is expected that the computational model developed during the present study would help during the design process of new models of refrigerators, avoiding high number of physical prototypes.

#### 5 ACKNOWLEDGEMENTS

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