APPLICATION OF NURBS FOR OBTAINING FREE FORM CURVES AND SURFACES IN SPUR AND HELICAL GEARS BY USING COORDINATE MEASUREMENT MACHINES (CMM)

Luiz Henrique Melo Silva Nóbrega, henriquenobrega.ifpb@gmail.com
Carlos Henrique Alencar Almeida, henrique_alencar1@hotmail.com
Instituto Federal de Educação da Paraíba, IFPB – Rua José Antônio da Silva, 300, Óasis - CEP 58.900.000 – Cajazeiras – PB, Brasil
Esly César Marinho da Silva, eslymarinho@ifba.edu.br
Instituto Federal de Educação da Bahia – IFBA, Via Universitária, s/n. Pitanguinha. CEP 43700-000 - Simões Filho – BA, Brasil
Verílton Nunes da Silva, verilton@yahoo.com.br
Igor Lucena Peixoto Andrezza, igorlpa90@gmail.com
José Carlos de Lima Júnior, limajrcarlos@hotmail.com
João Bosco de Aquino Silva, jbosco@ct.ufpb.br
Universidade Federal da Paraíba, Centro de Tecnologia, Bloco "F" – Campus I – CEP 58.059-900 – João Pessoa – PB, Brasil
Rafael Franklin Alves Silva, rafael.falves@hotmail.com
Instituto Federal de Educação da Paraíba, IFPB – Av. 1º de Maio, 720, Jaguaribe - CEP 58.900.000 – João Pessoa – PB, Brasil

Abstract. *Currently, engineering processes require reduced manufacturing time and low cost, in addition to the growing demand for workpieces with high accuracy. Workpieces with complex geometries and free forms has been a common practice in industries from different sectors such as: automotive, aeronautics, bioengineering, among others. One way to satisfy the market requirements satisfactorily is making measurements more efficient to make the production process faster, in other words, it is necessary inspection system more accurate and flexible. The coordinate measuring evolved over the past three decades and today it is the technology that best meets the requirements of modem manufacturing through Coordinate Measurement Machines (CMMs). The CMMs are important tool for design, fabrication and inspection of manufactured products, also used in the application of reverse engineering. These machines are also used by engineers in order to produce an accurate digital model in a virtual space for later use in CAD / CAM. It is worth mentioning that the accuracy of the modeling process of given piece depends on the number of control points that are captured on the workpiece surface. Consequently, the laser inspection systems are the best tools for use in reverse engineering, but more expensive when compared to contact measurement systems that use the touch trigger probe, also used by CMMs. In this case, this paper aims to present an approach based on NURBS (Non Uniform Rational B-Splines) to obtain free form curves and surfaces from a group of points obtained by using a contact sensor, the touch trigger probe. NURBS is an important mathematical tool and consist of generalizations of Bezier curves and surfaces and B-Splines. The approach proposed in this paper can be applied for obtaining free form curves and surfaces in spur and helical gears. Experimental results, obtained by measuring spur gears, showed that the NURBS technique contributes for application of CMMs with touch trigger probe in reverse engineering.*

Keywords: *Coordinate Measurement Machine, touch trigger probe, NURBS, complex geometries, reverse engineering.*

1. INTRODUCTION

The advancement of innovative technologies in the several engineering's areas has generated new demands/requirements with regard to ensuring the reliability and quality of manufactured parts. Consequently, inspection operations of workpieces and products in the industries have become essential and they can be done manually or by computerized machines. This last one is most commonly used to two reasons, the achievements more complex operations, once the execution carried out by a human being would mean a greater use of time and can occur various errors, and due the increasing interest in using the computer aided design and manufacture's concept CAD/CAM, the reverse engineering.

According Bagci (2009), reverse engineering is a process that intends to generate a copy of a physical prototype at industrial level. This process involves the prototype measurement and the construction of a Computer-Aided Design (CAD) model, followed by a manufacturing of the parts using Computer-Aided Manufacturing (CAM) process through rapid prototyping or automatic milling machines. This process is also applied in industry to design new parts, to reproduce an existing product, to recover damaged or defective parts and to design molds.

Nowadays, stringent inspections and accurate are needed, mainly when it refers to workpieces for been used in the aeronautical industries, space, medicine etc, and therefore, it is considered a major ally to conduct inspections of complex surfaces, the Coordinate Measuring Machines (CMMs), Silva (2009).

The CMMs operate according the principles of coordinating metrology. Its basic function is measure the actual shape of the workpiece, comparing to desired shape and the information about of metrological evaluation is carried out. The actual shape of the part is obtained by contact of its surface in discrete measurements points, in which these are

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expressed in terms of its coordinates (Nóbrega, 2011). According to Varady *et al* (1997), the CMM are not only used for the inspection, but also to reverse engineering.

Nowadays, the CMM have two different measurement systems applied to dimensional control: measurement for contact and non-contact. Papers published by Ferreira (2007) show that contact measurement systems, commonly known as probing systems, aims to make the acquision of a given point through contact with the surface to be measured, with the coordinate axis X, Y and Z relative to a coordinate system defined on the machine. While the mentioned system collects discrete points, on the other hand, non-contact measurement systems are able to collect a cloud of points, providing greater fidelity workpieces digitalization. According to Silva (2011), today is more used CMMs by using contact sensors, since these operations has great versatility, and low cost, when compared to non-contact measurement systems.

Following the changes in the shapes of products on the environment metrological is not an easy task, it must use specific software like a tool for design. The modeling can be considered as accurate and detailed creating of the free and complex forms. These are considered free forms when they are different from models with known characteristics and geometry, for example, planes, circles, cylinders, etc (Silva *et al, 2011*). As CMMs have CAD itself, they are able to identify, visualize and interpret data collected and thus characterize known geometries. For these types of measurements, the Coordinate Measuring Machines have a good accuracy, but for free form workpiece the inspections become harder. Since then, research has been developed in the area of data acquisition for surfaces of complex geometry using CMMS and CMA (Coordinate Measuring Arm).

Li and Gu (2004) mentioned, it is considered complex surface the format presented by piece and it has not the features of known geometry, within the mathematical analysis, but defined by control points through Bézier, B-spline, NURBS (Non Uniform Rational B-splines) techniques, etc.

NURBS are parametric functions that can represent any type of curve. NURBS are used in graphic computation in CAD/CAMs industry and are being considered a standard to create and to represent complex objects (automobile industry, aviation and vessel). The tools of more sophisticated graphic creation provide an interface to use NURBS, who are enough flexible to project a great variety of forms. Today it is possible to verify the use expanded of NURBS modeling objects for virtual arts, art and sculpture; they also are being used to model scenes for applications of virtual reality (Minetto, 2003).

Also according to Minetto (2003), NURBS result from a mathematical base, unified to represent analytic and free forms to besides maintaining exactness and independence of mathematical resolution.

Then, this paper aimed to present the measurement of spur gears by using a Coordinate Measuring Machine with contact probe. Additionally, the measuring points were used to generate CAD models by applying the NURBS technique. Then, error analysis was performed comparing measuring points to respective NURBS ones. The results showed that the proposed methodology/strategy provides accurate CAD models.

2. NURBS: CURVES AND SURFACES

B-Splines are polynomial curves. They are flexible and offer many useful properties for the design of curves, but they do not have the capacity to represent the simplest ones, for example, circles and ellipses. For dealing with the mentioned above geometries, it is necessary an extension of B-Spline curves. Considering the way to introduce rational homogeneous coordinates in a B-Spline curves, it is derived the definition of NURBS, which it is called Non-Uniform Rational B-Spline.

A NURBS curve is a combination of a set of piecewise rational functions with $n+1$ control points P_i and associated weights w_i, defined by Eq. (1):

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) \omega_i P_i}{\sum_{i=0}^{n} N_{i,p}(u) \omega_i}, \quad a \leq u \leq b \tag{1}$$

The function $\{N_{i,p}(u)\}$ is the basis function defined by B-Spline function and it is determined by Eq. (2) and Eq. (3). The knot vector non uniform and non periodic is defined by Eq. (4).

$$N_{i,0}(u) = \begin{cases} 1 \; if \; u_i \leq u \leq u_{i+1} \\ 0 \; otherwise \end{cases} \tag{2}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \tag{3}$$

$$U = \left\{ \underbrace{a,...,a}_{p+1}, u_{p+1},..., u_{m-p-1}, \underbrace{b,...,b}_{p+1} \right\} \tag{4}$$

Nodes in a NURBS curve are points on spaced parameters in which the rational polynomial curves are joined to form a segment multicurve (Minetto, 2003).

The Equation (1) can be rewrite into the following equivalent form:

$$C(u) = \sum_{i=0}^{n} R_{i,p}(u) P_i , \qquad a \le u \le b \tag{5}$$

and the $R_{i,p}$ is known by Eq. (6) below:

$$R_{i,p}(u) = \frac{N_{i,p}(u)\omega_i}{\sum_{j=0}^{n} N_{j,p}(u)\omega_j} \tag{6}$$

Then, the equation of the NURBS curve can be represented by Eq. (7):

$$C(u) = \sum_{i=0}^{n} R_{i,p}(u) P_i , \qquad a \le u \le b \tag{7}$$

It is NURBS curve of p degree defined by control points p_0, p_1, ..., p_n, knot vector u = {u_0, u_1, ..., u_m}, and weights w_0, w_1, ..., w_n. It is important to noted that because of the weight to be associated with the control point as its fourth component, the number of weights and control points must be identical. This result in a very important characteristic of NURBS curve called *local approximation*, it means that: if a control point is moved or a weight is changed, it will affect only that portion of the curve on the interval $u \in [u_i, u_{i+p+1})$.

According to Piratelli-Filho *et al* (2009), the data processing involved adjustment (approximation) of NURBS curves following the measurement path. This means that the curve does not cross exactly over the data points and there are errors related to the fitting procedure (Piegl and Tiller, 1997).

Silva (2011) has published that the representation of NURBS surfaces are very similar with the development of parametric representation of NURBS curve. A NURBS surface of degree p in the u direction and degree q in the v direction is a bivariate vector-valued piecewise rational function of the form (Piegl and Tiller, 1997):

$$s(u,v) = \frac{\sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) \omega_{i,j} P_{i,j}}{\sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) \omega_{i,j}} , \quad a \le u,v \le b \tag{8}$$

The {$P_{i,j}$} form a bidirectional control net, the {$w_{i,j}$} are the weights, and the {$N_{i,p}(u)$} and {$N_{j,q}(v)$} are the norational B-Spline basis functions defined on the knot vectors (Piegl and Tiller, 1997):

$$U = \left\{ \underbrace{a,...,a}_{p+1}, u_{p+1},..., u_{m-p-1}, \underbrace{b,...,b}_{p+1} \right\} \tag{9}$$

$$V = \left\{ \underbrace{a,...,a}_{q+1}, u_{q+1},..., u_{m-q-1}, \underbrace{b,...,b}_{q+1} \right\} \tag{10}$$

3. GEARS AND MEASUREMENT METHODOLOGY

It is very common for industries that manufacture gears require equipment capable of measuring profile of those ones with high accuracy. Nowadays, one of the instruments commonly used for measuring gears profile is called profile reproducer.

In this topic, a methodology to measure spur gears is proposed. It involves the measurement of spur gears using a Coordinate Measuring Machine with contact probe and the development of CAD models using Non-Uniform Rational

B-Splines (NURBS) modeling techniques. One aim is to generate a profile envolvental of cylindrical spur gear using a few control points. The error analysis was carried out comparing the data points with the respective CAD points. Dumoulin *et al* (1994) presented a methodology to recover the design of a Francis hydraulic turbine runner and Piratelli-Filho *et al* (2009) published a paper that a measurement strategy was proposed to reconstruct a Kaplan hydraulic turbine runner. All these authors aimed to capture a minimum amount of points.

3.1 SPUR AND HELICAL GEARS

According the gear geometry, the kinematics relations and the forces transmitted, there are two types of teeth gears: spur and helical teeth.

Spur gears are used to transmit rotary motion between parallel shafts. Helical gears are used to transmit motion between parallel and nonparallel shafts, as shown Fig. 1 below.

(a)   (b)
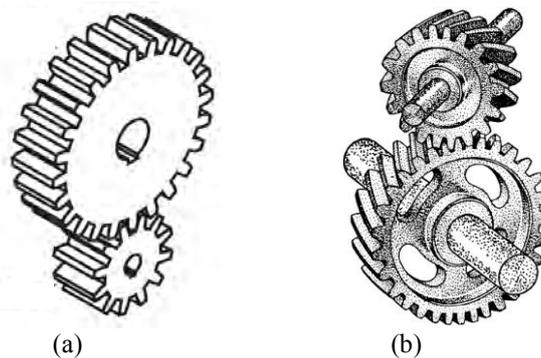
Figure 1. Types of gears: (a) spur and (b) helical gears (TC2000)

The terminology of spur-gear and helical teeth are illustrated in Fig. 2 and 3 below.

(a)   (b)
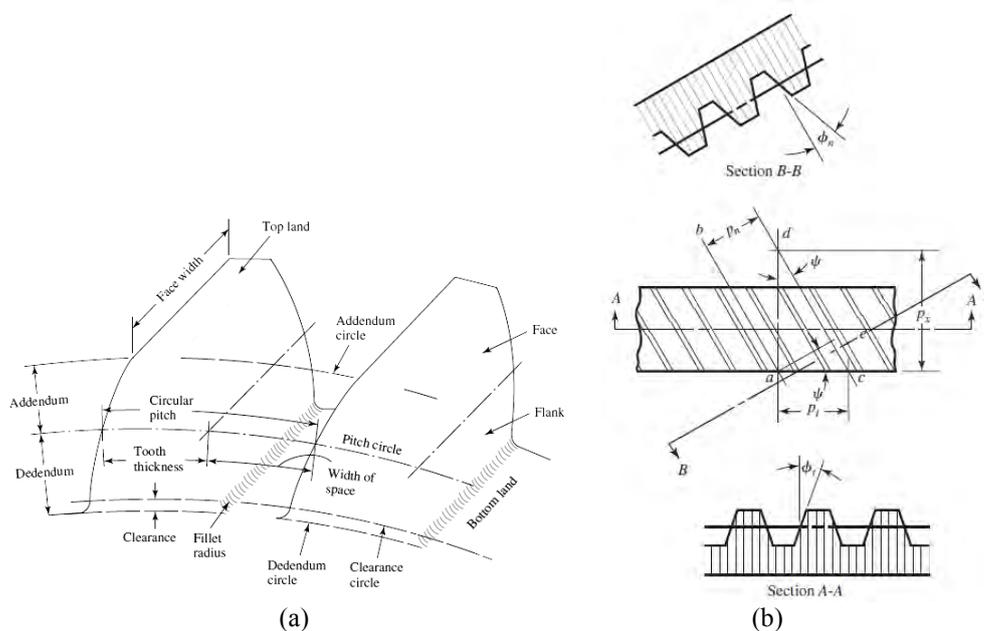
Figure 2. Spur (a) and helical (b) gears terminology (Shigley, 2008)

Figure 2(b) represents a portion of the top view of a helical rack. Lines *ab* and *cd* are the centerlines of two adjacent helical teeth taken on the same pitch plane. The angle ψ is the helix angle. The distance *ac* is the *transverse circular pitch* p_t in the plane of rotation (usually called the circular pitch). The distance *ae* is the *normal circular pitch* p_n (Shigley, 2008).

The initial contact will take place when de flank of the driver comes into contact with the tip of the driven tooth, as shown Fig. 3 (Shigley, 2008).
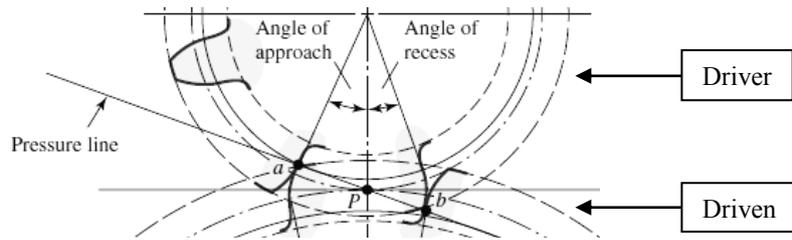
Figure 3. Tooth action (Shigley, 2008)

This contact occurs at point *a*, where the addendum circle of the driven gear crosses the pressure line. As the teeth go into mesh, the point of contact will slide up the side of the driving tooth so that the tip of the driver will be in contact just before contact ends. The final point of contact will therefore be where the addendum circle of the driver crosses the pressure line. The sum of the angle of approach and the angle of recess for either gear is called the *angle of action*, and the line *ab* is called *the line of action* (Shigley, 2008).

The envolvental teeth gear profile can be obtained by prior knowledge of the radius of the primitive circle, *r*, and the envolvental angle, *β*. Through these values, it is possible to determine the coordinates (x,y) of any point that belong to the envolvental. The coordinate are calculated by Eq. (10) and (11) below.

$$x = r.(\cos\beta + sen\beta)$$

(11)

$$y = r.(sen\beta - \cos\beta)$$

(12)

3.2 MEASUREMENT METHODOLOGY

According to Equation (11) and (12) was developed a code using MATLAB software to generate control points, simulating points collected through a CMM. The gear parameters used were: pressure angle 20º, module equal to 5 and number of tooth equal to 20 (Lima Jr, 2007). It is important to emphasize that this step was carried out in order to develop a measurement strategy to collect points using a CMM.

With the analytically points generated, the theoretical and NURBS curves were plotted, in order to establish a comparison of values between the two curves, for the envolvental profile in a spur gear. For the elaboration of NURBS curve, was used a toolbox developed in MATLAB ambient, by Zhang (2008).

Generating curves and surfaces using that toolbox, it is necessary insert the control points, the weights for NURBS function and the knot vector. In additionally, for this paper was also developed specific algorithms to generate the building of vector knots and errors analysis.

According Silva *et al* (2011), is possible to observe that the accuracy of the modeling process of a part will be higher the larger the number of points collected. Searching a convenient measurement strategy, that seeks to optimize the number of points collected, the theoretical curves were compared to NURBS ones using 5 and 6 control points, in order to achieve an error range less than 100μm. That strategy was developed to generate a envolvental curve of a spur gear.

ISO 10360 dictates that the accuracy ranges of measuring equipment have measurement uncertainties ranging from 20μm to 100μm. Among the equipment specified by the standard are the CMM with laser head. If the results of a system for contact point is within this uncertainty level for the errors found, the following results are considered acceptable, because was used a lower equipment cost and still managed to meet the uncertainties with a smaller number of points collected.

A similar methodology was developed for helical gear. In this case, there were collected a number of control points which compose the surface of the tooth. To collect the points, the gear was divided in a mesh. To facilitate the computational procedure on building of multicellular matrices is important to collect the coordinates points to form a square matrix. Silva (2011) highlighted in his work a methodology for curves and surfaces measurement. In this case, the author showed a (x,y) plane view and specified the direction that the points should be collected through a machine contact measurement, as shown Fig. 4.
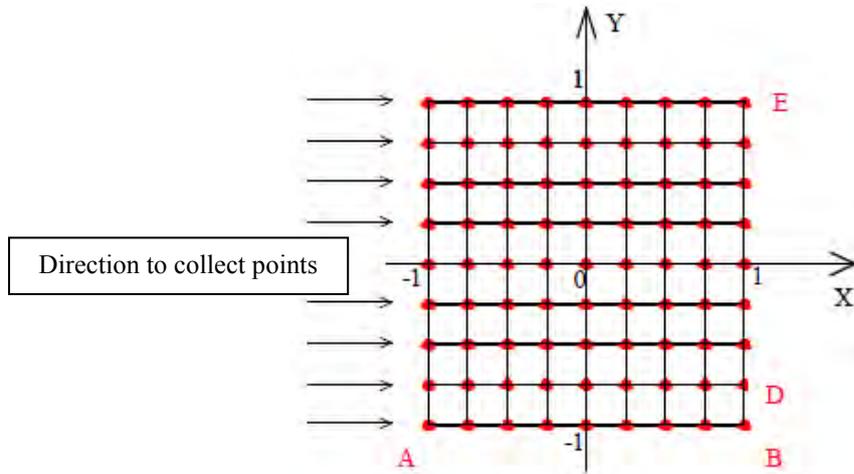
Figure 4. Selected points for NURBS surface representation (Silva, 2011).

In Figure 4 the red points indicates the collected coordinates. The sequence to be followed for the storage of data starts at point "*A*" and end at point "*E*", observing the direction chosen, simulating the use of CMM. The sequence starts at A = (-1, -1), ..., B = (1, -1), C = (-1, 0.75), ..., D = (1, -0. 75), and ends in E = (1,1).

4. RESULTS

4.1 THEORETICAL RESULTS

The Table 1 presents the comparative values between the theoretical and NURBS curve. To plot the NURBS curve, was utilized as computational tool a MATLAB toolbox. The input data was utilized the points of control simulated and the knot vector. Specific algorithms were generated to determine the knot vector and the error analysis.

Table 1. Comparative values of representation theoretical curve X NURBS curve for the profile envolvental tooth spur gear

Collected points	Theoretical curve		NURBS curve		Error [mm]
	x	y	x	y	
Point 1	47.632	-4.384	47.632	-4.3840	0.0000
Point 2	47.727	-4.389	47.727	-4.3866	0.0024
Point 3	48.013	-4.385	48.013	-4.3794	0.0056
Point 4	48.488	-4.343	48.488	-4.3360	0.0070
Point 5	49.150	-4.235	49.150	-4.2252	0.0098
Point 6	49.991	-4.031	49.991	-4.0190	0.0120
Point 7	51.005	-3.706	51.005	-3.6886	0.0174
Point 8	52.180	-3.231	52.180	-3.2091	0.0219
Point 9	53.504	-2.583	53.504	-2.5673	0.0157
Point 10	54.964	-1.736	54.964	-1.7360	0.0000

The set of points generated analytically was used to plot the gear envolvental profile on MATLAB software. The Figure 9 shows the envolvental teeth curve using 10 control points (points of the Tab. 1), presenting a comparative of the theoretical curve and the NURBS curve, using the same points cited before.
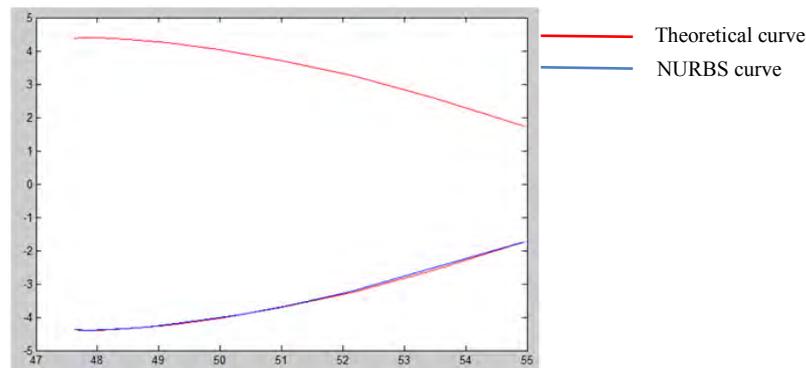
Figure 9. Theoretical and NURBS curves (values in "mm")

The Tables 2 and 3 presents a comparative between the theoretical and NURBS curve for a determined number of control points, generated using MATLAB.

Table 2. Comparative values of representation theoretical curve X NURBS curve – 5 points

Collected points	Theoretical curve		NURBS curve		Error [mm]
	x	y	x	y	
Point 1	47.727	-4.389	47.727	-4.3890	0.000
Point 2	48.488	-4.343	48.488	-4.3064	0.0366
Point 3	49.991	-4.031	49.991	-3.9598	0.0712
Point 4	52.180	-3.231	52.180	-3.1387	0.0923
Point 5	54.964	-1.736	54.964	-1.7360	0.000

Table 3. Comparative values of representation theoretical curve X NURBS curve – 6 points

Collected points	Theoretical curve		NURBS curve		Error [mm]
	x	y	x	y	
Point 1	47.632	-4.384	47.632	-4.3840	0.000
Point 2	47.727	-4.389	47.727	-4.3823	0.0067
Point 3	48.013	-4.385	48.013	-4.3377	0.0473
Point 4	52.180	-3.231	52.180	-3.3184	0.0874
Point 5	53.504	-2.583	53.504	-2.5383	0.0447
Point 6	54.964	-1.736	54.964	-1.7360	0.000

The next section will detail the results of the cases studied, experimentally, with different points collected.

4.2 EXPERIMENTAL RESULTS

To generate the evolvental profile, control points were collected in a spur gear with external diameter equal 25.4 mm. For this procedure, it was used a point-to-point sensor, known as touch trigger probe, in which possessed a ruby ball diameter 2.0 mm. Bosch (1995) highlights that the sphere located at the stylus tip, element that comes in contact with the object to be measured, is usually made of ruby, since this material is enough rigid and resistant. Figure 5 shows the gear used in the experimental procedure.

Measurement with CMM always presuppose in doing the following steps: system qualification probe, identifying the position of the workpiece relative to the machine coordinate system, measurement coordinate points on the surface of the part, and finally, the presentation of measurement results. All of them are required by the machine.

Nóbrega, L. H. M. S.; Silva, E. C. M.; Silva, V. N.
Application of NURBS for Obtaining Free Form Curves and Surface in Spur and Helical Gears by Using CMMs.

Figure 5. Gear used in the experimental procedure

Based in a methodology described in item 3.2, the input data to the computational ambient developed in MATLAB *software* by Zhang (2008) were the control points, which in this case correspond to the points obtained in the CMM. For this piece, three measurement cases were carried out with 5 and 6 points on the curve length evolvental.

For each measurement case, a point was selected to carry out the validation using NURBS.

Carrying out the experimental procedure described previously, it was utilized a CMM with the specifications TESA Micro-Hite 3D, of the Hexagon Metrology, as shown Fig. 6.

Figure 6. CMM TESA Micro-Hite 3D, Hexagon Metrology

For each case studied (7, 6 and 5 points collected), was generated a knot vector. In this case, was developed a computational ambient using MATLAB to generate the vector, as shown Fig. 7,

```
function vetor = calc_knots(numberPoints, grau)

  intervalo = qntintervalo(numberPoints, grau);

  vetor = zeros(1, 2*(grau+1) + intervalo);

  vetor(1, grau+1:grau+intervalo+2 ) = 0:1/(intervalo+1):1;
  vetor(1, size(vetor,2)-grau+1:size(vetor,2)) = ones(1, grau);
```

Figure 7. Knot vector routine

If the knot vector does not have any particular structure, the curve generated will not touch the first and the last segment of the control polygon, as shown Fig. 8 below. These types of curves are called open curves, as shown Fig. 8-(b). May want to achieve such that the curve is tangent the first and the last segment, to do this, the first and the last nodes must be repeated $p+1$ (i.e., the multiplicity $p+1$), this will generate the denominated curves semi-enclosed, see Fig. 8-(b) (Minetto, 2003).

(a)   (b)
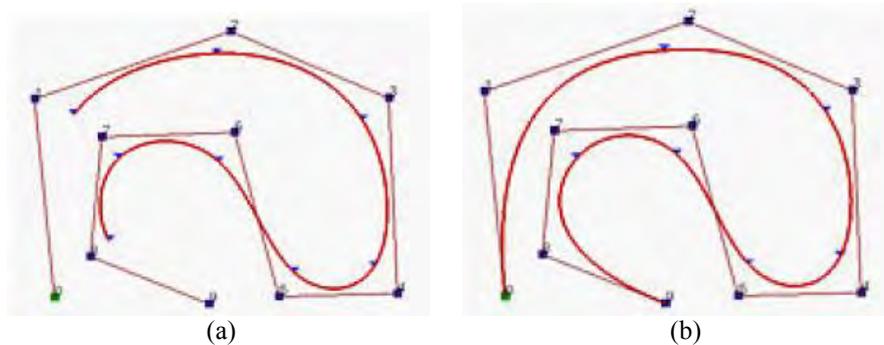
Figure 8. Types of curves: (a) open curves, and (b) semi-enclosed (Minetto, 2003)

The input data presented on routine are: the control points and the degree of NURBS function. The knot vectors for 7, 6 and 5 control points are shown respectively below:

7 points - $U = \{0\ 0\ 0\ 0\ 0.25\ 0.5\ 0.75\ 1\ 1\ 1\ 1\}$ (13)

6 points - $U = \{0\ 0\ 0\ 0\ 0.33\ 0.66\ 1\ 1\ 1\ 1\}$ (14)

5 points - $U = \{0\ 0\ 0\ 0\ 0.5\ 1\ 1\ 1\ 1\}$ (15)

a) Case 1: 5 control points

The Table 4 shows the values of the coordinates of the points probed by CMM that correspond the evolvental profile to the left side gear, to the case 1 with 5 points. In the Table are added the values of the weight function, required to input data to construct the profile evolvental using NURBS.

Table 4. Points collected for case 1

Collected points	$Z_{MEASURED}$ CMM [mm]	Weight used	Z_{NURBS} curve	Error [mm]
PT 1	12.154	1.00	12.154	0.000
PT 2	12.840	1.00	12.644	-0.195
PT 3	14.123	1.00	13.987	-0.135
PT 4	14.976	1.00	14.904	-0.071
PT 5	16.078	1.00	16.078	0.000

b) Case 2: 6 control points

The Table 5 shows the values of the coordinates of the points probed of the evolvental profile to the left side gear, to the case 2 with 6 points.

Table 5. Points collected for case 2

Collected Points	$Z_{MEASURED}$ (CMM) [mm]	Weight used	Z_{NURBS} curve	Error [mm]
PT 1	12.877	1.00	12.877	0.000
PT 2	13.377	1.00	13.343	-0.0331
PT 3	13.742	1.00	13.724	-0.0175
PT 4	14.170	1.00	14.150	-0.0192
PT 5	14.705	1.00	14.649	-0.0551
PT 6	15.947	1.00	15.947	0.000

Nóbrega, L. H. M. S.; Silva, E. C. M.; Silva, V. N.
Application of NURBS for Obtaining Free Form Curves and Surface in Spur and Helical Gears by Using CMMs.

The Figure 9 shows the envolvental teeth curve using the control points studied in all the cases. The values of the axis on figure are defined in "mm".

| (a) | (b) |
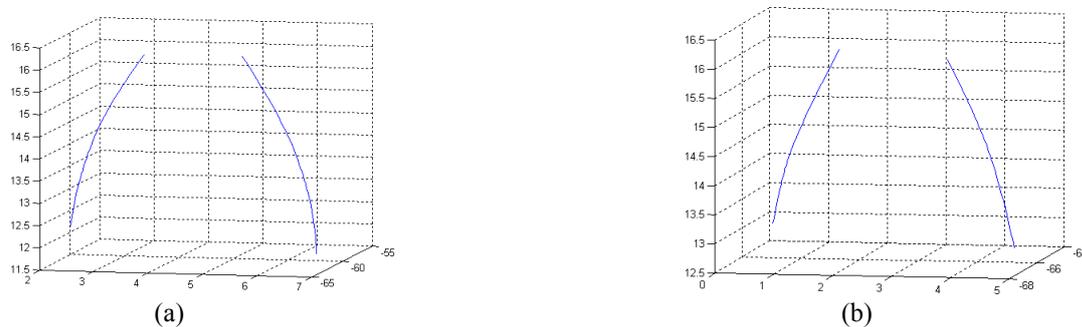
Figure 9. Evolvental teeth using: (a) 5 and (b) 6 control points

5. CONCLUSIONS

This article presents the definition for generation of curves and surfaces using Non Uniform Rational B-Splines (NURBS). To generate NURBS curves, was presented a measurement strategy for measuring the profile of a spur gear. In generating surfaces, was presented a methodology by Silva (2011). Before starting the experimental procedures, coordinates points was generated using Eq. (1) and (2). These equations were established from the primitive radius, r, and the evolvental angle, β. Initially, with 10 control points generated analytically, were drawn the theoretical and NURBS curves, to compare the values of the "y" axis. For the above case, the maximum error was found to be 21.9μm.

For developing a measurement strategy, the same procedure was carried out using 5 and 6 control points. The errors were: 92.3μm and 87.4μm, respectively. After analysis of the theoretical data, were carried out the experimental procedures, in the CMM using the methodology described. With the touch trigger probe were carried out the practical procedures e obtained the following values for error: 195μm and 55μm, for 5 and 6 coordinates points, respectively. In this case, the modeling accuracy of a part as will be greater when the numbers of points collected or measured on the surface are greater.

About the diameter of the probe ball, the choice of the diameter 2 mm of the sphere allowed a better captures of the points desired for the analysis. The results showed by the machine were compensated automatically, in other words, the coordinates of the touch point sensor-piece are obtained by correction of the sphere radius at tip of the probe.

The results were considered satisfactory for 6 points, where the errors obtained varies between 18 and 55 μm, resulting in errors less than 100μm, according ISO 10360. One way to minimize the error value would be changing the weight function. A method for automatically changing and defining best weight is being developed as part of this research. Further experimental work will be developed with helical gears, to generate surfaces.

Following this methodology, intended to meet the requirements of measurement of parts with the most varied forms, to meet the project specifications using a smaller amount of points inspected, contributing to the areas of inspection and reverse engineering, apart from research on dimensional metrology using Coordinate Measuring Machines.

6. ACKNOWLEDGEMENTS

7. REFERENCES

Bagci, E., 2009. *Reverse Engineering applications for recorver of broken or worn parts and re-manufacturing: three case studies*. Advances in Engineering Software, v. 40, p. 407-418.

Bosch, J. A., 1995. *Coordinate Measuring Machines and Systems*. Marcel Dekker, New York.

Dumoulin, C., Avellan, F. and Henry, P., 1994. *Geometry recovery of a francis runner prototype at site. In: IAHR Symposium*. Association of Hydraulic Engineerring and Research, v. 3, p. 1125-1136.

Ferreira, F., 2007. *Medição por Contato versus Medição Óptica em Máquinas de Medir por Coordenadas (3D)*. CATIM – Centro de Apoio Tecnológico à Industria Metalomecânica, Portugal.

Lima Júnior, J. C., 2007. *Aplicação de Redes Neurais Artificiais para determinar parâmetros de geometrias substitutas em MMCs*. Tese de doutorado. Programa de Pós-Graduação em Engenharia Mecânica, PPGEM. Universidade Federal da Paraíba, UFPB, João Pessoa.

Li, G. and Gu, P., 2004. *Automatic Localization and Comparison for Free-Form Surface Inspaction*. Journal of Manufacturing System. Vol.25.

Minneto, C. F., 2003. *Um Estudo sobre Curvas NURBS*. Dissertação de mestrado. Programa de Pós-Graduação em Matemática Aplicada.Universidade Federal do Rio Grande do Sul, UFRGS, Porto Alegre.

Nóbrega, L. H. M. S., 2011. *Projeto e Desenvolvimento de um Apalpador Touch Trigger de Baixo Custo para Medição por Coordenadas*. Dissertação de mestrado. Programa de Pós-Graduação em Engenharia Mecânica, PPGEM. Universidade Federal da Paraíba, UFPB, João Pessoa.

Piratelli-Filho, A., Araújo, J. A. and Brasil Júnior, A. C. P., 2009. *Reverse Engineering of Hidraulic Turbine Runners using Coordinate Measuring Arms and NURBS Modeling*. Revista Tecnologia, Fortaleza, v. 30, p. 114-122.

Piegl, L. and Tiller, W., 1997. *The NURBS Book*. 2nd ed. Berlin: Springer.

Shigley, J. E. and Budynas. N., 2008. *Mechanical Engineering Design*. McGraw-Hill.

Silva, E. C. M. and Silva, J. B. A., 2009. *Medição de Superfícies de Formas Livre: Um Estudo Comparativo com Uso de Medição com Contato e Sem Contato para Fins de Uso de Inspeção e Engenharia Reversa*. V Congresso Brasileiro de Metrologia, Metrologia para a Competitividade em Áreas Estratégicas. Salvador, Bahia.

Silva, E. C. M., Silva, J. B. A., Lima Júnior, J. C. and Andrezza, I. L. P., 2011. *Aplicação de NURBS para Modelagem de Curvas e Superfícies de Formas Livres Utilizando MMCs com Apalpadores do Tipo Touch Trigger*. II Congresso Internacional de Metrologia Mecânica, II CIMMEC.

Silva, E. C. M., 2011. *Aplicação de NURBS em MMCs, com Apalpador Touch Trigger, para Escaneamento de Superfícies de Formas Livres e Geometrias Complexas*. Tese de doutorado. Programa de Pós-Graduação em Engenharia Mecânica, PPGEM. Universidade Federal da Paraíba, UFPB, João Pessoa.

TC2000 – Profissionalizante. *Elementos de máquinas: Engrenagens*. vol. 1, aula 32.

Zhang, P., 2008. "NURBS Toolbox by D.M. Spink". 15 Feb. 2013 <http://www.mathworks.com/matlabcentral/fileexchange/authors/82397>.

8. RESPONSIBILITY NOTICE