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CLOSED B-SPLINE FOR RECONSTRUCTION OF THREE-DIMENSIONAL SURFACES

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Abstract. *In recent years, the techniques of scanning and reconstruction of three-dimensional objects of complex shapes have shown a great development. Depending on the object, thousands or even millions of samples must be acquired. The result consists of a "mass" of data that require efficient and reliable algorithms that can generate computational models from these samples. These data form, in reality, a cloud of points, not always organized and accurate reconstruction of surfaces from this cloud is a difficult problem. From the cloud of points, must rebuild the surface by selecting the type of curve that best fits the set of points read. The method divides the point of cloud to build sections of the object and, from these sections, build the three-dimensional surface. The representation of a curve as a succession of straight sections can be sufficient for many applications, and the B-spline very useful in modeling these surfaces. Although it is common to find in the literature about the subject, both in tracing curves and surfaces using B-splines, that there are several advantages of its use, but there are still many difficulties in its implementation, especially in reconstructing closed surfaces. The aim, therefore, of this paper is to develop efficiently a reconstruction of surface cross sections using B-spline closed.*

Keywords: *scanning, point of cloud, closed B-spline, reconstruction of three-dimensional objects*

1. INTRODUCTION

The digitalization and reconstruction techniques of objects having complex shape and/or free-form have been presented an important development. The velocity and precision of these techniques are due to the improvement in the areas of physics, electrical, engineering, the lasers' development, CCD cameras, and the high velocity acquisition cards. Such technologies have enabled profiles accurately measure the ratio of 1 per 1.000 and above rates of 20.000 samples per second. In the acquisition data process, thousands or millions of data must be acquired. These acquired data consist in a cloud of points in general, not organized, with gaps and noise, acquiring efficient and trustful algorithms capable to obtain computational models from those samples (Curless, 1997).

The precise surface reconstruction from unorganized point of cloud is a difficult problem, not solved completely and very troublesome in cases where the data are incomplete, with noisy and / or sparse. The goal of reconstruction is always to obtain a computational model of an object that resembles as closely as possible to the actual object (Díaz et al, 2007).

Despite major advances in this area, the correct modeling of closed surfaces or objects of complex shapes is not fully resolved and is still an area of many research activities (Remondino, 2003).

The representation of a curve as a series of straight sections may be sufficient for many applications. However, curves and complex surfaces usually require a more efficient representation. The application of B-splines and NURBS have been very helpful in shaping these surfaces.

The B-splines, the original functions of Bezier and splines, consist of curves segments whose polynomial coefficients do not depend on all the control points. Thus, to move a control point, the affected area of the curve is lower and the time of calculation of the coefficients is reduced. It is a fitting approximate the curve generated does not pass the control points. However, the control points can be calculated to realize interpolation or approximation at the desired points.

Scanning profiles of three-dimensional objects and reconstruction can be applied in several areas such as manufacturing (casting, stereolithography, etc..), reverse engineering (reconstruction of machine components that have no designs), the collaborative project (enabling interaction between the real model and the virtual model), inspection (allowing you to check if the object is as designed), virtual simulation (special effects in movies, games), in print of available pieces in museums, medicine (reconstruction of parts of the human body), in scientific exploration and in the consumer market.

There are several methods for surface reconstruction, among them there is the reconstruction by slices (cross sections) of the object, which will be treated in this article. A methodology will be presented for this reconstruction of cross sections using closed B-spline.

2. DEVELOPMENT METHOD

The goal of the reconstruction of a surface can be defined as: "given a set of points **P** assumed to define a surface **S**, to create a model surface **S** **'**closest to **S"**. The surface reconstruction process cannot confirm that it will be exactly equal to **S**, as it is known a finite number of points. An increase in the number of points does not certificate the accuracy of the surface (Remondino, 2003).

The surface reconstruction of an object by scanning has been widely used nowadays. However, when it comes to a closed surface and complex, there are still many difficulties in implementation by B-spline curve.

This proposal will be continued to the method of reconstruction from data acquired by a laser sensor unifilar (distance sensor). In this case the sensor gives the coordinates of points on the surface from the parameters of the structure where the scanner is mounted. This scanner consists of a robotic structure cooperative-type PR + RRP. Although the laser distance sensor, gives the distance between him and the object actually your processor handles data acquired in a statistical way to define this distance. When data from the surface are read what you get is a cloud of points as shown in Fig. 1, whose number of points depends on their constructive characteristics.

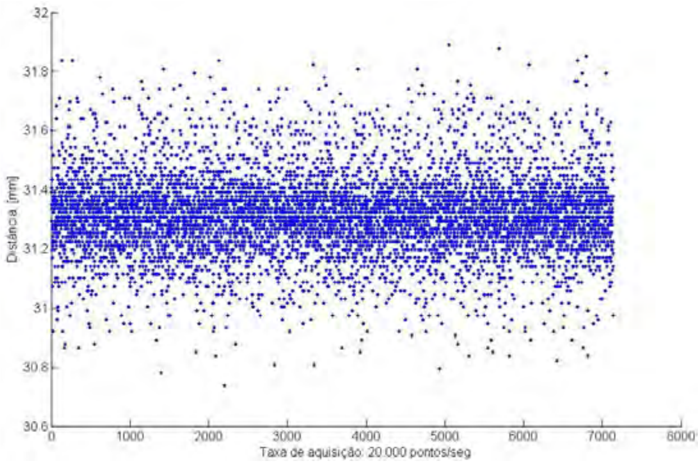

Figure 1. Example point cloud acquired by laser sensor for a fixed distance

Obtaining a B-spline curve or NURBS from this cloud of points is computationally unfeasible for processing time. To facilitate the attainment of the curve, a methodology was developed that allows, from the point cloud "read" by the sensor, making the reduction of points and order them. In the literacture [Aquino et al, 2009; Fonseca e Carvalho, 2005] can be consulted, for details of this methodology. A major advantage of this method is that the noise data acquisition does not affect the result of the curve as in other processes. Another important aspect is that the data do not need to be acquired in an orderly and some voids do not require being "filled" manually (with data created by the operator). This allows automate the process of surface reconstruction.

The method divides the point of cloud to build sections of the object and, from these sections, build the three-dimensional surface.

With ordered points can then apply the B-spline closed in order to reconstruct the cross sections of the surface. A B-spline is applied via the curve fit approximation by using the method of least squares. As it is approximation, the curve does not necessarily pass through the points previously defined, but to approximate this curves to these points is possible to develop methods imposing weights to each predefined point.

2.1 B-spline

Authors like (Piegl; Tiller, 1997), (De Boor, 2000) and (Rogers, 2001) define the B-spline as a spline version that implements local control curve, so that changing a control point modifies the curve only in the region nearest neighbors of the points in the order in continuity. The continuity of B-splines is the same as natural cubic splines, but not interpolates their control points.

The B-spline curve of p- degree is defined by:

$$C(u) = \sum_{i=0}^{n} N_{i,p}(u) P_i \quad (1)$$

Where:

- P_i are the control points (n+1 points);
- $N_{i,p}(u)$ are the basis functions of p-degree defined, in a knot vector, by:

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \le u < u_{i+1} \\ 0 & \text{otherwise.} \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u) \tag{2}$$

The knots are represented by a list of numbers that is commonly called a knot vector.

$$U = \{\underbrace{a, \dots, a}_{p+1}, u_{p+1}, \dots, u_{m-p-1}, \underbrace{b, \dots, b}_{p+1}\}$$

The knot vector must be a string of numbers equal or increasing (uniform or non-uniform) and the repetition of each knot does not exceed the degree of the surface in the respective direction. The number of times that the knot is duplicated is called multiplicity, and when the knot does not repeat called single knot. Furthermore, when the number of repetitions of a knot is equal to the degree of the surface in the respective direction has a knot maximum multiplicity. When the knot vector starts and ends with knot maximum multiplicity, intermediate knot are simple and equally spaced has become a uniform knot vector (exemple: 0, 0, 0, 1, 2, 3, 4, 4, 4). Otherwise, the knot vector is called non-uniform (exemple: 0, 0, 0, 1, 4, 7, 7, 7).

The degree p, the number of control points *(n +1)*, and the knnot number *(m +1)* are related as follows:

$$m = p + n + 1 \tag{3}$$

More details of B-spline curves can be found in Piegl and Tiller (1996).

2.2 Least squares curve approximation

When defining a B-spline it does not pass the control points. However, if it is required that the curve passing through points at or near predetermined points (Q_k), it takes to calculate a set of control points and knot a new vector from the data points to be able to trace the curve.

Assume that $p \ge 1$, $n \ge p$, and $Q_0, \dots, Q_m$ $(m > n)$ are given, and then seek a pth degree curve, defined by Eq. (1), where Q_k are approximated by the method of least squares, ie:

$$\sum_{k=1}^{m-1} |Q_k - C(\bar{u}_k)|^2 \tag{4}$$

is a minimum with respect to the $n+1$ variables, P_i; the $\{\bar{u}_k\}$ are the precomputed parameter values. It is emphasized that the resultant curve typically does not pass precisely through Q_k.

The control points are calculated by Eq. (5):

$$(\boldsymbol{N^T W N})\boldsymbol{P} = \boldsymbol{R} \tag{5}$$

where N is the *(m+1) x (n+1)* matrix of scalars

$$N = \begin{bmatrix} N_{0,p}(\bar{u}_0) & \cdots & N_{n,p}(\bar{u}_0) \\ \vdots & \ddots & \vdots \\ N_{0,p}(\bar{u}_m) & \cdots & N_{n,p}(\bar{u}_m) \end{bmatrix} \tag{6}$$

W is the *(m+1) x (m+1)* matrix

$$W = \begin{bmatrix} w_0 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & w_n \end{bmatrix} \tag{7}$$

R is the vector of $n+1$ points

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$$R = \begin{bmatrix} N_{0,p}(\bar{u}_0)WQ_0 + & \dots & +N_{0,p}(\bar{u}_m)WQ_m \\ \vdots & & \vdots \\ N_{n,p}(\bar{u}_0)WQ_0 + & \dots & +N_{n,p}(\bar{u}_m)WQ_m \end{bmatrix} \quad (8)$$

In order to set up Eqs. (6) e (8), a knot vector $U = \{u_0, \dots, u_r\}$ and parameters $\{\bar{u}_k\}$ are required.
The $\{\bar{u}_k\}$ can be computed using the technique chord length. Let d the total chord length, given by:

$$d = \sum_{k=1}^{n} |Q_k - Q_{k-1}| \quad (9)$$

Then,

$$\bar{u}_0 = 0 \qquad \bar{u}_n = 1$$

$$\bar{u}_k = \bar{u}_{k-1} + \frac{|Q_k - Q_{k-1}|}{d} \qquad k = 1, \dots, n-1 \quad (10)$$

The knot vector can be obtained by the following method:
Let $\bar{d}$ a positive real number, denoted by $i = int(\bar{d})$ the largest integer such that $i \le \bar{d}$. Let:

$$\bar{d} = \frac{m+1}{n-p+1} \quad (11)$$

Then it defines the knots internal by:

$$i = int(j\bar{d}) \qquad \alpha = j\bar{d} - i$$

$$u_{p+j} = (1-\alpha)\bar{u}_{i-1} + \alpha\bar{u}_i \qquad j = 1, \dots, n-p \quad (12)$$

2.3 Obtaining the number of control points

The number of control points *(n +1)* is a pre-defined data to the use of the method of least squares.
Having as base plane cross section, with the points representing the contour of the object, are caught three consecutive points belonging to this plan, as outlined in Fig 2.

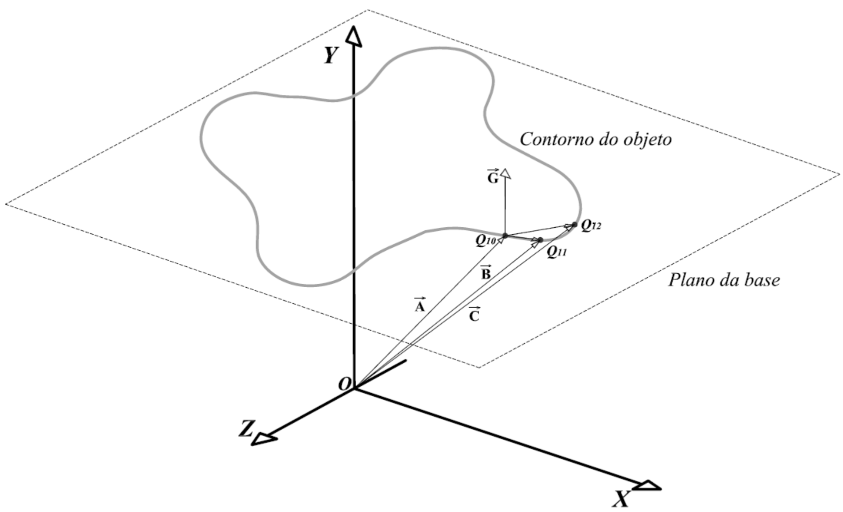

Figure 2. Base plane and the three initial points (Díaz, 2011)

It made an algorithm that runs along the contour and define the three points the normal vector, so has defined the normal vector of each three points in the whole section. The change of the signal vector is used as a measure showing how the curvature of the contour varies.
Thus, the number of points that represent the change in curvature represents the number of control points. The Fig. 3 illustrates these points:

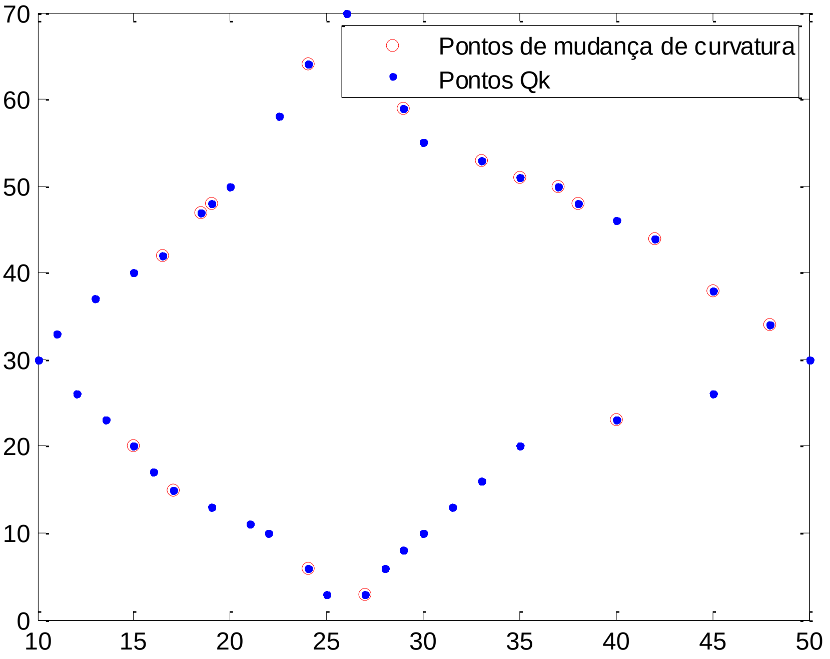

Figure 3. Points representing the change of curvature of the edge

2.4 Closed B-spline

The control points found using the method of section 2.2, are validated for an open curve, but for a closed B-spline curve the knot vector and the control points found must be adapted.

For the control points, should be added to the points already defined *p+1* points equal to *p+1* points first. Let *p*, the degree of curve.

With the new amount of control points, the knots number of knot vector is defined by Eq. (3). In this case the first and last p +1 nodes, the knot vector, should not be the same as defined in section 2.1. For a closed curve knot vector must be open (no repeat) and this is again calculated using the equations (11) and (12). Finally, this new knot vector, the last p +3 intervals between the knots must be equal to the p +3 first, always respecting the relation in Eq (3).

Figures 4 and 5 illustrate the method used for the open curve and adaptation, the knot vector and control points for the closed curve, respectively.

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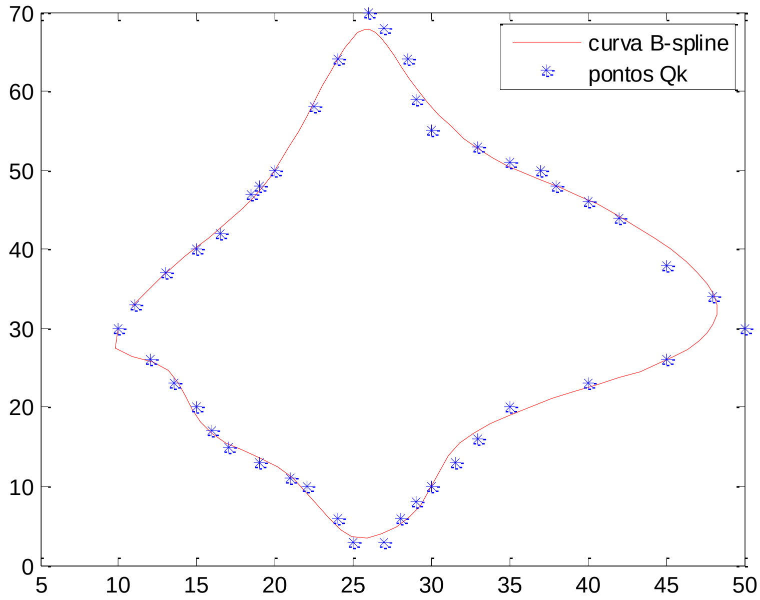

Figure 4. Least square approximation for an open B-spline

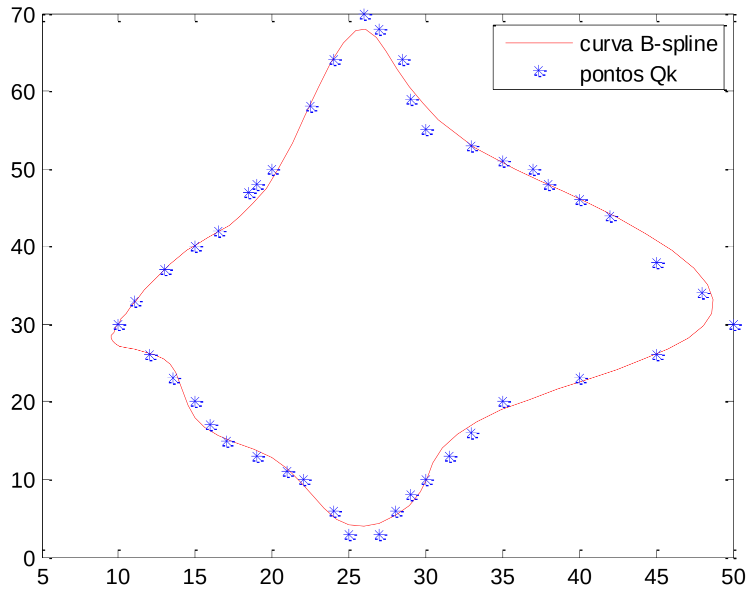

Figure 5. Least square approximation for a closed B-spline

It appears that the repetition of knots and control points for defining the closed B-spline curve, is effectively and continuously, reproducing a cross closed section of a surface. From these curves obtained, methods can be developed based settings specific features of the curve, such as radius or diameter of the curve, derived at specific points, weights and errors between the curve and the real profile of the object. These adjustments are made so that the curve becomes a model as close to the real. With the curves of the cross sections obtained and properly adjusted, it will be held the connection between points that are in different planes, producing a bidirectional mesh to define the object's surface.

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