ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

A COMPUTATIONAL TECHNIQUE APPLIED TO COOLING SYSTEMS MODELING IDENTIFICATION AND REDUCTION

Jose Maria Galvez
Department of Mechanical Engineering
Federal University of Minas Gerais
Av. Antonio Carlos 6627, Pampulha,
Email: jmgalvez@ufmg.br

Abstract. *Control of dynamic systems is a model-accuracy dependent task. This is the case of cooling systems control in which the model exactness plays a fundamental role in the control loop performance. As in any other thermal process, exact models are formulated as distributed parameter systems (DPS) that have dynamic realizations on infinity-dimension spaces and there are no means to formally ensure that a finite-dimension controller can produce closed loop stability and good performance. The most widely used methods for modeling of thermal processes are finite element based methods, however they usually produce models that do not accurate represent the actual system and may cause poor control performance. Also, cooling systems are multi-input, multi-output, cross-coupled, time-varying, nonlinear processes whose inputs (manipulated variables) and outputs (controlled variables) present saturation and rate constrains; additionally, the non co-collocated temperature sensors produce time-delays that must be included in the control loop for design purposes. In any case, the controller design must be performed based on some reduced order model of the plant. Up to the present there is no clean design procedure that formally ensures closed loop stability of ROM based controllers, thus, it remains a challenging research field. This paper considers a computational procedure for model identification, model reduction and control design based on the Eigensystem Realization Algorithm (ERA). An extension of the ERA technique is proposed and applied to a cooling system experimental model used as a benchmark. Numerical results are presented and discussed.*

Keywords: *Cooling Systems, Model Identification and Reduction, Eigensystem Realization Algorithm.*

1. INTRODUCTION

Power consumption efficiency of industrial devices is one of the main issues for the incoming centuries. Since the beginning of the industrial age, the world population has virtually exploded, nature has been almost devastated and energy resources have been depleted. In spite of that, the human living comfort has become essential for the world population and because of that energy per-capita consumption should continuously increase in the future. It is a fact that the next decades are going to testify a continuous and strenuous search for new devices and technologies to save energy resources. The energy consumption by heating and cooling systems in commercial and industrial buildings corresponds to 50% of the world energy consumption (Imbabi, 1990). Heating and cooling systems are high-energy consumption processes (Arguello-Serrano, 1999) and their operation in commercial and industrial buildings still are inefficient.

It is already known that the solution for efficient operation of heating and cooling systems relies on the proper choice and design of automatic control system. Low cost conventional single-input single-output (SISO) controllers such as On/Off control and PID control are currently used as the standard controllers in the heating, ventilation and air conditioning (HVAC) industry. However, their low energy efficiency causes an extra-undesired energy burning. Furthermore, due to the I/O cross coupling, conventional SISO control is not capable of controlling the freezing power and super-heating independently. Multi-input multi-output (MIMO) control strategies to deal with the control problem of time varying processes, time delays and I/O cross coupling must be further explored by the control community. Particularly, to face I/O cross coupling, MIMO control strategies such as state feedback can be used but this requires exact models of the plant.

This paper introduces a MIMO modeling procedure based on the step response of the cooling machines. Figure 1 shows the schematic diagram of a cooling machine based on the vapor compression cycle used in this work. The modeling procedure is based on the Eigensystem Realization Algorithm (ERA). The ERA procedure uses the plant impulse response as the data source and delivers a state realization model of the plant. In this paper the results are extended to model identification from the step response of the system. Numerical results are presented to illustrate the modeling procedure. A computational model of a cooling system is used to generate clean and noisy experimental data. The modeling results are then compared with the ones from the original data and the results are analyzed. Finally, it is shown that the technique delivers accurate results from the plant step response even in the presence of noisy data.

2. THE EIGENSYSTEM REALIZATION ALGORITHM – A SHORT REVIEW

Control of dynamic systems is usually a model-accuracy dependent task. This is the case of cooling and heating systems (distributed parameter systems) that have dynamic realizations on infinity-dimension spaces and there are no means to ensure that a finite-dimension controller can produce closed loop stability. In order to assure good control performance, the model must include as much plant dynamics as possible. Models for control design purposes are developed either from: physical laws, from finite element methods (FEM's) or from experimental data. Due to the usual complexity of these processes, the direct use of physical laws becomes a strenuous task. Thus, the most widely used modeling methods for heat involving processes are the finite element method (FEM) and model identification from experimental data. The FEM methods lead to good simulation models that are not fitted to control analysis purposes. Model identification from experimental data is usually affected by measurement noise that makes difficult to distinguish information from noise. In any case, the design of the control system must be performed based on some reduced order model (ROM) of the plant. On the other hand, the placement of the sensors and actuators might negatively affect the observability and controllability properties of the system. Residual dynamics (dynamics not included in the model) causes undesired performance deterioration and even instability of the control loop. Up to the present there is no clean design procedure that ensures closed loop stability of ROM based designed controllers, so the ROM based control design problem remains a challenging research field.

Several interesting algorithms for model reduction and identification have been proposed in the last decades, however, there is need for further experimentation to surely conclude for the best algorithm. On the other hand, some very powerful algorithms, due to their relatively high computational requirements, have been also overlooked. With the current development of computational resources, those modeling procedures should be addressed in more detail. This paper tries to partially fulfill this lack of experimentation and with this goal in mind it presents an application of the Eigensystem Realization Algorithm originally proposed by Juang and Pappa (1985,1986) to the modeling problem of cooling systems. This section shows just the very fundamental ideas of the Eigensystem Realization Algorithm (ERA) as proposed in Juang and Pappa (1985). Besides that, Juang and Pappa (1986) also proposed two quantitative criteria to eliminate frequency components created by measurement noise. In the following, less important derivation steps and some results have also been omitted in order to save printed space.

Consider a state space realization for a linear time-invariant discrete-time dynamic system given by

$$\begin{aligned} x(k+1) &= A(\theta)x(k) + B(\theta)u(k) \\ y(k) &= C(\theta)x(k) + v(k) \end{aligned} \tag{1}$$

where [***A, B, C***] defines a discrete-time state space realization, ***x*** is a n-dimensional state vector, ***u*** an m-dimensional control input, ***y*** a p-dimensional measurement vector and ***v*** represents measurement noise.

the system impulse response sequence is given by

$$h(k) = \{\ h(0) \quad h(1) \quad h(2) \quad h(3) \quad . \ . \ . \ . \ . \ . \ \} \tag{2}$$

in compact form the impulse response can be written as

$$h(k+1) = y(k+1) = CA^k B \tag{3}$$

a Hankel matrix can be constructed from the impulse response sequence as

$$H(k) = \begin{bmatrix} h(k+1) & h(k+2) & h(k+3) & . & . \\ h(k+2) & h(k+3) & h(k+4) & . & . \\ h(k+3) & h(k+4) & h(k+5) & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix} = \begin{bmatrix} CA^kB & CA^{k+1}B & CA^{k+2}B & . & . \\ CA^{k+1}B & CA^{k+2}B & CA^{k+3}B & . & . \\ CA^{k+2}B & CA^{k+3}B & CA^{k+4}B & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix} \tag{4}$$

also

$$H(0) = \begin{bmatrix} CA^0B & CA^1B & CA^2B & . & . \\ CA^1B & CA^2B & CA^3B & . & . \\ CA^2B & CA^3B & CA^4B & . & . \\ & & & . & . \\ & & & . & . \end{bmatrix} = \begin{bmatrix} \overline{C} & \overline{B} \end{bmatrix} \tag{5}$$

where $\overline{C}$ and $\overline{B}$ are de system observability and controllability matrices, respectively.

ISSN 2176-5480

$$\overline{C} = \begin{bmatrix} C \\ CA \\ CA^2 \\ . \\ . \\ CA^{n-1} \end{bmatrix} \quad \text{and} \quad \overline{B} = \begin{bmatrix} B & AB & A^2B & . & . & A^{n-1}B \end{bmatrix} \tag{6}$$

where n is the order of the system, m is the number of system outputs and p is the number of system inputs

From the singular value decomposition (SVD)

$$H(0) = M\Sigma N^T \tag{7}$$

$$H(0) = M\left[\begin{array}{c|c} D & 0 \\ \hline 0 & 0 \end{array}\right]N^T \tag{8}$$

$$H(0) = M\left[\begin{array}{c} In \\ \hline 0 \end{array}\right]D\left[In \mid 0\right]N^T \tag{9}$$

then

$$H(0) = PDQ^T = \overline{C}\,\overline{B} \tag{10}$$

and

$$H^+ = N\Sigma^+M^T = QD^{-1}P^T \tag{11}$$

where M, N, P and Q are orthogonal matrices; D is a diagonal matrix and In is a nxn identity matrix; usually, $H(0)$ is not square and $dim(H(0)) = np \times nm$ with $\text{rank}(H(0)) \le n$.

We know that, there exist constant matrices *Ep* and *Em* such that

$$h(k+1) = Ep^T H(k) Em \tag{12}$$

and that

$$H(k) = \overline{C} A^k \overline{B} \tag{13}$$

and also that

$$\overline{C}\,\overline{B} = PDQ^T = H(0) \tag{14}$$

then

$$h(k+1) = Ep^T H(k) Em = Ep^T \overline{C} A^k \overline{B} Em \tag{15}$$

$$h(k+1) = \left[Ep^T\right]\left[PD\right]\left[D^{-1}P^T\right]\left[H(k)\right]\left[QD^{-1}\right]\left[DQ^T\right]\left[Em\right] \tag{16}$$

hence

$$h(k+1) = \left[Ep^T PD^{1/2}\right]\left[D^{-1/2}P^T H(1) QD^{-1/2}\right]^k\left[D^{1/2}Q^T Em\right] \tag{17}$$

finally, a minimal order realization is given by

$$C = \left[E_p{}^T P D^{1/2} \right]$$

$$A = \left[D^{-1/2} P^T H(1) Q D^{-1/2} \right] \quad (18)$$

$$B = \left[D^{1/2} Q^T E_m \right]$$

3. THE COOLING SYSTEM

This paper is concerned with the modeling of a system constituted by an expansion valve, an evaporator and a compressor as shown in Fig.1.

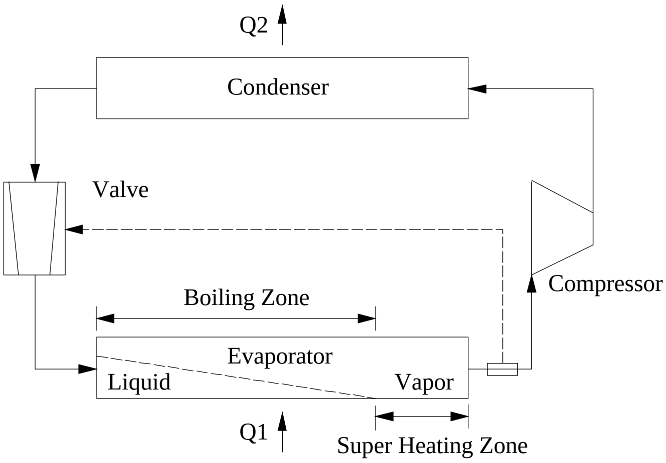

Figure 1. The Cooling System.

The system inputs are the expansion valve opening position, which defines the mass flow rate (MFR) and the compressor speed, which controls the volume flow rate (VFR). The system outputs are the super heating, ΔT, and the freezing power, Q_1, (Fig.2).

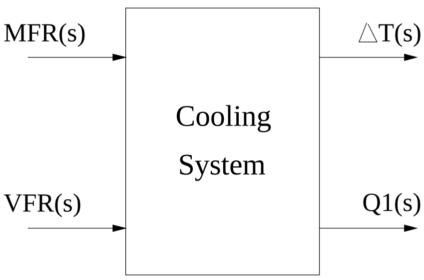

Figure 2. The Open Loop System.

In this case, the system dynamics is defined by a matrix transfer function of the form:

$$\begin{bmatrix} \Delta T(s) \\ Q_1(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} MFR(s) \\ VFR(s) \end{bmatrix} \leftrightarrow \quad [Y(s)] = [G(s)]\,[U(s)] \quad (19)$$

Ideally, the expansion valve would be used to regulate the super heating and the variable-speed compressor to control the generation of freezing power (Fig.3 with $G_{12}(s) = G_{21}(s) = 0$). Unfortunately, this is not the case. Actually, each of the outputs is a function of both inputs (the valve opening position and the compressor velocity) as shown in Fig.3. In practice, $G_{12}(s)$ and $G_{21}(s)$ can't be neglected. A strong cross-coupling interaction among inputs and outputs characterizes this type of system (Fig. 3).

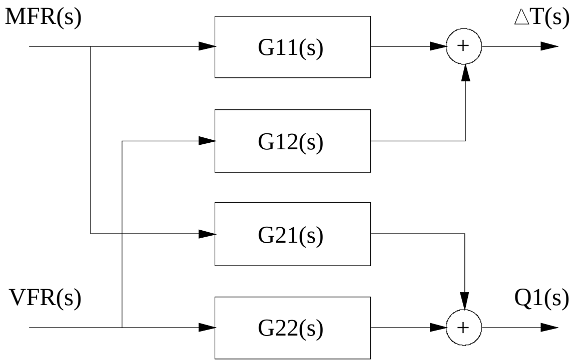

Figure 3. The Cooling System Cross Coupling.

Several linear and non-linear computational models for cooling systems can be found in the technical literature (Koury, 1998; Machado, 1996; Rocha, 1995; Outtagarts, 1994). A model based on the one proposed by Machado (1996) is used in this work as a benchmark for the proposed modeling procedure:

$$[G(s)] = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{-5.6}{(45s+1)} & \dfrac{2.5}{(60s+1)} \\ \dfrac{34(-36s+1)}{(26s+1)(68s+1)} & \dfrac{22(630s+1)}{(80s+1)(90s+1)} \end{bmatrix} \tag{20}$$

It should be notice that the plant is non-minimal phase. A discrete-time state-space realization can be found as:

$$\left[\begin{array}{c|c} Ad & Bd \\ \hline Cd & Dd \end{array}\right] = \left[\begin{array}{cccccc|cc} 0.9852 & -0.0213 & 0.0000 & 1.7255 & 0.0000 & -0.9307 & 0.4733 & 0.5182 \\ 0.0004 & 0.9951 & 0.0000 & 0.9112 & 0.0000 & 1.7366 & 0.5157 & -0.4813 \\ -0.0000 & 0.0000 & 0.9829 & 0.0000 & 0.0016 & 0.0000 & 0.3183 & 0.9480 \\ 0.0000 & 0.0001 & -0.0000 & 0.9721 & 0.0000 & 0.0035 & 0.6752 & 0.2344 \\ -0.0000 & 0.0000 & 0.0016 & 0.0000 & 0.9786 & 0.0000 & -0.9480 & 0.3183 \\ 0.0000 & -0.0000 & 0.0000 & -0.0180 & 0.0000 & 0.9718 & 0.2329 & -0.6670 \\ \hline -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.1298 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0000 & -2.8543 & 0.0000 & 0.0000 \end{array}\right] \tag{21}$$

Also notice that its discrete-time state space realization leads to ill-conditioned observability and controllability matrices as shown in Table 1:

Table 1. Conditioning Numbers

Conditioning Number	Controllability Matrix	Observability Matrix
Continuous Time Realization	8.6478×10^{4}	5.4764×10^{8}
Discrete Time Realization	2.9896×10^{5}	1.9499×10^{9}

4. SIMULATION RESULTS

The ERA algorithm, as originally proposed by Juang and Pappa (1985), requires the plant impulse response as its source of information data. However, impulse response is a theoretical definition used for analysis and can’t be generated in practice. Thereby, the plant step responses were differentiated with respect to time and obtained, in this way, the required impulse responses to be used with the ERA procedure. It is well known the differentiating a noisy signal amplifies noise. Fortunately, Juang and Pappa (1986) also proposed two quantitative criteria to eliminate frequency components created by measurement noise such that the algorithm deals fairly well with experimental data corrupted by noise as it has been shown in Juang and Pappa (1986).

ISSN 2176-5480

To assess the performance of the proposed modeling procedure an ill-conditioned (Tab. 1) non-minimal phase 2x2 MIMO plant was chosen as a benchmark. A MIMO step response test was applied to the system defined by Eq. (20) and/or Eq. (21) that was assumed to be the exact model of the plant. In this case, the sampling frequency was chosen to be 2π rd/sec (T_s = 1 sec). Through differentiating the plant step response the impulse response was generated and 10% of white noise was added to the signal. The resulting signal is shown in Fig. 4.

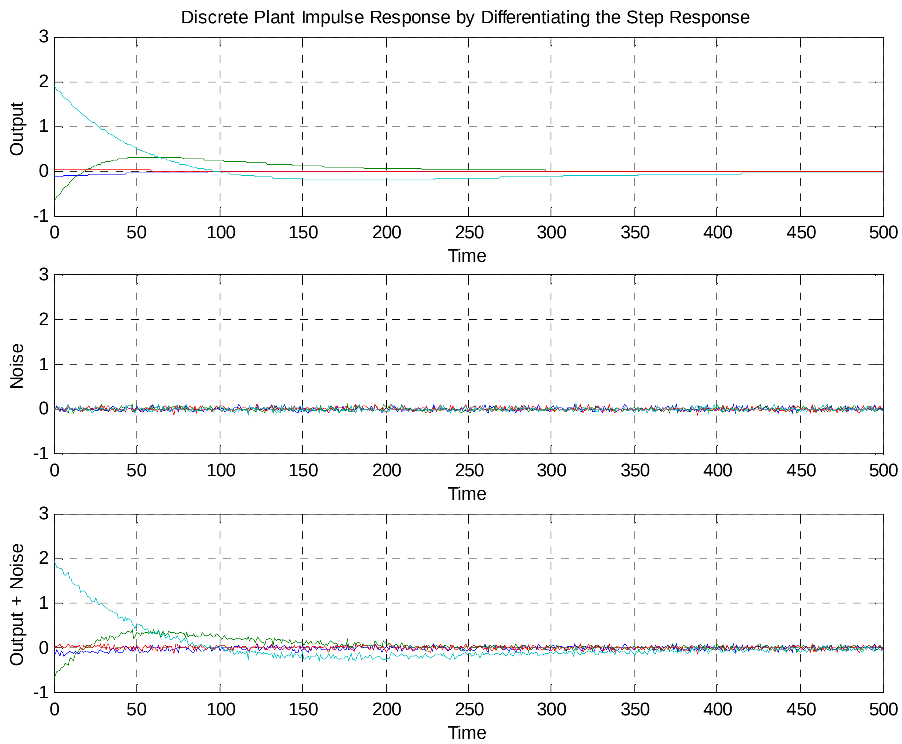

Figure 4. Source Data for the Modeling Procedure.

A quite large 1000x1000 Hankel matrix was built from the plant impulse responses obtained by differentiation of the plant step responses. Next, the singular values of the Hankel matrix are computed and used as a quantitative parameter to define the model order. The procedure continues with the factorization of the Hankel matrix, as shown in Section 2, to finally reach a state space realization given by Eq. (18). To assess the modeling procedure performance, two sets of data were generated. The first one is the noise-free impulse response of the system and the second one is the impulse response with 10% white noise added.

Figures 5, 6, 7 and 8 show the simulation results from noise-free experimental data.

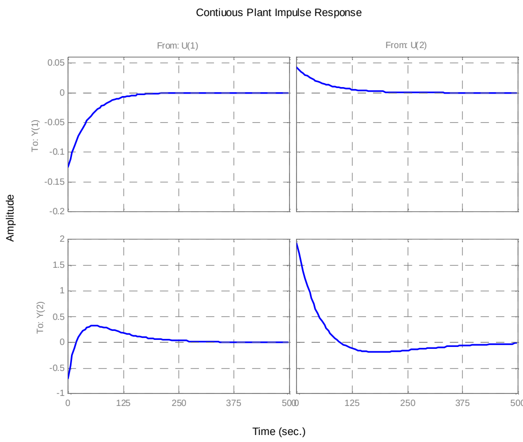

Figure 5a. Continuous Plant Impulse Responses.

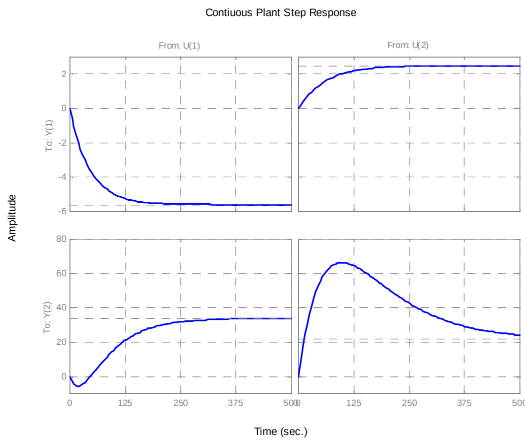

Figure 5b. Continuous Plant Step Responses.

ISSN 2176-5480

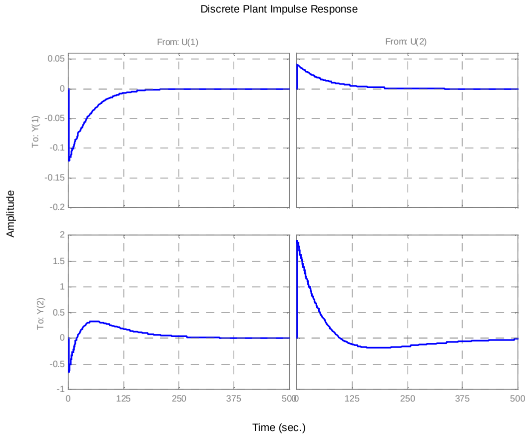

Figure 6a. Discrete Plant Impulse Responses.

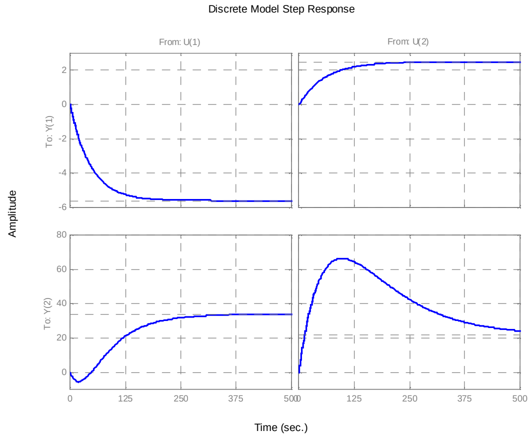

Figure 6b. Discrete Plant Step Responses.

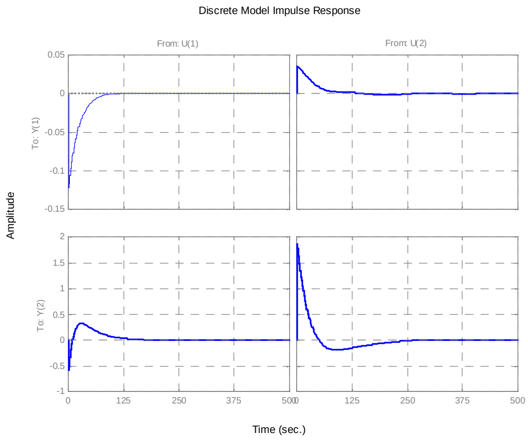

Figure 7a. Discrete Model Impulse Responses
(from noise-free data).

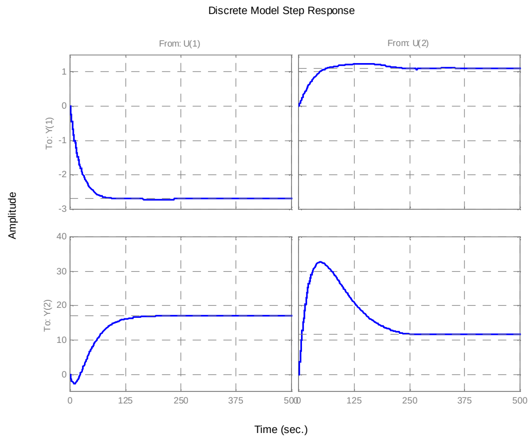

Figure 7b. Discrete Model Step Responses
(from noise-free data).

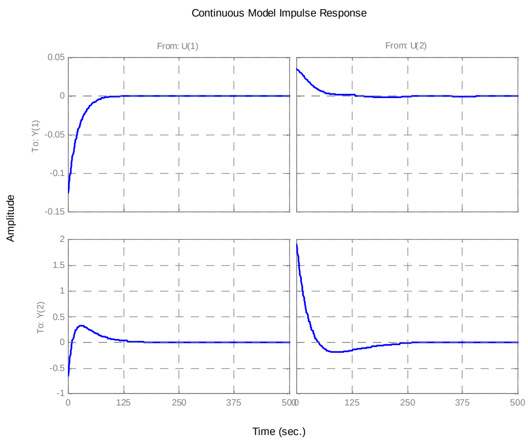

Figure 8a. Continuous Model Impulse Responses
(from noise-free data).

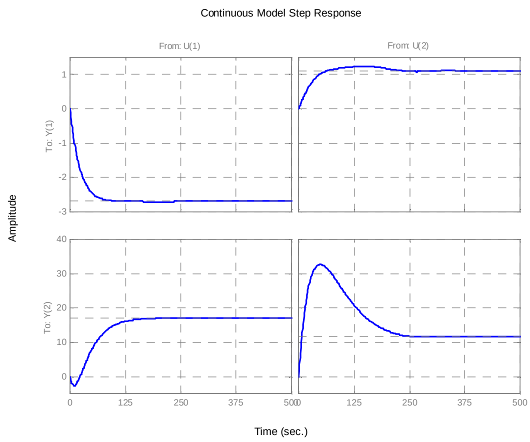

Figure 8b. Continuous Model Step Responses
(from noise-free data).

Figures 9 and 10 show the first 10 singular values (from a total of 998 computed singular values) for noise-free data and noisy data, respectively.

ISSN 2176-5480

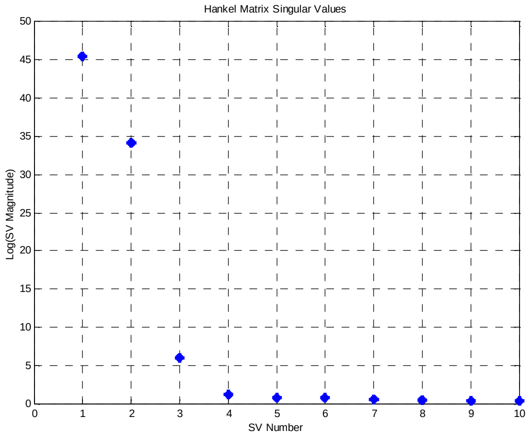

Figure 9. Discrete Model Impulse Responses (from noise-free data).

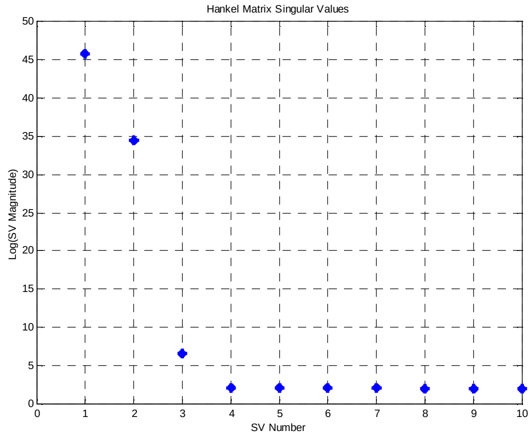

Figure 10. Discrete Model Impulse Responses (from noisy data).

And finally, Figs 11 and 12 show the simulation results from noisy experimental data.

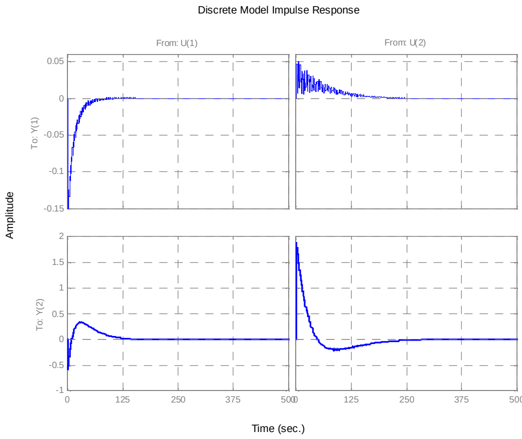

Figure 11a. Discrete Model Impulse Responses (from noisy data).

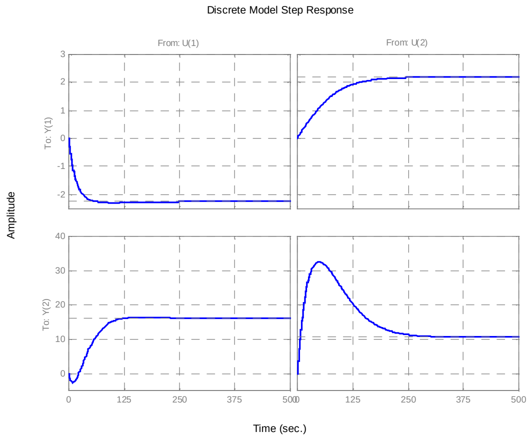

Figure 11b. Discrete Model Step Responses (from noisy data).

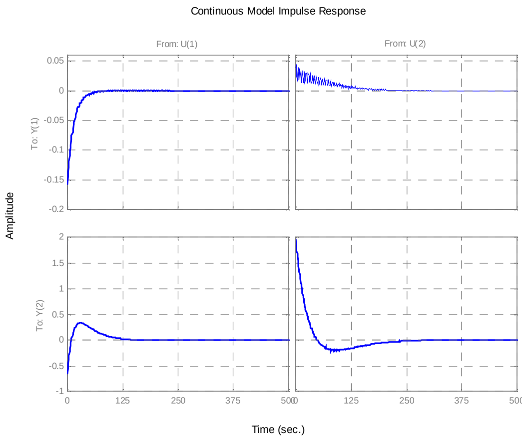

Figure 12a. Continuous Model Impulse Responses (from noisy data).

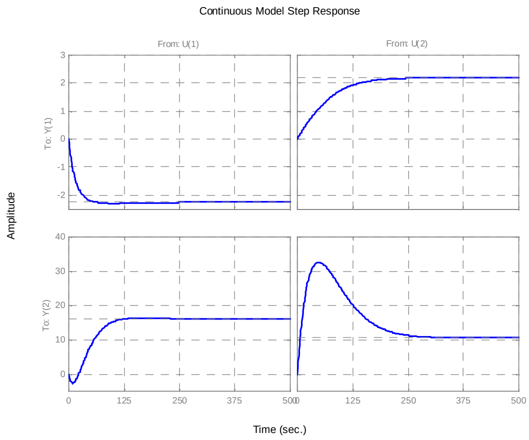

Figure 12b. Continuous Model Step Responses (from noisy data).

5. FINAL COMMENTS

The proposed modeling procedure based on the ERA algorithm has shown outstanding performance in the case of noise-free data and has a truly acceptable performance in the presence of noise.

Have been observed that high frequency model components are difficult to identify with accuracy. At high frequency, even with high resolution A/D converters, the contribution of high frequency components are hidden by numerical round off and truncation (even in the hypothetical case of a noise-free environment). Also, the signal-noise relation of the experimental data at high frequency becomes too low for experimental purposes.

For accuracy of model identification, the noise level plays an important role at high frequency. However, under regular noise presence, the sampling frequency appears to be more restrictive than the presence of noise in experimental data. This is because widely spread natural frequencies require a very fast sampling rate producing huge Hankel matrices. The difficulty of getting numerical accuracy with large dense matrices is well known by the technical community and there is no need for further comments.

Finally, model validation is a main difficulty in model identification from experimental data that is frequently overlooked by inattentive experimentalists. Time sequences are usually exactly reproduced by the identification technique but the resulting model is not the one that produced the data. This can be easily verified in simulation but it can be overlooked in experiments since the plant model is actually unknown. The reason for that is the improper choice of sampling rates and the aliasing characteristic of sampled signals.

In any case and based on this author’s experience the ERA algorithm is surely one of the most interesting tools for model identification and reduction of systems such as cooling systems.

6. ACKNOWLEDGEMENTS

The author thanks FAPEMIG for its financial support of this work.

7. REFERENCES

Arguello-Serrano, B. & Vélez-Reyes, A., 1999, “Nonlinear Control of a Heating, Ventilating, and Air Conditioning System with Thermal Load Estimation”, IEEE Transactions on Control Systems Technology, vol. 7, pp. 56-63.

Bristol, E.H., 1966, “On a New Measure of Interaction for Multivariable Process Control”, IEEE Transactions on Automatic Control, Vol. 11, pp.133-134.

Doyle & G. Stein, 1981, "Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis", IEEE Transactions on Automatic Control, Vol. 26, pp. 4-16.

Doyle, J.C., 1982, Analysis of Feedback Systems with Structured Uncertainties, Proceedings of the IEE, Part D, vol 129, pp 242-250.

Grosdidier, P. and Morari, M., 1986, “Interactions Measurements for Systems under Decentralized Control”, Automatica, vol. 22, pp. 309-319.

Ho, W.K. and Xu, W., 1998, “Multivariable PID Controller Design Based on the Direct Nyquist Array Method”, Proceedings of the American Control Conference, Philadelphia, Pennsylvania, pp. 3524-3528.

Imbabi, M.S., 1990, “Computer Validation of Scale Model Test for Building Energy Simulation”, Int. J. Energy Res., vol. 14, pp. 727-736.

Juang, J.N. and Pappa, R.S., 1985, “An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction”, Journal of Guidance, Control and Dynamics, Vol. 8, No. 5, pp. 620-627.

Juang, J.N. and Pappa, R.S., 1986, “Effects of Noise on Modal Parameters Identified by the Eigensystem Realization Algorithm”, Journal of Guidance and Control, Vol. 9, No. 3, pp. 294-303.

Juang, J.N., Cooper, J.E. and Wright, J.R., 1988, “An Eigensystem Realization Algorithm Using Data Correlations (ERA/DC) for Modal Parameter Identification”, Control-Theory and Advanced Technology, Vol. 4, No. 1, pp. 5-14.

Koury, R., 1998, Modelo Numérico de uma Máquina Frigorífica de Compressão de Vapor, Tese de Doutorado, UNICAMP, Campinas, SP, Brasil.

Kouvaritakis, B., 1979, "Theory and Practice of the Characteristic-Locus Design Method", IEE Proceedings, Vol. 126, pp. 542-548.

Ljung, L., 1987, System Identification: Theory for the User. Prentice Hall, Englewood Cliffs, New Jersey, USA.

Machado, L., 1996, Modele de Simulation et Etude Experimentale d’un Evaporateur de Machine Frigorifique en Regime Transitoire, Thése de Doctorat, L’Institut National des Sciences Appliquees de Lyon, France.

Maciejowski, J.M., 1989, “Multivariable Feedback Design”, Addison-Wesley Publishing Company.

McAvoy, T. J., 1983, “Interaction Analysis - Principles and Applications”, Instrument Society of America.

Mees, A.I., 1981, “Achieving Diagonal Dominance”, Systems and Control Letters, vol. 1, pp. 155-158.

Niu,, S.S. and Mijares, G., 1999, "Blind Identification of Large Multivariable Systems", Proceeding of the American Control Conference, São Diego, California, USA.

Outtagarts, A., 1994, Comportement Dynamique d'un Evaporateur de Machine Frigorifique Alimente par un Detendeur Electronique, Thése de Doctorat, L'Institut National des Sciences Appliquees de Lyon, France.

Rocha, R., 1995, Contribuição ao Estudo de um Sistema de Refrigeração com Compressor Operando a Velocidade Variável, Dissertação de Mestrado, PPGEE-UFMG, Brasil.

Skogestad, S. and Postlethwaite, I., 1996, "Multivariable Feedback Control - Analysis and Design", John Wiley & Sons.

Zilouchian, A., Agaee, P.K. and Nickravesh, K., 1997, "Model Reduction of Large Scaled Systems via Frequency-Domain Balanced Structures", Proceeding of the American Control Conference, Alguquerque, New Mexico, USA.

8. RESPONSIBILITY NOTICE

The author is the only responsible for the printed material included in this paper.