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DYNAMIC AND CONTROL OF A NONLINEAR OSCILLATORY ELECTROMECHANICAL SYSTEM

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Abstract. *Electromechanical systems are increasingly gaining applications replacing purely mechanical devices. The simpler control of electrical parameters and variables make this type of equipment interesting for use in actuators, sensors, vibration absorbers and energy harvesting devices. The wide range of devices, appliances and equipments that can be built using the principles presented on the electromechanical system is what makes it relevant for research and improvement. This work has a focus on an electromechanical macro scale device, according to the classification most commonly found in the literature. However, the model can be adjusted in order to represent systems of nano and micro scales, according to a desired application. In this work, the electromechanical system functions as a vibration absorber. The dynamic stability of the system, in particular the electrodynamics stability, is assessed for a better understanding of the general behaviour of the system and enabling the correlation of electrical variables with the mechanical damping of the device. The discussion includes the study of the conditions and criteria of stability and the influence of the nonlinearities on the behaviour of the system.*

Keywords: *Electromechanical Absorber, Nonlinear Oscillations, Dynamic Stability*

1. INTRODUCTION

Devices which rely on different domains of Physics to perform a specific task are currently the study object of many researchers. The areas of applications of these multi-Physics devices is very wide, including, for example, harvester devices (Priya and Inman, 2009), dampers or absorbers (Yamapi, 2006), actuators and piezoelectric devices (Erturk and Inman, 2008), and they are used in automotive, energy, aerospace, naval and medical industries, among others. There is a tendency for such devices to get more space because of the possibility of converting mechanical energy into electrical energy (Preumont, 2006). These devices makes possible any motion or vibration into electricity, this feature is extremely interesting, because we can draw, for example, energy of a vehicle dampers, we can take energy from a vibration of a full people stadium or, charge batteries with piezoelectric energy harvesting (Sodano, 2005).

Some of these devices is already investigated and also has many results about that, these electromechanical devices studied is reference for this investigation and future research. However the wide range of variations, applications, parameters, modeling methods and another variables that can bring alterations in the way to see the electromechanical system show us that the investigation about this devices has a large field of research and results yet. Investigations in electromechanical devices, electrical devices and mechanical devices are availed for discussion and reference of this paper.

The specific application to energy harvesting is currently an important subject. Abedelkefi *et al.* (2012), for example, investigated piezoelectric energy harvesting from freely oscillating cylinders, and showed that from vortex-induced vibration of cylinders submerged in fluid and attached to a piezoelectric transducer, it is possible to generate energy for use in low-power consumption devices, maybe replacing small batteries. This study also shows the interference caused by load resistance and capacitance present in the piezoelectric device. The authors shown results in the sub-harmonic and super-harmonic resonance cases.

Tusset and Balthazar (2012) studied a device which uses magnetorheological damping in a Duffing based vibrating system, focusing on the control and suppression of chaotic behavior in the dynamic system.

The magnetorheological damper is a kind of electromechanical transducer. Other authors worked in a research of a nonlinear response of vibrational conveyers with non-ideal vibration exciter (Bayiroğlu *et al*., 2011), this system is purely mechanical, however, has in their modeling the deductions of the equations shown in a simple form, also has a schematic disposition nearly like which we used in the present article. This Investigation show also the analytical analysis and numerical analysis of the system obtained by the method of multiple scales for super-harmonic and sub-harmonic resonances.

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Felix *et al*. (2012) investigated a viscoelastic material with essential nonlinear stiffness and time-dependent damping properties, considering non-ideal excitation. In this case, the non-ideal excitation comes from a limited power-supply, which is connected to a DC-motor. The viscoelastic material was developed to reduce the Sommerfeld effect and to enhance the response of the structure. This problem was investigated with numerical simulations.

The present work focuses on the modeling of an electromechanical system with two degrees of freedom, one related to the mechanical subsystem and the other related to the electrical subsystem. The mechanical subsystem in this case is represented by a body with mass, an elastic stiffness and a viscous damper. The other subsystem is a RLC electrical circuit represented by an electrical resistance, inductance and capacitance These two subsystems are connected by a movable coil transducer, which works as a coupling mechanism between the two subsystems. The components of the electromechanical system are explained in Section 2. The aim of this work is to acquire a better understanding of the electromechanical system and its control parameters.

2. ELECTROMECHANICAL SYSTEM

This session presents the subsystems considered in this electromechanical system. The following, each subsystems is described with relation to their specific physics domain.

In this work the mechanical subsystem is represented as a system with mass m, elastic stiffness k, and viscous damping coefficient d. An external harmonic excitation force is applied to the mass. The schematic representation of the mechanical subsystem is shown in Fig 1.

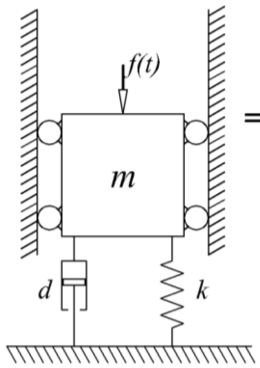

Figure 1. Schematic representation of the mechanical subsystem

The parameters representing the electrical subsystem are the inductance L, the capacitance C and the resistance R. The electric symbols in a series circuit are shown in Fig. 2.

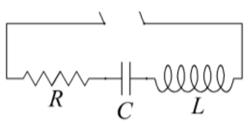

Figure 2. Schematic representation of the electrical subsystem

Transducer is a device which converts energy of one nature to another. A microphone is an example of transducer, which turns sound into electrical signal. Other types of transducers are known. Yamapi (2003) studied the conversion of mechanical energy into electrical energy using piezoelectric devices, while Sinclair (2001) worked with electrostatic transducers. In the present work, the type of transducer used is a movable coil transducer whose objective is to convert mechanical movement into electrical current.

The movable coil transducer shown in Fig. 3 is an electromechanical device compound by a mobile part linked to the mechanical subsystem and the coil at the same time, the coil is immersed into a permanent magnet magnetic field, the magnetic field $\vec{B}$ is perpendicular to wires of coil. The characters N and S represent the conventional designation for North Pole and South Pole of the magnet respectively.

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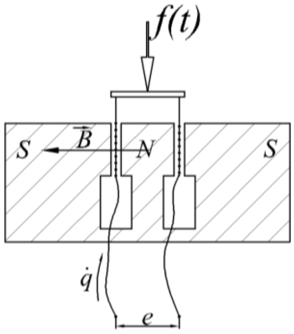


Figure 3. Schematic representation of the mobile coil transducer

Respecting the transducer geometry, disposal and operation shown in the Fig. 3, the transducer equations responsible for relation between mechanical excitation and electrical energy can be written as linear relations.

The transducer focused in study is composed by a radial permanent magnet whit your North Pole in the center, this type of transducer is already known in literature, the transducer equations according by Preumont (2006) are the Eq. (1) and the Eq. (2).

$$e = -B\ l\ v \tag{1}$$

$$f = B\ l\ i \tag{2}$$

Where e is the electric tension in the coil terminals, v is the velocity of mass, f is the mechanical force produced for the coil, i is the electric current, l is the fulllength of the coil and B is the magnetic field.

3. GOVERNING EQUATIONS OF DYNAMIC OF THE ELECTROMECHANICAL SYSTEM

The electromechanical system is classified as being a multi-domain system with two subsystems: the mechanical and the electrical. The external harmonic excitation $f(t)$ is applied to the mechanical subsystem. The fact that the external excitation is applied to the mechanical assembly causes the transducer connected to a RLC circuit behave as a damper. Figure. 4 shows the schematics of the two sub systems connected forming the electromechanical system.

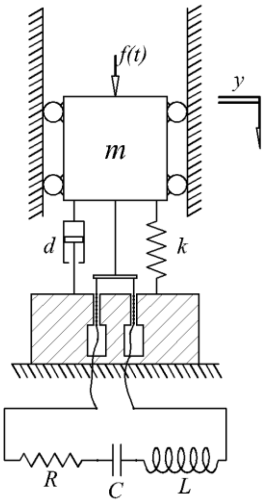


Figure 4. Schematic representation of the full electromechanical system

3.1 Linear Electromechanical System

The physical parameters present in the electromechanical system are divided into two different domains. The use of generalised coordinates becomes important because the treatment makes the problem less tied to a domain or another and puts the equations in a better format to use in the computational simulation. With these coordinates, x_1 is the position, x_2 is the velocity, x_3 is the electrical charge and x_4 is the electrical current. The governing equations of the motion are expressed in Eq. (3).

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = [-k\, x_1 - d\, x_2 + B\, l\, x_4 + A_{ex}\ \cos(\omega_{ex}\, t)]/m \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = (-C^{-1}\, x_3 - R\, x_4 - B\, l\, x_2)/L \end{cases} \tag{3}$$

3.2 Electromechanical System With Nonlinear Duffing Stiffness

Based on Eq. (3), the nonlinear system has, in the mechanical subsystem, a nonlinear stiffness term of the Duffing type, k_1, associated to the square of the displacement. Therefore, the new mechanical stiffness is represented by Eq. (4).

$$k_{total} = k + k_1\, {x_1}^2 \tag{4}$$

Elastic force F_{el} and elastic potential energy Ep_{el} can be written as Eq. (5) and Eq. (6):

$$F_{el} = k\, x_1 + k_1\, {x_1}^3 \tag{5}$$

$$Ep_{el} = \frac{1}{2} k\, {x_1}^2 + \frac{1}{4} k_1\, {x_1}^4 \tag{6}$$

On the Fig. 5, shows a comparison between linear stiffness used in the model shown in section 3.1 and the nonlinear stiffness Duffing type shown in present section 3.2, with the parameters values presented on Tab. 1. Considering the displacement of the mass between -0.1 m and 0.1 m, the values of the elastic force and the elastic potential energy are relatively close for both linear and nonlinear cases.

Table 1. Stiffness values used in linear system and Duffing stiffness values used in nonlinear system

Stiffness type	Linear stiffness (k)	Nonlinear stiffness (k_1)
Linear	4E4	0
Duffing	-4E4	4E7

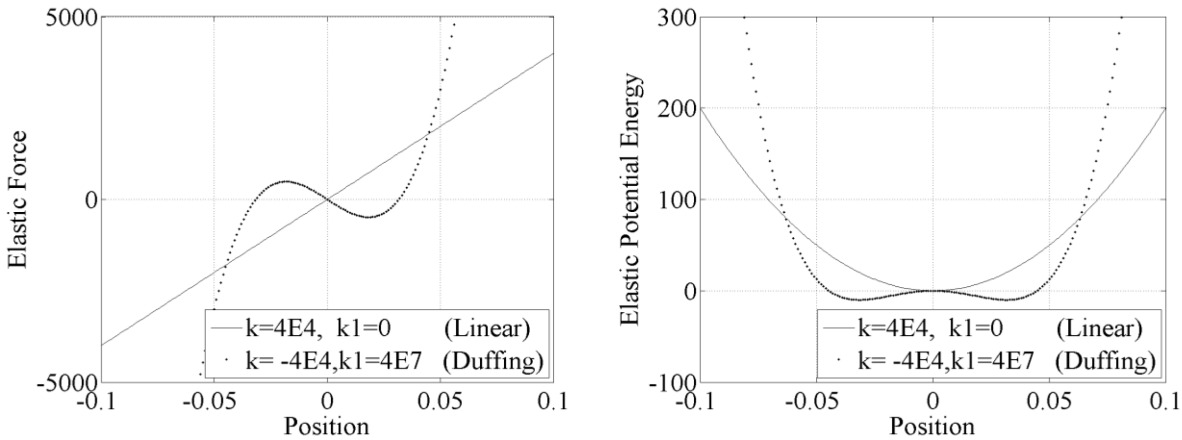


Figure 5. Curves of comparison of force and energy elastic between linear and Duffing stiffness

Using the Duffing stiffness values, we can get the governing equations of dynamic for the nonlinear electromechanical system. Follow the system of equations Eq. (7).

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = [-k\, x_1 - k_1\, {x_1}^3 - d\, x_2 + B\, l\, x_4 + A_{ex}\ \cos(\omega_{ex}\, t)]/m \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = (-C^{-1}\, x_3 - R\, x_4 - B\, l\, x_2)/L \end{cases} \tag{7}$$

ISSN 2176-5480

4. SIMULATIONS AND ELECTROMECHANICAL SYSTEM RESPONSE

The basic parameters for this electromechanical system were based on an application to passenger vehicles. From a quarter-car model, the parameters were chosen so that the electromechanical damping effect is close to a mechanical viscous damper. Table. 2 show de base parameters values used in linear and nonlinear electromechanical system.

Table 2. Base parameters values.

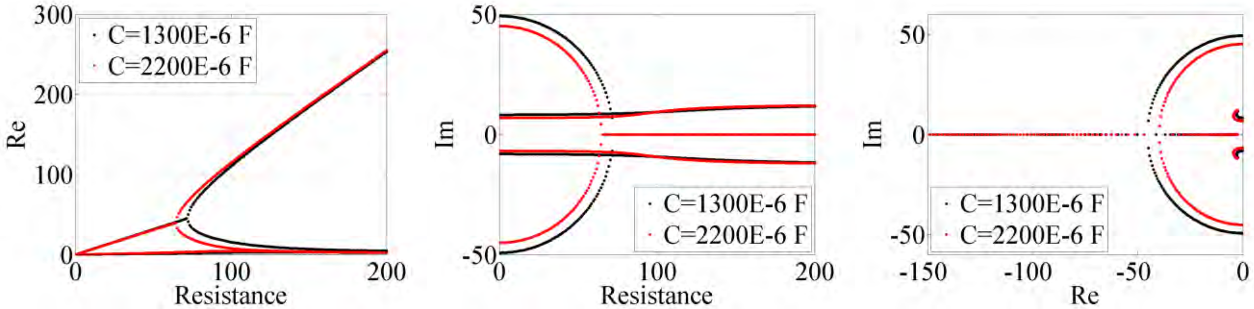
m	[Kg]	Mass	250.00
d	[N.s/m]	Mechanical viscous damper	0.00
L	[H]	Inductance	0.762749
R	[Ω]	Resistance	0.824354 – 200.00
C	[F]	Capacitance	(1300 – 2200)E-6
A_{ex}	[N]	Force amplitude	5000.00
f_{ex}	[Hz]	External frequency	2.00
Bl	[T.m]	Electromechanical coupling	502.6548

4.1 Stability Analysis of the Linear Electromechanical System

To perform a stability analysis, the characteristic equation with four eigenvalues is used as in Eq. (8).

$$\lambda^4 + (d/m + R/L)\lambda^3 + (1/C + Rd/Lm + k/m + B^2l^2/Lm)\lambda^2 + (d/CLm + Rk/Lm)\lambda + k/CLm = 0 \quad (8)$$

From this characteristic equation, with parameters as in Tab. 2 and letting R and C vary, the system is stable for R>0 and C>0. Therefore, the eigenvalues were calculated for R ranging between 0.824354 Ω and 200 Ω and for two values of C, 1300E-6 F and 2200E-6 F. The eigenvalue analysis shows that the behavior of the electromechanical system changes considerably in the range of R between 50 Ω and 100 Ω, as can be observed in Fig. 6. The equivalent damping coefficient changes with the variation of R. More specifically, the system has larger electromechanical damping for the range of R between 50 Ω 100 Ω. Values for R smaller than 50 Ω or larger than 100 Ω cause the electromechanical damping to decrease. The system is less sensitive to variations of C.



(a) (b) (c)

Figure 6. Eigenvalues vs. resistance (a) and (b), root locus (c), the capacitance values shown in the figures in black color for 1300E-6 F and in red color for 2200E-6 ¨F

ISSN 2176-5480

4.2 Response of the Nonlinear Electromechanical System

On a linear electromechanical system, the influence of C in the behaviour of the system is negligible. However, there are different dynamical responses of the nonlinear electromechanical system for varying values of C. This can be seen in the bifurcation diagram of Fig. 7. This bifurcation diagram was obtained varying C between 1300E-6 F and 2200E-6 F, while R is fixed at 0.824354 Ω. Based on the different regions observed in this diagram, seven representative values of capacitance were chosen for a more detailed analysis. There are apparently three regions with different types of behaviour. One region is associated with the 1st and 6th capacitance values (1300E-6 F and 2050E-6 F). Another region is associated with the 2nd, 3rd, 4th and 5th capacitance values (1350E-6 F, 1400E-6 F, 1429E-6 F and 1600E-6 F). And the final region is associated to the 7th capacitance value (2200E-6 F).

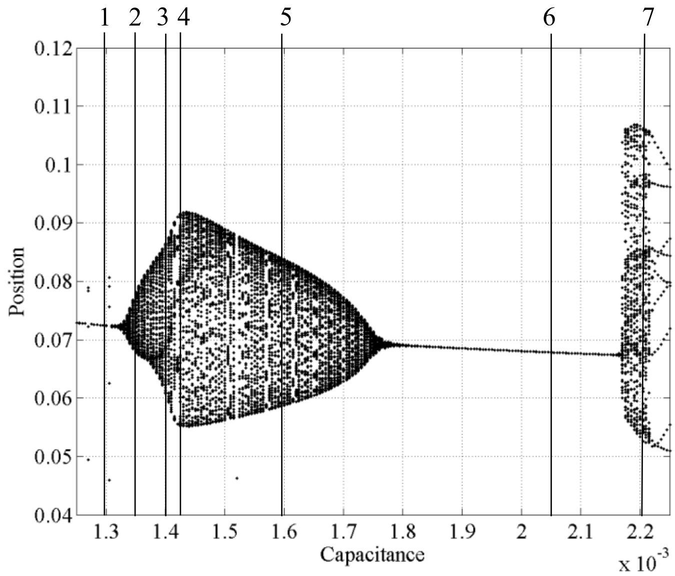


Figure 7. Bifurcation diagram for capacitance, indicated values: C=1300E-6 F (1), C=1350E-6 F (2), C=1400E-6 F (3), C=1429E-6 F (4), C=1600E-6 F (5), C=2050E-6 F (6) and C=2200E-6 F (7)

Figure 8 shows the response for C=1300E-6 F. The system presents period-1 response relative to external excitation frequency f_{ex}. However, the displacement completes three oscillations for one period of the excitation force, which can be observed in the time history and the phase plane. The behavior found for C=1300E-6 F is similar for C=2050E-6 F.

Figure 9 shows the simulation for C=1600E-6 F. A different response can be seen. Relative to f_{ex}, the system seems to have quasi-periodic behaviour. This can be observed in the phase plane, which shows a broadband aspect. Also, the Poincaré map seems like a closed loop. The behavior found for this capacitance value can also be found for 1350E-6 F, 1400E-6 F and 1429E-6 F, which comprises the hazelnut shape in the bifurcation diagram of Fig. 7.

The last set of simulations shows on Fig. 10 the response of the system for C=2200E-6 F. This behaviour is different from what was observed for the other capacitance values, and is representative of the region on the far right portion of the bifurcation diagram. The dynamics observed is considered chaotic.

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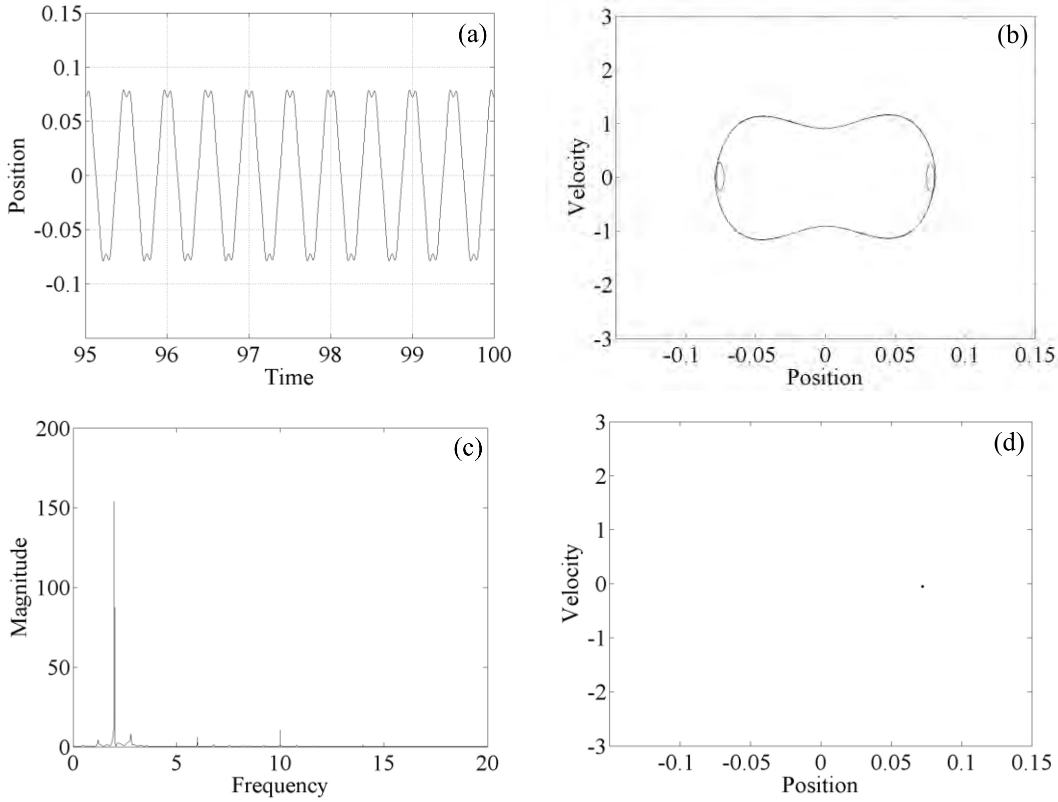

Figure 8. Electromechanical system response for 1st capacitor value 1300E-6 F, the figures shown are: Historic (a), Phase Plane (b), FFT (c) and Poincaré Map (d)

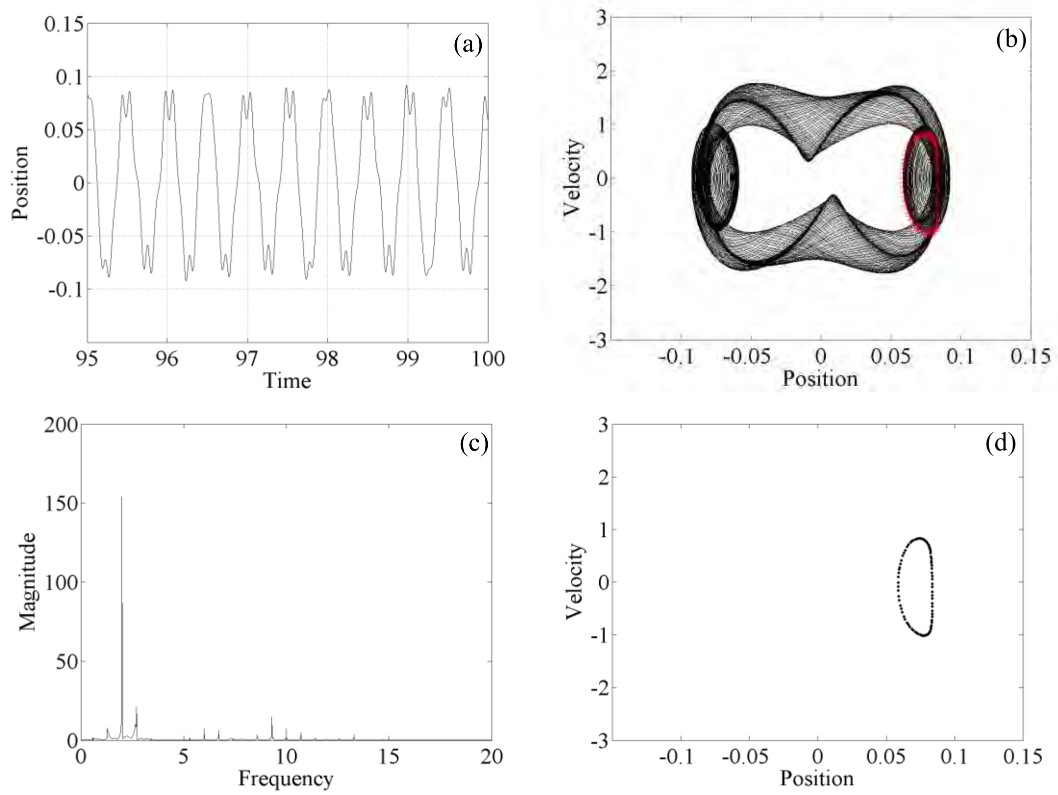

Figure 9. Electromechanical system response for 5th capacitor value 1600E-6 F, the figures shown in order are: Historic (a), Phase Plane (b), FFT (c) and Poincaré Map (d)

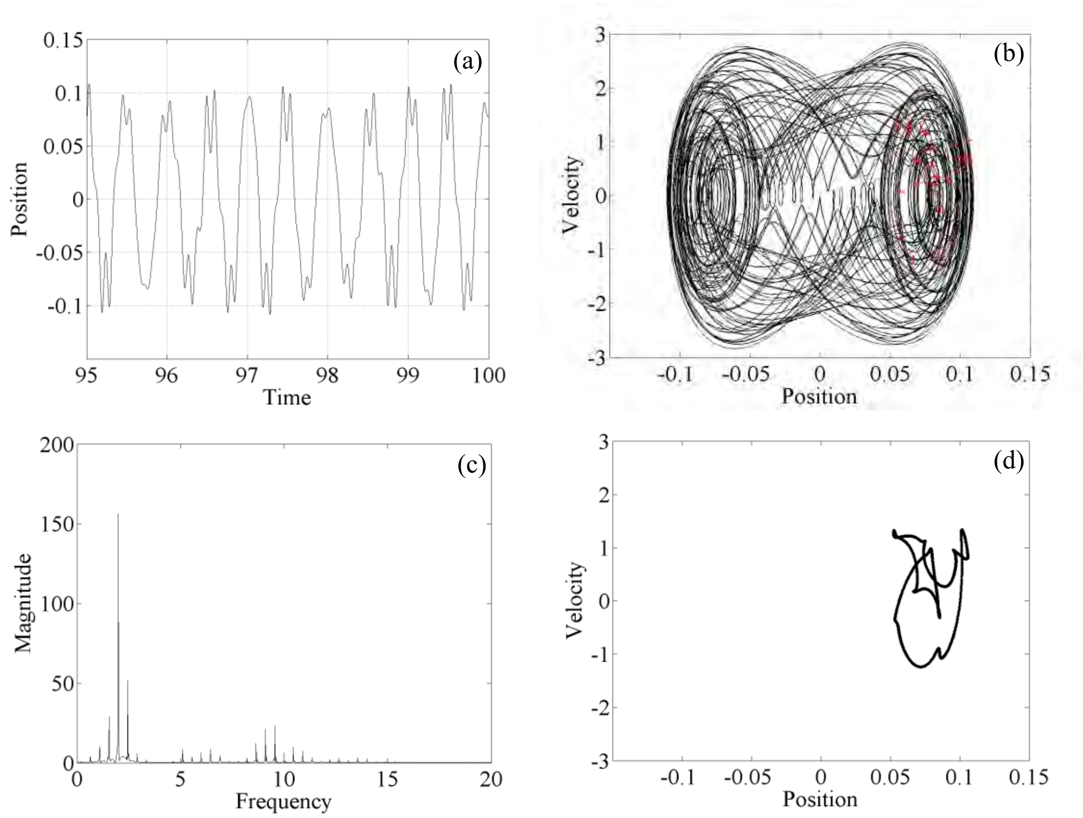

Figure 10. Electromechanical system response for 7th capacitor value 2200E-6 F, the figures shown in order are: Historic (a), Phase Plane (b), FFT (c) and Poincaré Map (d)

5. CONCLUSION

The behaviour of the system is influenced by both control parameters used, capacitance and resistance. However, the resistance has larger influence on the mechanical damping characteristics. This damping effect is larger for values of resistance between 50 Ω and 100 Ω, and has a maximum value within this region. This maximum may change for different values of the capacitance. There are different qualitative responses of the system, depending on the value of the capacitance, as shown in the bifurcation diagram. The response can be periodic, quasi-periodic or chaotic. Within the studied range of parameters, two period-1 regions can be identified as more interesting for operation of the system.

6. ACKNOWLEDGEMENTS

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