STATIC ANALYSIS OF BURIED PIPES USING COUPLING BETWEEN A LAYERWISE FINITE ELEMENT AND BOUNDARY ELEMENT METHOD

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Abstract. This work deals with the analysis of soil-structure interaction modeling of pipeline problems in static behavior using the coupling between Finite Element Method (FEM) and Boundary Element Method (BEM).The representation of the pipe is made by MEF using one finite element in the cylindrical panel formulated from the theory of equivalent discrete layers (Layerwise Theory), proposed by J. N. Reddy. The soil is represented by elastic continuum infinite or semi-infinite and modeled using boundary elements with special curved surface, associated with cylindrical panel, used to represent the soil-structure interaction within the soil, especially at the contact surface with the pipe.

Keywords:Finite Elements, Boundary Elements, Pipe-Soil Coupling

1. INTRODUCTION

The industry in many sectors, has in recent years implement and improve its infrastructure, with use of buried and submerged pipelines. These structures to be interacting with other media (soil, liquid, etc.), have a high degree of complexity, which demands special attention from initial conception until the final operation. Thus, in recent decades designers have used some tools to help to produce projects of buried pipelines more efficient. One of the important lines is based on the accumulated experience that are usually described in norms already established. However, this is not enough, since each project has its own peculiarities. Then, in many cases a set of experimental tests is specified to extract some behavior parameters of the buried pipe. However, even with the great help of the experimental apparatus, there are importante unknowns in problem behavior that are generally not accessible appropriately, such as the state of deformation and stress in the pipes and its interfaces with the ground. Thus, the third family of tools that can potentially be used are the numerical simulation techniques enabling the structural analysis of the problem. The mathematical models commonly used are usually restricted to two-dimensional space. If the analysis region of the line along its length is extended without significant change, the two-dimensional analysis results generally no significant differences when compared to the three-dimensional model. However, it is not always possible to ensure that the proximity of pipe remains unchanged, as it usually has to go through urban areas, cross roads, railroads, etc. Thus, the development and implementation of three-dimensional models for the analysis of buried pipes is undoubtedly a real need for improvement projects. In recent decades, many researchers have been dedicated to studying of mathematical models of interaction, consisting of a set of sub-models associated respectively to the pipeline, soil and their interfaces. In general, the analytical solutions of the governing equations of buried pipes are available only for some special cases of interaction. This is the case in the one dimensional model, when it admits the buried pipes as infinite beam supported by elastic base, which solutions are found in the work of Biot (1937), Hetenyi (1946). Already Burns and Richard (1964) analyzed a pipe-soil system subjected to horizontal and vertical pressures, using the principles of continuum mechanics, where was developed closed solutions (analytical) for a two-dimensional elastic model (plane strain condition). There is also the work of Hoeg (1968), Moore (2000), Dhar (2004) that has presented two dimensional analytical solutions expressed more smoothly than techniques employed by Richard (1964).

There are many semi-empirical methods which are obtained by the incorporation of some correction factors in the two-dimensional analytical solutions, which are calibrated experimentally. Among the techniques most known semi-empirical has: Iowa Method (Spangler 1941) and its modified versions in Watkins and Spangler (1958), Greenwook and Lang(1990). However, not all the analytical solutions of pipelines are available, so an alternative is the discretization of continuous medium and the systematization of the discrete problem, to obtaining approximate solutions using numerical techniques, among which has been the Finite Element Method (FEM) and Boundary Element Method (BEM). In this context, for the one dimensional model there are some numerical solutions where the domain is usually discretized by 1D finite element (bars) and the effect of elastic foundation is introduced into the problem by potential energy. This produces an additional contribution to the stiffness matrix of the structure. For the two dimensional model there are several numerical solutions for pipeline and soil represented by continuous media: Finite Elements Methods (FEM), Boundary Elements Method (BEM) (Freitas 2008) and FEM-BEM Combination (Vieira 2009). For numerical analysis of the three dimensional models, some work can be found in the literature for each of the sub-systems involving isolated pipe, isolated soil and soil-pipe interaction. Already numerical models for analysis insulated idealized as shells. Several researchers, Sydenstricker and Landau (1995), Chen (1979), Bathe and Ho (1981), Fafart et al. (1989),

formulated plane shells finite elements, for example the membrane finite element caled CST (Constant Stress Triangle), combined with plate elements, such as discussed in DKT Batoz et al.(1980) or DST described in Batoz and Lardeur (1989). A research group showed a alternative finite element family for cylindrical shells analysis called assumed strain-based model (Aswell and Sabir 1972), (Sabir 1983), (Bull 1984), (Sabir et al. 1994) were the first works with analysis of deep cylindrical shells. Already Djoudi and Bahai (2002,2004) presented an extension of the ideas of the model to cylindrical shells proposed by Sabir employing the theory of shells from Donnel (1933,1938). Against this background, this paper presents a mathematical model for the analysis of three-dimensional problems of buried homogeneous pipes.

2. LAYERWISE CYLINDRICAL SHELL FINITE ELEMENT

In this section we describe the formulation of the laminated cylindrical shell based on Layerwise theory by Reddy (1984) and systematized by some researchers Barbero et al. (1990), and Savoia and Reddy (1992), Kassegne (1992) Gerhard et al. (1994). The laminated cylindrical shell has layers with same or different thickness, composed by materials with different orientation angles to the fibers. The representation of a Cylindrical Shell with length L, thickness h and deformable surface with radius R (where $h << R$), is shown in Figure 1.
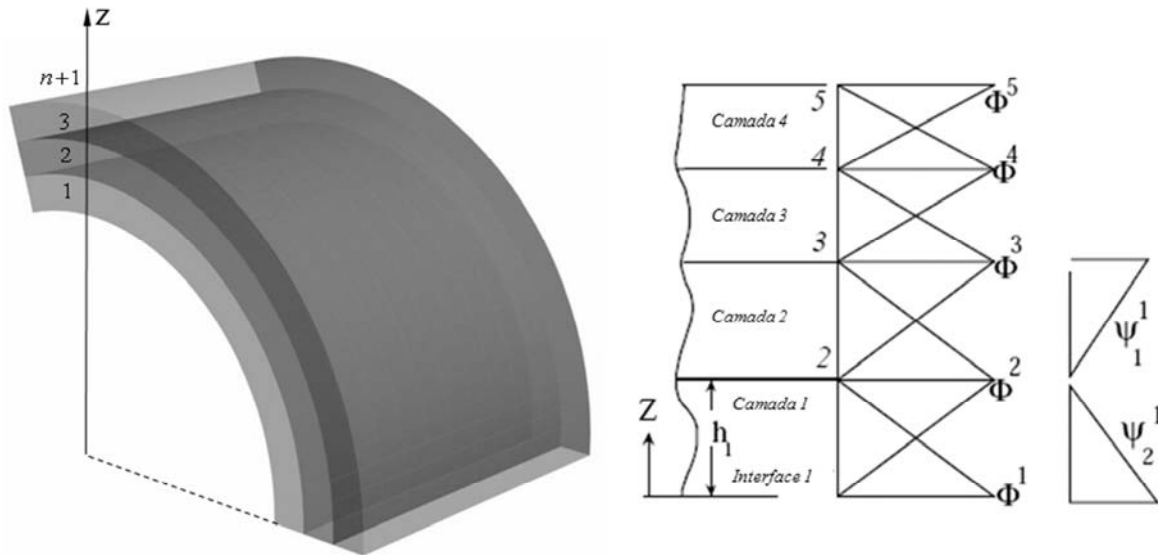

Figure 1 – Finite Element of Laminated Cylindrical Shell

The displacement field (u, v, w) at a generic point (x, y, z) of a layer in one Laminated cylindrical shell is approximated by the following Lagrangian interpolation functions (Kassegne 1992).

$$u(x,y,z) = \sum_{i=1}^{n+1} u_i(x,y)\Phi^i(z)$$

$$v(x,y,z) = \sum_{i=1}^{n+1} v_i(x,y)\Phi^i(z)$$ $\qquad (1)$

$$w(x,y,z) = \sum_{i=1}^{n+1} w_i(x,y)\Phi^i(z)$$

3 – NODAL LOAD EQUIVALENT VECTOR

The nodal load equivalent vector is deducted for uniformly and linearly distributed self-weight for the finite element RLS. The consistent nodal vector is obtained through the work done by external loads that can be expressed by:

$$W = \int_A \left(p_x u + p_y v + p_z w \right) dA = \int_A \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}^T \begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} dA$$ $\qquad (2)$

The external loads due the linear distribution has the matrix representation as follows:

$$\begin{Bmatrix} p_x \\ p_y \\ p_z \end{Bmatrix} = [R]\{\hat{p}\} \tag{3}$$

Where the $[R]$ matrix depends of kind of distribution of the pressure in the element with nodal forces vector.

$$\{\hat{p}\}^T = \{\hat{p}_{x1} \quad \hat{p}_{y1} \quad \hat{p}_{z1} \quad \hat{p}_{x2} \quad \hat{p}_{y2} \quad \hat{p}_{z2} \quad \hat{p}_{x3} \quad \hat{p}_{y3} \quad \hat{p}_{z3} \quad \hat{p}_{x4} \quad \hat{p}_{y4} \quad \hat{p}_{z4}\} \tag{4}$$

Substituting the nodal force vector in (4) in the external work equation in (2), the equivalent nodal force vector $\{F\}$ is:

$$\{F\} = [C_d]^{-T}\{P\} \tag{5}$$

Where:

$$\{P\} = [A]\{\hat{p}\}$$

$$[A] = \int_A [Z]^T [R] \, dA$$

The $[A]$ matrix calculation in (5) results in the explicit form.

$$
A = \frac{ab}{9}
\begin{bmatrix}
4 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 \\
0 & 4 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 2 & 0 \\
0 & 0 & 4 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 2 \\
2 & 0 & 0 & 4 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\
0 & 2 & 0 & 0 & 4 & 0 & 0 & 2 & 0 & 0 & 1 & 0 \\
0 & 0 & 2 & 0 & 0 & 4 & 0 & 0 & 2 & 0 & 0 & 1 \\
1 & 0 & 0 & 2 & 0 & 0 & 4 & 0 & 0 & 2 & 0 & 0 \\
0 & 1 & 0 & 0 & 2 & 0 & 0 & 4 & 0 & 0 & 2 & 0 \\
0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 4 & 0 & 0 & 2 \\
2 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 4 & 0 & 0 \\
0 & 2 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 4 & 0 \\
0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 4
\end{bmatrix}
\tag{6}
$$

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For the case of uniform pressure in each direction, then the vector in (4) takes the following form:

$$\{\hat{p}\}^{T} = \{\hat{p}_{x1} \quad \hat{p}_{y1} \quad \hat{p}_{z1} \quad \hat{p}_{x2} \quad \hat{p}_{y2} \quad \hat{p}_{z2} \quad \hat{p}_{x3} \quad \hat{p}_{y3} \quad \hat{p}_{z3} \quad \hat{p}_{x4} \quad \hat{p}_{y4} \quad \hat{p}_{z4}\}$$

(7)

Where:

$$\hat{p}_{x1} = \hat{p}_{x2} = \hat{p}_{x3} = \hat{p}_{x4} = \hat{p}_{x}$$

$$\hat{p}_{y1} = \hat{p}_{y2} = \hat{p}_{y3} = \hat{p}_{y4} = \hat{p}_{y}$$

$$\hat{p}_{z1} = \hat{p}_{z2} = \hat{p}_{z3} = \hat{p}_{z4} = \hat{p}_{z}$$

Where explicit values of subvetores $\{\hat{p}\} = \begin{Bmatrix} \{F_1\} \\ \{F_2\} \end{Bmatrix}$ are:

(8)

$$\{F_1\} = ab \begin{Bmatrix} p_x \\ p_y \\ p_z \\ p_x \\ p_y \\ p_z \end{Bmatrix} \qquad\qquad \{F_2\} = ab \begin{Bmatrix} p_x \\ p_y \\ p_z \\ p_x \\ p_y \\ p_z \end{Bmatrix}$$

Considering the effect of the self weight of the shell divided into two stages, where in the first, it is assumed that the longitudinal axis of the shell is orthogonal to the gravity vector, then the $[R]$ matrix in (3) takes the following form:

$$[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(y / r + \theta_0) & 0 \\ 0 & 0 & sen(y / r + \theta_0) \end{bmatrix}$$

(9)

$$\{\hat{p}\}^{T} = \{0 \quad g \quad g\}$$

Where $g = \gamma \cdot h$, $\gamma =$ Specific weigth.

The vector $\{\hat{p}\} = \begin{Bmatrix} \{P_1\} \\ \{P_2\} \end{Bmatrix}$ is given by.

$$\{P_1\} = \begin{Bmatrix} 0 \\ p_{y1} \\ p_{z1} \\ 0 \\ p_{y1} \\ p_{z1} \end{Bmatrix} \qquad\qquad \{P_2\} = \begin{Bmatrix} 0 \\ p_{y3} \\ p_{z3} \\ 0 \\ p_{y3} \\ p_{z3} \end{Bmatrix}$$

(10)

Where p_{y1} to p_{y4} and p_{z1} to p_{z4} are:

$$p_{y1} = -\frac{ra}{2b}\left(r \sin\left(\frac{b}{r}\right)\sin(\theta) - b\cos\left(\frac{b}{r}\right)\sin(\theta) - b\sin\left(\frac{y}{r}\right)\cos(\theta) \right)\rho g t$$

$$p_{y1} = p_{y2}$$

$$p_{y3} = -\frac{ra}{2b}\left(r\sin\left(\frac{b}{r}\right)\sin(\theta) - b\cos\left(\frac{b}{r}\right)\sin(\theta) + b\sin\left(\frac{y}{r}\right)\cos(\theta) \right)\rho gt$$

$$p_{y4} = p_{y3}$$

$$p_{z1} = -\frac{ra}{2b}\left(r\sin\left(\frac{b}{r}\right)\cos(\theta) - b\cos\left(\frac{b}{r}\right)\cos(\theta) - b\sin\left(\frac{y}{r}\right)\sin(\theta) \right)\rho gt$$

$$p_{z2} = p_{z1}$$

$$p_{z3} = -\frac{ra}{2b}\left(r\sin\left(\frac{b}{r}\right)\cos(\theta) - b\cos\left(\frac{b}{r}\right)\cos(\theta) - b\sin\left(\frac{y}{r}\right)\sin(\theta) \right)\rho gt$$

$$p_{z4} = p_{z3}$$

If the longitudinal axis of the shell is parallel to vector gravity then.

$$\{P_1\} = ab\begin{Bmatrix} p_x \\ 0 \\ 0 \\ p_x \\ 0 \\ 0 \\ p_x \\ 0 \\ 0 \\ p_x \end{Bmatrix} \qquad \{P_2\} = ab\begin{Bmatrix} p_x \\ 0 \\ 0 \\ p_x \\ 0 \\ 0 \\ p_x \\ 0 \\ 0 \\ p_x \end{Bmatrix} \qquad (11)$$

Where: $p_x = \sum \dfrac{\gamma_i \cdot th_i}{2}$

4 – USUAL BOUNDARY ELEMENTS

Triangular plans elements for discretization of contact interfaces of machine foundation and soil, see Figure 2.
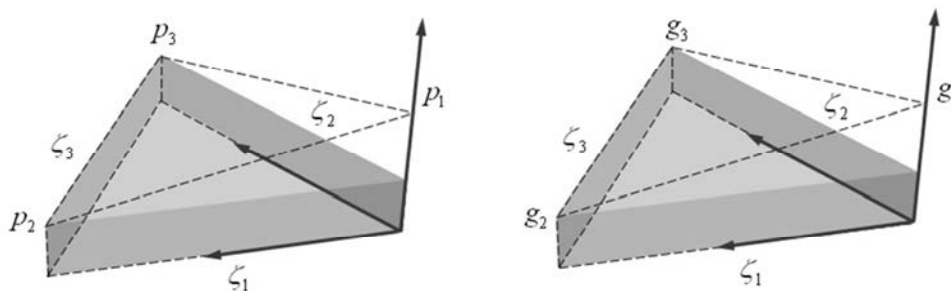

Figure 2 – Triangular Boundary Element

The interpolation variable geometry is made linearly from three nodes functional positioned at the vertices of the triangle.

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$$
\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & 0 \\ 0 & N_1 & 0 \\ 0 & 0 & N_1 \end{bmatrix} \begin{Bmatrix} u_1^1 \\ u_2^1 \\ u_3^1 \end{Bmatrix} + \begin{bmatrix} N_2 & 0 & 0 \\ 0 & N_2 & 0 \\ 0 & 0 & N_2 \end{bmatrix} \begin{Bmatrix} u_1^2 \\ u_1^2 \\ u_3^2 \end{Bmatrix} + \begin{bmatrix} N_3 & 0 & 0 \\ 0 & N_3 & 0 \\ 0 & 0 & N_3 \end{bmatrix} \begin{Bmatrix} u_1^3 \\ u_1^3 \\ u_3^3 \end{Bmatrix} \tag{12}
$$

Where the functions are : $N_1 = 1 - \xi - \eta$; $N_2 = \xi$; $N_3 = \eta$

5 – SPECIAL BOUNDARY ELEMENTS

In the literature (Beskos 1987), (Brebia and Dominguez 1977), most of the boundary elements are flat or approximated by polynomials, usually up to second degree. Given the specificity of the subject discussed in this work, these would not be very appropriate. Therefore, broke for the proposed new boundary element in which cylindrical surfaces, flat surfaces with polygonal boundaries could not be accurately represented. The elements proposed: Elements associated with a cylindrical panel to discretization the cylindrical shaft: this element there are four nodal points with interpolation of variables, see Figure 3.
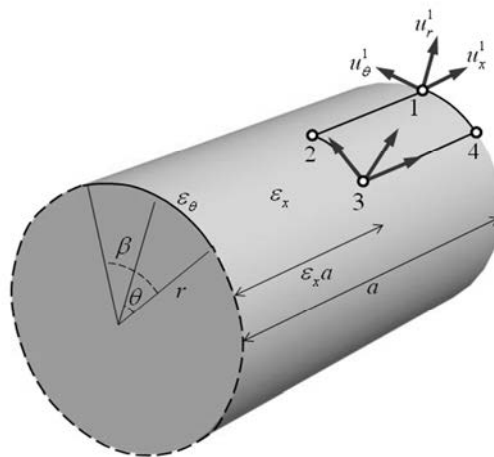

Figure 3 - Cylindrical Boundary Element

If considered a linear distribution of the field components within a cylindrical boundary element panel, their shape functions are interpolated:

$$
\begin{Bmatrix} u_x \\ u_\theta \\ u_r \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & 0 \\ 0 & N_1 & 0 \\ 0 & 0 & N_1 \end{bmatrix} \begin{Bmatrix} u_x^1 \\ u_\theta^1 \\ u_r^1 \end{Bmatrix} + \begin{bmatrix} N_2 & 0 & 0 \\ 0 & N_2 & 0 \\ 0 & 0 & N_2 \end{bmatrix} \begin{Bmatrix} u_x^2 \\ u_\theta^2 \\ u_r^2 \end{Bmatrix} + \begin{bmatrix} N_3 & 0 & 0 \\ 0 & N_3 & 0 \\ 0 & 0 & N_3 \end{bmatrix} \begin{Bmatrix} u_x^3 \\ u_\theta^3 \\ u_r^3 \end{Bmatrix} + \begin{bmatrix} N_4 & 0 & 0 \\ 0 & N_4 & 0 \\ 0 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} u_x^4 \\ u_\theta^4 \\ u_r^4 \end{Bmatrix} \tag{13}
$$

Where the interpolation functions are given by:

$$ N_1 = \varepsilon_r \varepsilon_\theta; N_2 = (1 - \varepsilon_r)\varepsilon_\theta; N_3 = (1 - \varepsilon_r)(1 - \varepsilon_\theta); N_4 = \varepsilon_r(1 - \varepsilon_\theta) $$

com $\varepsilon_r = \dfrac{\overline{r}}{\Delta r}$; $\varepsilon_\theta = \dfrac{\overline{y}}{r\beta} = \dfrac{r\theta}{r\beta} = \dfrac{\theta}{\beta}$

Sectoral plans boundary elements formed by two straight sides and two curved, destined for discretization of axial extremity shaft. The fields are linearly interpolated over the nodes of the functional element, see Figure 4.
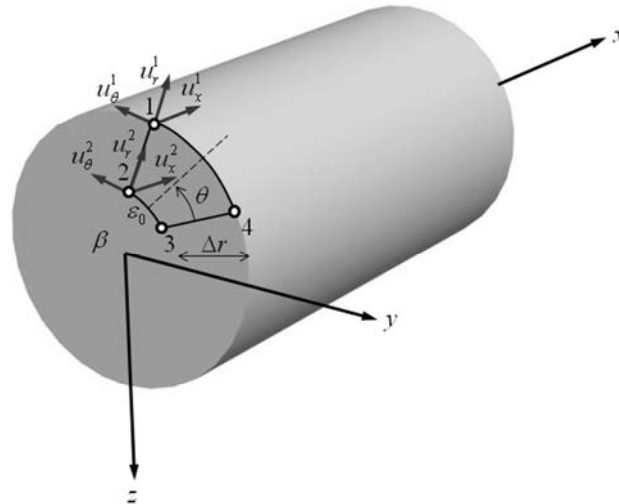

Figure 4 - Plane Sector Boundary Element

Discoid plans boundary elements, used for discretization of regions in proximity of centers of the circles in axial extremity of the shaft, see Figure 5.
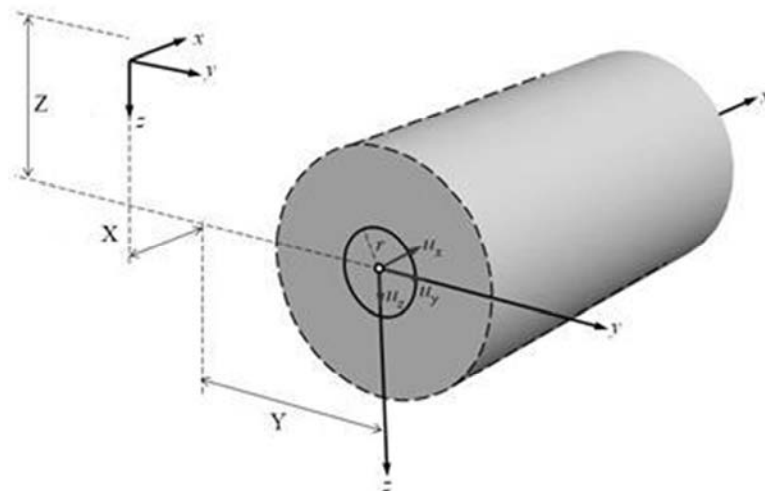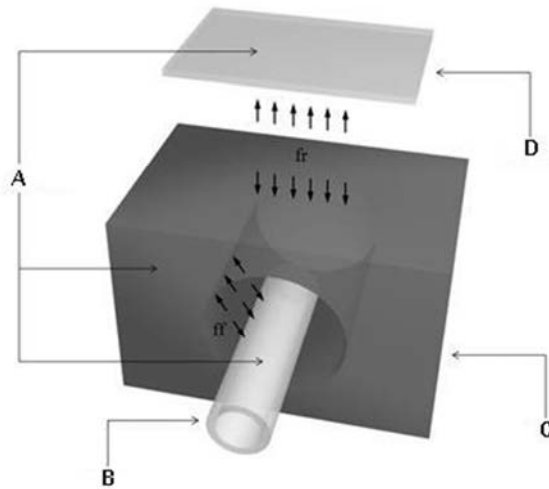

Figure 5 - Discoid Boundary Element

Note that a single nodal point is placed at the centers.

6 – FORMULATION OF INTERACTION ANALYSIS FOR SOIL-PIPE

Figure 6 presents a brief description of the problem to be fully analyzed and their interaction forces.

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A = Coupling; B = Pipe; C = Soil; D = Radie
Figure 6 – Interaction Forces for Buried Pipe Problem.

For the fundamental solution of kelvin, the p_{ij} is not null in the soil region in contact with radie, which requires defining points outside the region of contact, including the boundary region for the soil that envelops a cylindrical shaft. Thus, are placed sources in the shaft (E), at the end of the shaft (B), the contact interface with the radie (C) and the boundary contour (D). The algebraic system that represent this condition is:

$$
\begin{bmatrix} \tilde{H}_{ff} & \tilde{H}_{fe} & \tilde{H}_{fl} \\ \tilde{H}_{ef} & \tilde{H}_{ee} & \tilde{H}_{el} \\ \tilde{H}_{lf} & \tilde{H}_{le} & \tilde{H}_{ll} \end{bmatrix} \begin{Bmatrix} \tilde{u}_f \\ \tilde{u}_e \\ \tilde{u}_l \end{Bmatrix} = \begin{bmatrix} \tilde{G}_{ff} & \tilde{G}_{fe} & \tilde{G}_{fl} \\ \tilde{G}_{ef} & \tilde{G}_{ee} & \tilde{G}_{el} \\ \tilde{G}_{lf} & \tilde{G}_{le} & \tilde{G}_{ll} \end{bmatrix} \begin{Bmatrix} \tilde{p}_f \\ \tilde{p}_e \\ \tilde{p}_l \end{Bmatrix}
\tag{14}
$$

The pipe-soil interaction can be represented mathematically from the following assumptions:

• Contact between the pipe and the surface soil is ideal (there are no relative displacement at any point on the surface of interaction);

• Compatible only the degrees of freedom of displacements and forces in the axial, radial and circumferential contact surface.

The algebraic system of structure obtained from finite element discretization of cylindrical shells RLS, is estabilished with numeration of the first interface coupled with the soil, folow by the others interfaces not coupled. Thus, the system can be particioned in the matrix form:

$$
\begin{bmatrix} \bar{K}_{LL} & \bar{K}_{LN} \\ \bar{K}_{NL} & \bar{K}_{NN} \end{bmatrix} \begin{Bmatrix} \bar{U}_L \\ \bar{U}_N \end{Bmatrix} = \begin{Bmatrix} \bar{F}_L \\ \bar{F}_N \end{Bmatrix} - \begin{Bmatrix} P_L \\ 0 \end{Bmatrix}
\tag{15}
$$

The configuration of degree of freedom for the finite element RLS to be coupled with boundary element of soil is shown in Figure 7 (a) and (b). For the sake of conciseness only nodes 1 and 4 are represented with the degrees of freedom associated with it.
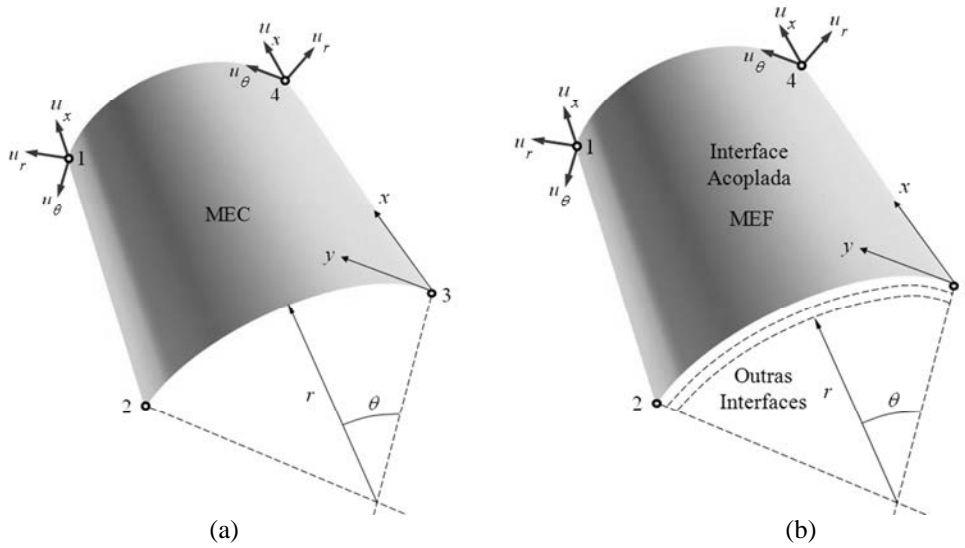
(a)   (b)

Figure 7 – (a) Soil Boundary Element, (b) RLS Finite Element

Making the force equilibrium $\{\bar{P}_L\} = \{P_F\}$ and the compatibility of desplacements $\{\bar{U}_L\} = \{U_F\}$, and substituting the soil system in (14) in the RLS finite element system in (15), the soil-pipe interaction is:

$$\begin{bmatrix} \begin{bmatrix} \bar{K}_{LL} + K_{ff} \end{bmatrix} & \bar{K}_{LN} \\ \bar{K}_{NL} & \bar{K}_{NN} \end{bmatrix} \begin{Bmatrix} \bar{U}_L \\ \bar{U}_N \end{Bmatrix} = \begin{Bmatrix} \bar{F}_L \\ \bar{F}_N \end{Bmatrix} + \begin{Bmatrix} P_F^{**} \\ 0 \end{Bmatrix} \qquad (16)$$

7 – RESULTS

7.1 – ANALYSIS OF A SHALLOW LAMINATED PIPE

Whether a cavity how the radius is 2.3 m in a rock mass whose mechanical properties are indicated in **Table 1**:

Table 1 – Physical Properties of Soil-Pipe Coupling			
E	v	Thickness	Structure
12,50 GPa	0,20	-	Soil
25,70 GPa	0,15	0,3 m	Pipe

We considered the pipe initially homogeneous and subjected to a confining pressure due to the weight of the rock mass $p = 1000 kPa$ as shown in Figure 8.

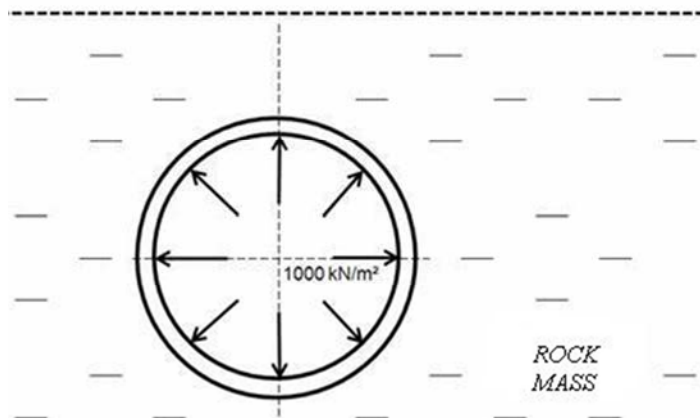Figure 8 – Low Pressure Pipe Depth

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In **Table 3** are listed the results for the radial displacement at the point of cavity 2 which are compared with analytical solution of Timoshenko (1959) and the numerical solution of BEM in Freitas (2008) (using 40 elements), see **Table 2**. In order to recover the two dimensional analysis applying the 3D model, it was considered a shaft / cylinder with length greater than 10 times the diameter of the shaft, resulting in $L = 45m$. Furthermore displacements at the axial extremities of the shaft were prescribed. The graph of Figure 9 describes the behavior Convergence of the results for the analyzes.

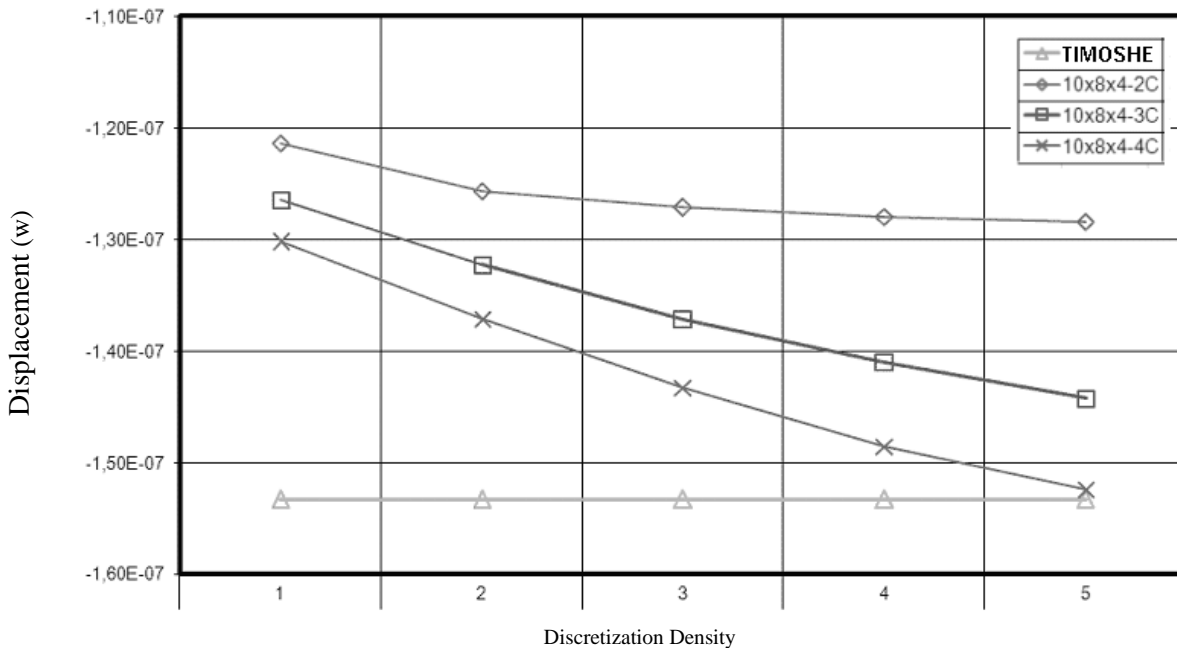| Table 2 – Results of Other Authors	
Timoshenko (1959)	MEC 2D – Freitas (2008)
1.53333e-007 (m)	1.53000e-007 (m)

Figure 9 – Laminated **Low Pressure Pipe Depth** – Graphic

The results (Table 3), where: RLS 2C [2 Layer] RLS 3C [3 Layer] 4C RLS [4 layers]), show that the RLS finite element, when coupled to ground, present convergence to values in the literature. Should be emphasized that the RLS element appears more effective with more layers present in thickness of the model, however, the computational analysis becomes more costly.

Table 3 – Laminated Low Pressure Pipe Depth

	1	2	3
TIMOSHE	-1,53300000000000000000E-07	-1,53300000000000000000E-07	-1,53300000000000000000E-07
MEC 2D	-1,51779220244818000000E-07	-1,52800000000000000000E-07	-1,53000000000000000000E-07
MEC 3D	-1,48040631303902000000E-07	-1,51779220244818000000E-07	-1,53216451717576000000E-07

8 – CONCLUSIONS

In this paper we studied the problem analysis of insulated and buried and their neighborhood. From the standpoint of mathematical and computational some original contributions were given, and listed below :

a) Contour elements with special geometries (cylindrical, flat and discoid sector);

b) Analysis of soil-pipeline interaction with the accurate representation of the laminated pipe surface;

9 – ACKNOWLEDGMENTS

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