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DYNAMIC STABILITY ANALYSIS OF AEROVISCOELASTIC SYSTEMS

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Abstract. : *Viscoelastic materials have been extensively used for the purpose of passive vibration mitigation in various types of mechanical systems. In this paper, the authors investigate the use of those materials in aeroelastic systems and evaluate the influence of the viscoelastic behavior on flutter speeds. Viscoelastic elements are introduced to replace traditional elastic springs associated to heave and pitch motions of a typical section model. The equations of motion of the aeroelastic system are modified to account for the dependence of the viscoelastic behavior on frequency and temperature, by using the concepts of complex modulus and shift factor. The aerodynamic forces and moments are modeled by using Theodorsen's method. The results of numerical simulations enable to assess the influence of the addition of viscoelastic elements to the airfoil support and the influence of temperature on flutter speed.*

Keywords: *Viscoelasticity; Damping; Aeroelasticity; Flutter.*

1. INTRODUCTION

Viscoelastic materials have frequently been used for the mitigation of vibrations in various types of mechanical systems, such as buildings, automobiles, airplanes and industrial equipment (Samali and Kwok, 1995; Rao, 2001). The main advantages provided by such strategy are inherent stability, typical of passive control approaches, and moderate installation and maintenance costs. However, the viscoelastic behavior entails some modeling difficulties, especially due to the dependence of the dynamic behavior of those materials with respect to a number of operational and environmental factors, among which the vibration frequency and temperature are the most relevant (Jones, 2001). As a result, evolved models and specialized experimental procedures are necessary for the accurate prediction and characterization of dynamic systems containing viscoelastic materials.

On the other hand, techniques of aeroelastic control have received a great deal of attention from the aerospace community in the quest for increased safety margins and improved operational and cost effectiveness. Most of the reported studies have addressed active approaches (Song, Feng-Ming, 2012). Surprisingly enough, few works have investigated the use of viscoelastic materials as a passive strategy of aeroelastic control.

In this paper, the influence of viscoelastic damping on flutter speeds of is addressed. For this purpose, a typical section model of two degrees-of-freedom (pitch and heave motions), containing viscoelastic resilient elements is proposed. Based on the concept of complex modulus and reduced frequency, these elements are modeled as frequency- and temperature-dependent complex translational and rotational springs. Using Theodorsen's aerodynamic model, the equations of motion of the **aeroviscoelastic** system are derived. Stability analysis is performed using the classical K-Method coupled with an optimization routine. The simulation results enable to verify the possibility of increasing the flutter velocity by the inclusion of viscoelastic elements.

2. MATHEMATICAL MODELING

2.1 Derivation of the aeroelastic model

A typical section model with two degrees-of-freedom, h (heave, positive downwards) and α (pitch, positive clockwise) is assumed as illustrated in Fig. 2. Also indicated is the aerodynamic center *AC*, located at one-quarter of the chord. The positions of the elastic axis *EC* and the center of mass *MC* are identified, respectively, by the nondimensional parameters *a* and *e*, multiplied by semichord *b*. Moreover, k_h e k_α designate the translational and rotational rigidities of the elastic suspension and U_∞ designates the airflow velocity.

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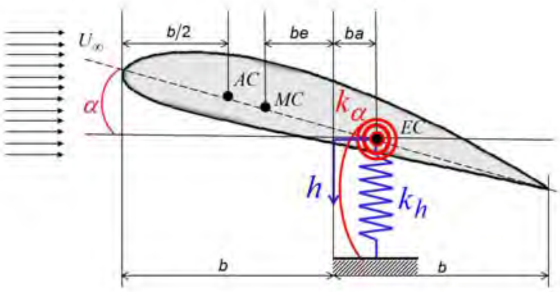

Figure 1 - Typical section aeroelastic model of two degrees-of-freedom

Under these conditions, the equations of motion of the aeroelastic system are given as:

$$
\begin{aligned}
&m_a\ddot{h}+S_\alpha\ddot{\alpha}+k_h h=-L\\
&I_\alpha\ddot{\alpha}+S_\alpha\ddot{h}+k_\alpha\alpha=M_\alpha
\end{aligned}
\tag{1}
$$

where m_a is the mass of airfoil; S_α and I_α are the static moment and the moment of inertia of the airfoil with respect to elastic axis. Moreover, L is the lift and M is the aerodynamic moment, both generated by the airflow. Assuming unsteady flow, the expressions for these aerodynamic loads are:

$$
\begin{aligned}
&L=\pi\rho b^2\left[\ddot{h}+U\dot{\alpha}-ba\ddot{\alpha}\right]+2\pi\rho UbC(k)\left[\dot{h}+U\alpha+b\left(\frac{1}{2}-a\right)\dot{\alpha}\right]\\
&M=\pi\rho b^2\left[ba\ddot{h}-Ub\left(\frac{1}{2}-a\right)\dot{\alpha}-b^2\left(\frac{1}{8}+a^2\right)\ddot{\alpha}\right]+2\pi\rho Ub^2\left(a+\frac{1}{2}\right)C(k)\left[\dot{h}+U\alpha+b\left(\frac{1}{2}-a\right)\dot{\alpha}\right]
\end{aligned}
\tag{2}
$$

where the Theodorsen Function $C(k)$ is a combination of Bessel functions of first and second types given by (Bisplinghoff, 1996; Fung, 1969):

$$
\begin{aligned}
&C(k)=1-\frac{0.165}{1-\frac{0.045}{k}i}-\frac{0.335}{1-\frac{0.30}{k}i}, \qquad k\le 0,5,\\
&C(k)=1-\frac{0.165}{1-\frac{0.041}{k}i}-\frac{0.335}{1-\frac{0.32}{k}i}, \qquad k> 0,5.
\end{aligned}
\tag{3}
$$

In the equations above, the parameter k indicates the reduced frequency, defined as:

$$
k=\frac{\omega b}{U}. \tag{4}
$$

A classical flutter analysis is conducted by assuming harmonic motions with frequency ω, as follows:

$$
\begin{aligned}
&h(t)=\bar{h}e^{i\omega t},\\
&\alpha(t)=\bar{\alpha}e^{i\omega t}.
\end{aligned}
\tag{5}
$$

As a consequence, the aerodynamic loads are also harmonic with the same frequency:

$$
\begin{aligned}
&L(t)=\bar{L}e^{i\omega t},\\
&M(t)=\bar{M}e^{i\omega t}.
\end{aligned}
\tag{6}
$$

Associating Eqs. (1) to (6), one obtains the following eigenvalue problem:

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$$\left[\mathbf{K}-\omega^2\left[\mathbf{M}+\frac{1}{\mu}\mathbf{A}\right]\right]\begin{Bmatrix}\frac{\bar{h}}{b}\\ \bar{\alpha}\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix} \tag{7}$$

where the stiffness, mass, and aerodynamic matrices are given, respectively, as:

$$\mathbf{K}=\begin{bmatrix}\omega_h^2 & 0\\ 0 & r_\alpha^2\omega_\alpha^2\end{bmatrix} \tag{8}$$

$$\mathbf{M}=\begin{bmatrix}1 & x_\alpha\\ x_\alpha & r_\alpha^2\end{bmatrix} \tag{9}$$

$$\mathbf{A}=\begin{bmatrix}L_h & L_\alpha-\left(\frac{1}{2}+a\right)L_h\\ M_h-\left(\frac{1}{2}+a\right)L_h & M_\alpha-\left(\frac{1}{2}+a\right)\left(L_\alpha+M_h\right)+\left(\frac{1}{2}+a\right)^2 L_h\end{bmatrix} \tag{10}$$

and the coefficients L_h, L_α, M_h and M_α represent the aerodynamical contribution and are given as:

$$\begin{aligned}L_h &= 1-\frac{2i}{k}C(k)\\ L_\alpha &= \frac{1}{2}-1i\frac{\left(1+2C(k)\right)}{k}-\frac{2C(k)}{k^2}\\ M_h &= \frac{1}{2}\\ M_\alpha &= \frac{3}{8}-\frac{1i}{k}\end{aligned} \tag{11}$$

For better visualization, Eq. (7) is expanded into the form:

$$\begin{bmatrix}\mu\left(1-\frac{\omega_h^2}{\omega^2}\right)+L_h & \mu x_\alpha+L_\alpha-\left(\frac{1}{2}+a\right)L_h\\ \mu x_\alpha+M_h-\left(\frac{1}{2}+a\right)L_h & \mu r_\alpha^2\left(1-\frac{\omega_\alpha^2}{\omega^2}\right)+M_\alpha-\left(\frac{1}{2}+a\right)\left(L_\alpha+M_h\right)+\left(\frac{1}{2}+a\right)^2 L_h\end{bmatrix}\begin{Bmatrix}\frac{\bar{h}}{b}\\ \bar{\alpha}\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}, \tag{12}$$

where the dimensionless parameters are:

$$r_\alpha^2=\frac{I_\alpha}{m_a b^2} \qquad x_\alpha=\frac{S_\alpha}{b m_a} \qquad \mu=\frac{m}{\pi\rho_\infty b^2}$$

$$\omega_h^2=\frac{k_h}{m_a} \qquad \omega_\alpha^2=\frac{k_\alpha}{I_\alpha}$$

From the characteristic equation

$$\det\left(\mathbf{K}-\omega^2\left(\mathbf{M}+\frac{1}{\mu}\mathbf{A}\right)\right)=0, \tag{13}$$

the stability analysis can be performed based on the examination of the complex eigenvalues.

2.2 Inclusion of viscoelastic damping

Viscoelastic damping is introduced into the aeroelastic model by including viscoelastic resilient elements to the suspension of the airfoil; those elements take the form of translational and rotational mounts, such as those illustrated in Fig. 3, in which the viscoelastic layer is deformed in pure shear.

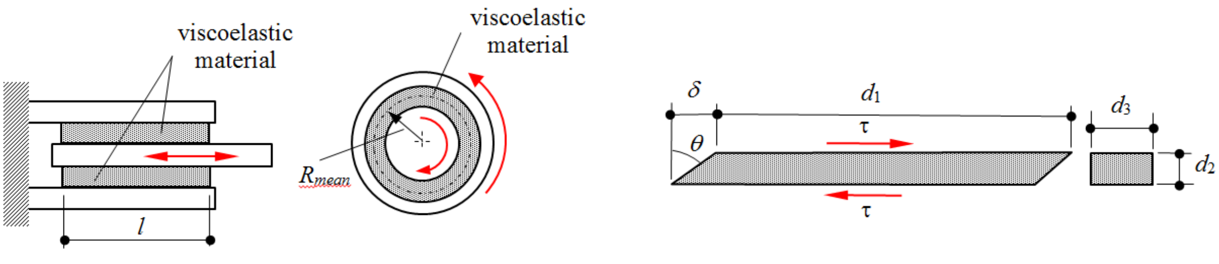

Figure 3 – (a): translational viscoelastic mount; (b) rotational viscoelastic mount; (c) deformation of the rectified viscoelastic layer

Assuming linear viscoelastic behavior, the relation between the shear stress τ and the shear angle θ in the viscoelastic layers is:

$$\tau = G_v(T,\omega)\theta. \tag{14}$$

where $G_v(T,\omega)$ is the complex shear modulus of the viscoelastic material, which will be discussed in the next sub-section.

Considering the dimensions of the viscoelastic layer shown in Fig. 3(c), the equivalent complex, frequency-and temperature-dependent viscoelastic stiffness of the translational and rotational mounts are, respectively:

$$k_{hv} = p_h G_v(T,\omega) \tag{15.a}$$

$$k_{h\alpha} = p_\alpha G_v(T,\omega) \tag{15.b}$$

where:

$$p_h = \frac{d_1 d_3}{d_2} = 2l\frac{d_3}{d_2}, \tag{16.a}$$

$$p_\alpha = \frac{d_1 d_3}{d_2} = 2\pi R_{mean}\frac{d_3}{d_2}, \tag{16.b}$$

In Eqs. (16), l is the length of the viscoelastic layer of the translational mount and R_{mean} is the mean radius of the viscoelastic layer of the rotational mount.

Associating Eqs. (14) to Eqs. (11) and (12), one obtains the following *nonlinear eigenvalue problem*, involving a frequency- and temperature-dependent stiffness matrix, which must be solved for performing stability analysis.

$$\left[\mathbf{K}(\omega,T) - \omega^2\left[\mathbf{M} + \frac{1}{\mu}\mathbf{A}\right]\right]\begin{Bmatrix}\frac{\bar{h}}{b} \\ \bar{\alpha}\end{Bmatrix} = \begin{Bmatrix}0 \\ 0\end{Bmatrix} \tag{17}$$

2.3. Computation of flutter speeds

In the present study, flutter predictions have been made by using the well-known K-Method (Cooper and Wrigth Jr., 2007, Pierce and Hodges, 2002), according to which the stiffness matrix is modified according to:

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$$\mathbf{K}^* = \mathbf{K}(1+ig). \tag{18}$$

where g is a parameter which simulate the structural damping.

The K-method has been adapted for the aeroviscoelastic problem, comprising the following steps:

1) Define the geometric parameters $\omega_\alpha, \omega_h, r_\alpha, \mu, a, e, b$.
2) Calculate $x_\alpha, m_a, k_h, k_\alpha, I_\alpha$.
3) Insert the viscoelastic associated parameters p_α, p_h, T.
4) Choose a value for the reduced frequency k;
5) Calculate the Theodorsen Function C(k) and the aerodynamics coefficients $L_h, L_\alpha, M_h, M_\alpha$.
6) Solve the eigenvalue problem for the undamped case and calculate the values of ω_i and associated g_i.
7) Calculate the velocity associated to *k*.
8) Calculate the complex viscoelastic modulus.
9) Insert complex modulus in the dynamic matrix and compute the characteristic equation $F(\omega, g)$.
10) Use an optimization algorithm to compute the eigenvalues, which are the roots of $F(\omega, g)$.
11) Repeat the steps 4 to10 for increasing values of the reduced frequencies.
12) Plot the values of damping factors and natural frequencies as functions of the flow velocity.

It is a good practice to evaluate the residuals associated to the optimization solutions obtained in Step 10.

3. VISCOELASTIC MATERIALS

The distinguishing feature of viscoelastic materials is that their mechanical behavior can be understood as the combination of pure elastic behavior, typical of elastic solids, and the viscous behavior of Newtonian fluids. As a result, those materials present memory effect. Moreover, when subjected to harmonic loads, viscoelastic materials exhibit hysteresis, which leads to energy dissipation. It is also observed that the harmonic behavior depends on the excitation frequency ω and more strongly on the temperature *T*.

Various models have been developed for describing the mechanical behavior of viscoelastic materials both in time and frequency domains (de Lima, 2007). Among them, the most frequently used is the *complex modulus approach*, which defines frequency-dependent complex moduli as follows:

$$G_v(\omega,T) = G'(\omega,T) + iG''(\omega,T) \tag{19}$$

or, alternatively,

$$G_v(\omega,T) = G'(\omega,T)[1 + i\eta(\omega,T)], \tag{20}$$

where $G'(\omega,T)$ is the storage modulus, $G''(\omega,T)$ is the loss modulus, and η is the loss factor. This later quantifies the capability of the viscoelastic material to dissipate energy.

Based on the experimental evidence that the influence of an increase in temperature is equivalent to a decrease in the excitation frequency, the Frequency-Temperature Equivalence Principle can been evoked (Lakes, 2009, Jones, 2001) to account for the influence of temperature on the viscoelastic behavior. According to this principle, the value of the modulus at any frequency ω and temperature *T* is equal to the value of the modulus at a shifted value of frequency, ω_r, known as *reduced frequency*, at prescribed reference value of the temperature, T_z, i.e.,

$$G_v(\omega,T) = G_v(\omega_r, T_z) \tag{21}$$

where $\omega_r = \alpha_T(T)\omega$.

In the equation above, $\alpha_T(T)$ is known as the shift factor, which must be identified from experimental tests for each viscoelastic material. Specifically for the case of the material ISD112™, produced by 3M®, which has been used in the present study, the model used to represent the complex moduli has been proposed by (Drake, Soovere, 1984), as follows:

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$$G_v(\omega_r, T_z) = B_1 + \frac{B_2}{1 + B_5\left(\frac{f_r i}{B_3}\right)^{-B_6} + \left(\frac{f_r i}{B_3}\right)^{-B_4}}; \tag{22}$$

$$\log_{10} \alpha_T = a\left(\frac{1}{T} - \frac{1}{T_z}\right) + 2{,}303\left(\frac{2a}{T_z} - b\right)\log_{10}\left(\frac{T}{T_z}\right) + \left(\frac{b}{T_z} - \frac{a}{T_z^{\,2}} - S_{AZ}\right)(T - T_z) \tag{23}$$

where:

$$a = (D_B C_C - C_B D_C)/D_E \qquad b = (C_A D_C - D_A C_C)/D_E$$

$$C_A = \left(\frac{1}{T_L} - \frac{1}{T_z}\right)^2 \qquad D_A = \left(\frac{1}{T_H} - \frac{1}{T_z}\right)^2 \qquad C_B = \frac{1}{T_L} - \frac{1}{T_z} \qquad D_B = \frac{1}{T_H} - \frac{1}{T_z}$$

$$C_C = S_{AL} - S_{AO} \qquad D_C = S_{AH} - S_{AO} \qquad D_E = D_B C_A - C_B D_A$$

The values of the coefficients appearing in the equations above are presented in Tab 1.

In Fig. 4(a), the real and imaginary parts of complex modulus, as well as the loss factor, are depicted as functions of the reduced frequency, while the curve for the shift factor is shown in Fig. 4(b).

In Fig. 5 the surface representing the loss factor as a function of temperature and frequency is presented.

Table 1 - Parameters for viscoelastic Material ISD112™.

Parameter	Value	Parameter	Value
$B_1\,[MPa]$	0.4307	$T_z\,[K]$	290
$B_2\,[MPa]$	1200	$T_L\,[K]$	210
$B_3\,[MHz]$	1.5403	$T_H\,[K]$	360
B_4	0.6847	$S_{A0}\,[1/K]$	0.05956
B_5	3.241	$S_{AL}\,[1/K]$	0.1474
B_6	0.180	$S_{AH}\,[1/K]$	0.009725

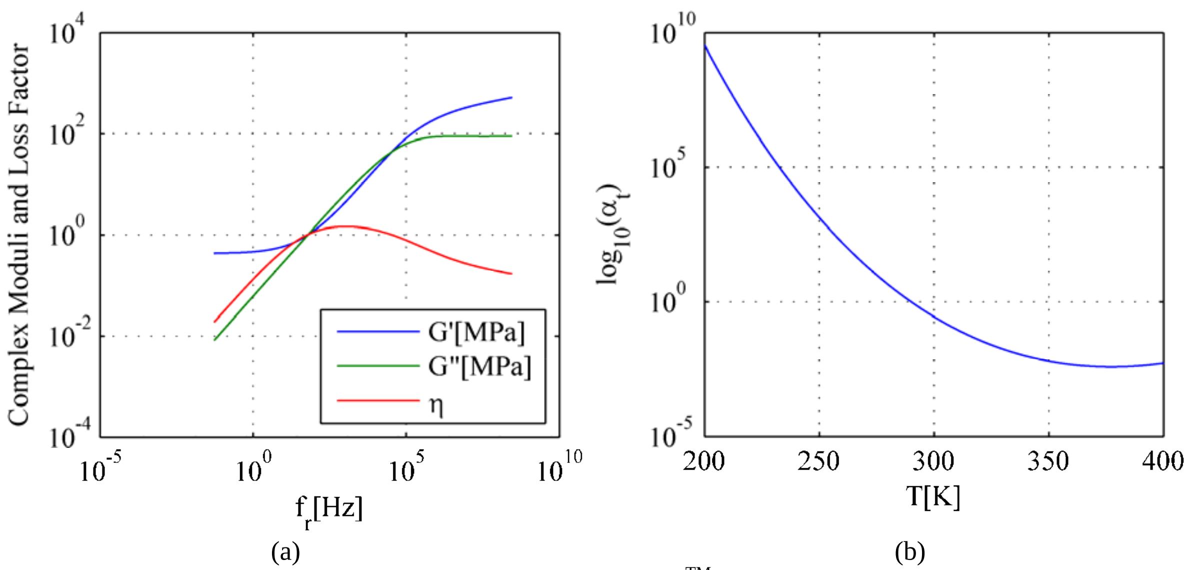

Figure 4 – Properties of the viscoelastic material ISD112™. (a) Complex moduli; (b) Shift Factor.

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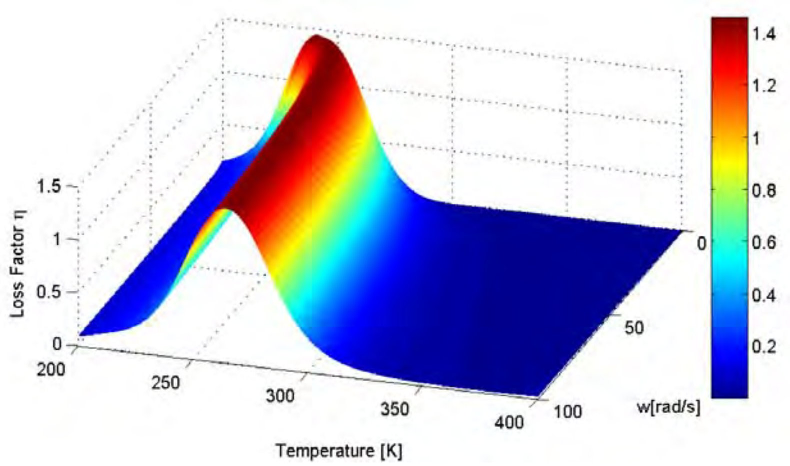

Figure 5 – Loss factor as function of temperature and frequency for ISD112™.

4. NUMERICAL SIMULATIONS

Based on the formulation presented in the previous sections, numerical simulations have been performed aiming at evaluating the influence of the viscoelastic dampers on the flutter speeds of a typical section model. For this purpose, the values of the parameters of the aeroviscoelastic system provided in Tab 2 have been used for the along with a MATLAB® computer code in which the steps presented in the sub-section 2.3 have been implemented.

Table 2 – Physical and geometric properties of typical section aeroviscoelastic model

Parameter	Value
Mass ratio (μ)	76
Semichord (b)	0.15 m
Natural frequency of heave (ω_h)	55 rad/s
Natural frequency of pitch (ω_α)	65 rad/s
Radius of gyration (r)	$\sqrt{0,5}\, Kg/m^2$
Air density (ρ_∞)	1.225 kg/m³
Dimensionless coefficient of position of mass center (e)	–0.1
Dimensionless coefficient of position of elastic center (a)	–0.2
Operation Temperature (T)	300 K
Geometric coefficient of viscoelastic spring in heave (p_h)	$1\times10^{-4} m$
Geometric coefficient of viscoelastic spring in pitch (p_α)	$1\times10^{-4} m$

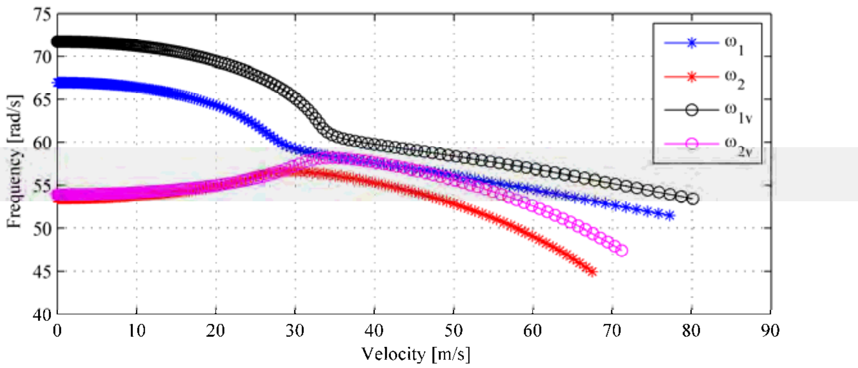

Figure 6 – Natural frequencies. Comparison between aeroelastic and aeroviscoelastic systems.

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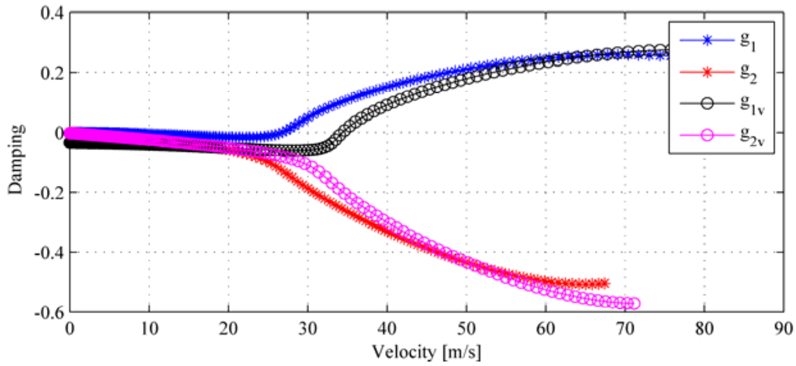

Figure 7 – Damping. Comparison between aeroelastic and aeroviscoelastic systems.

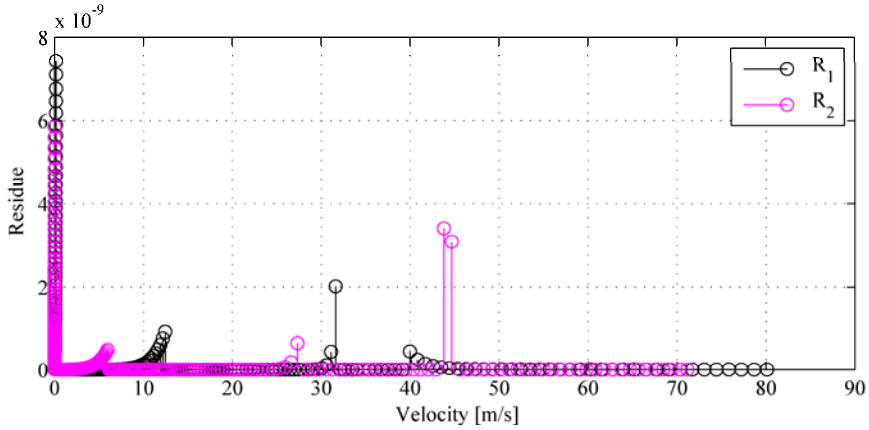

Figure 8 – Residuals of the optimization process.

As can be seen in Figs. 6 and 7, the flutter speed for the undamped system was found to be 27.33 m/s, while for the aeroviscoelastic model, the instability condition was achieved for a flow speed of 36.64 m/s.

For illustration, Figure 8 shows the residuals associated to the optimization process. As observed, these residuals are satisfactorily small, indicating that the roots of the characteristic equation have been correctly calculated.

In figures 9 and 10 one can assess the influence of temperature on the aeroviscoelastic behavior. Table 3 summarizes the values of the flutter speed for each value of temperature considered. It can be seen that the flutter speed monotonically decreases as temperature increases, exhibiting an asymptotical trend for higher temperature values. This trend is confirmed in Fig. 11, in which a larger number of temperature values have been considered.

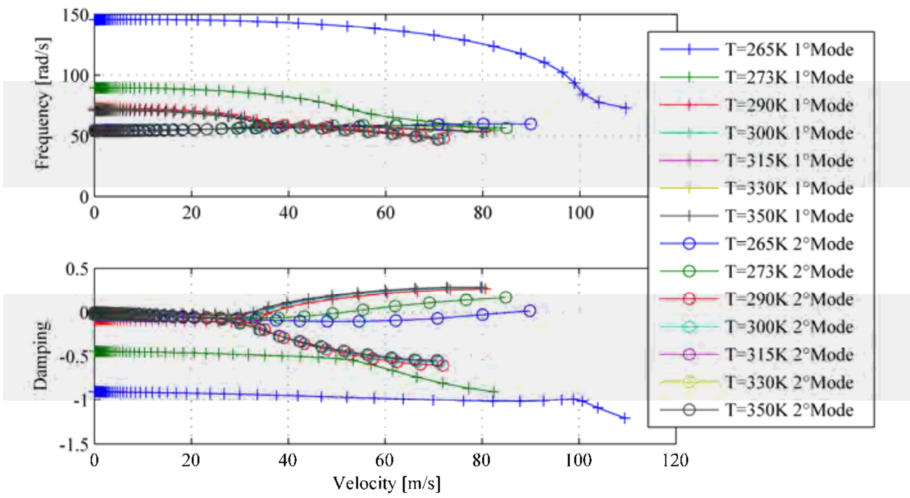

Figure 9 – Aeroviscoelastic system response for various temperature values

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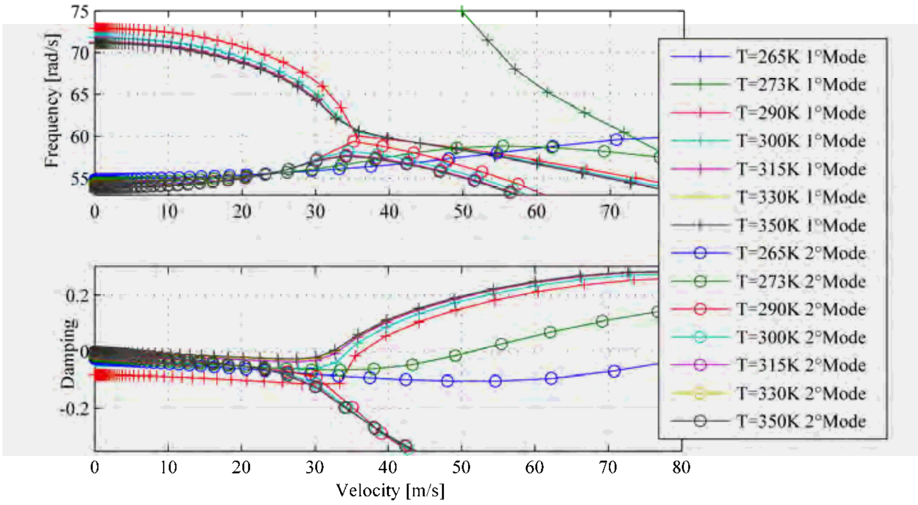

Figure 10 – Zoomed view of Fig. 9.

Table 3 – Temperatures associated to flutter speed.

Temperature [K]	Flutter Velocity [m/s]
265	-
273	51.81
290	36.56
300	34.64
315	33.48
330	32.92
350	32.39

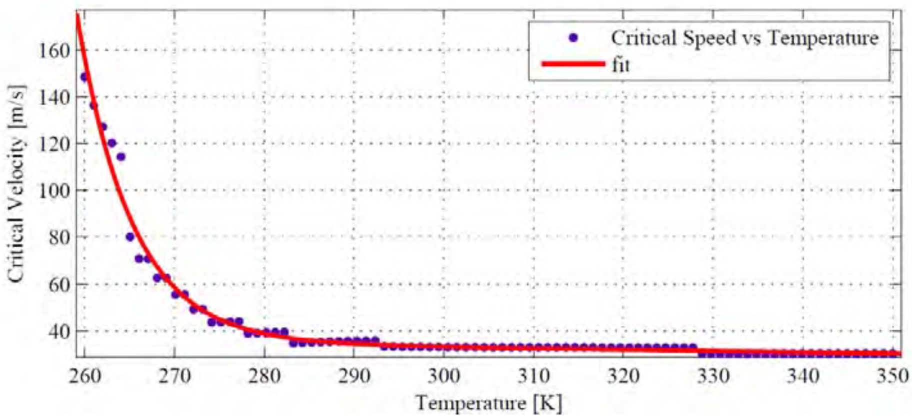

Figure 2 – Flutter velocity of the aeroviscoelastic system as function of temperature.

5. CONCLUSIONS

In this work the influence of the viscoelastic behavior associated to the resilient elements of a typical section aeroelastic model has been appraised. A numerical procedure has been suggested to compute the flutter speeds, accounting for the dependency of the viscoelastic material properties on frequency. Also, the modeling procedure enables to account for the influence of temperature on the aeroviscoelastic behavior. The results of numerical simulations lead to conclude that the inclusion of viscoelastic dampers do entails increased flutter speeds. Hence, the use of viscoelastic materials can be considered as valuable strategy for passive flutter control. However, the improvement of the stability range depends heavily on temperature and this issue must be properly accounted for.

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6. ACKNOWLEDGEMENTS

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