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FIREFLY ALGORITHM VARIATIONS FOR THE SOLUTION OF AN INVERSE RADIATIVE TRANSFER PROBLEM

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Abstract*.The study of inverse problems applied to radiative transfer has been the subject of numerous studies aiming at many applications ranging from medicine to industry, allowing, among others, for non-intrusive tests. In general, the formulation for the parameter estimation inverse problems in radiative heat transfer is achieved through the minimization of an objective function associated with the problem being analyzed. Stochastic optimization methods, usually inspired by nature behaviors, have been shown effective in recovering the global minimum, even for complex functions, given a sufficiently high number of iterations. This work is aimed at the investigation of a new heuristic, known as the Firefly Algorithm (FA), in the minimization procedure of the defined objective function. The inverse problem solution is also carried out for some new algorithms derived from the FA and proposed in this work. One of them, is the insertion of the technique known as the Opposition-Based Learning (OBL). Another variation implemented is the insertion of a fuzzy concept together with the definition of the center of mass of the firefly swarm. Each algorithm was tested in 30 runs, and the results obtained are critically compared. For the test case implemented, the fuzzy technique with the definition of the center of mass of the firefly swarm improved the FA canonical algorithm.*

Keywords: *optimization, firefly algorithm, opposition-based learning, inverse problem, radiative transfer.*

1. INTRODUCTION

Inverse problems are generally implicity formulated (Silva Neto, 2005), with the definition of an objective function, for example given by the sum of the squared residues between the experimental data and the predicted values from the direct problem solution. In this context, the main task becomes the minimization of the objective function.

Although gradient based methods are still largely employed, they are very sensitive to the choice of the initial guess, since many local minima may exist in those objective functions. Alternatively, it has been observed an increasing interest in the use of stochastic methods, which are likely to find the global minimum despite the initial guess, or the initial population. Those stochastic optimization methods, usually inspired by nature behaviors, have been demonstrated to be effective in recovering the global minimum (Silva Neto, 2009), even for complex functions, given a sufficiently high number of iterations. Even though such high number of iterations can be considerably computer intensive, the continuous development of computers hardware allowed for the development of new engineering models and faster computing, and it can be observed an increasing interest and use of stochastic methods.

Inverse problems in radiative transfer have many applications, ranging from medicine to industry (Arridge, 1999), allowing, among other issues, for non-intrusive tests (Oliva et al, 2004). Our research group has been publishing plenty of works on inverse radiative transfer problems aimed at the study of different optimization stochastic methods as well as proposing new hybrid and variations of the canonical algorithms. Just to mention a few, Knupp et al.(2009) investigated the Particle Collision Algorithm and the Luus-Jaakola method, proposing a hybrid approach, Becceneri el al. (2006) demonstrated the feasibility of using the Particle Swarm Optimization technique in identifying radiative properties, and many other heuristics, such as the Simulated Annealing, Genetic Algorithms, Ant Colony Optimization,

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Generalized Extremal Optimization and Differential Evolution, have been investigated for the solution of inverse radiative transfer problems and are compiled in (Silva Neto and Becceneri, 2009).

This work is aimed at the investigation of a recent heuristic, recently proposed by (Yang, 2008), known as the Firefly Algorithm (FA), for the solution of a radiative transfer problem, where the space-dependent single scattering albedo must be estimated (Stephany et al, 2010). It is also considered the simultaneous estimation of the optical thickness (Knupp, 2011). The inverse problem solution is also carried out for some new algorithms derived from the FA and proposed in this work. One of them, is the insertion of the technique known as the Opposition-Based Learning (OBL) (Tizhoosh, 2005). Another variation implemented is the insertion of a fuzzy concept together with the definition of the center of mass of the firefly swarm (Gandomi, 2012).

2. MATHEMATICAL FORMULATION AND SOLUTION OF THE DIRECT PROBLEM

For this work, it is considered a one-dimensional medium, gray, heterogeneous, isotropic scattering, with optical thickness τ_0, and with transparent boundary surfaces. The boundaries at $\tau = 0$ and $\tau = \tau_0$ are subjected to the incidence of isotropic radiation with intensities given by A_1 and A_2, respectively, as illustrated schematically in Fig. 1, where $\rho_1 = \rho_2 = 0$.

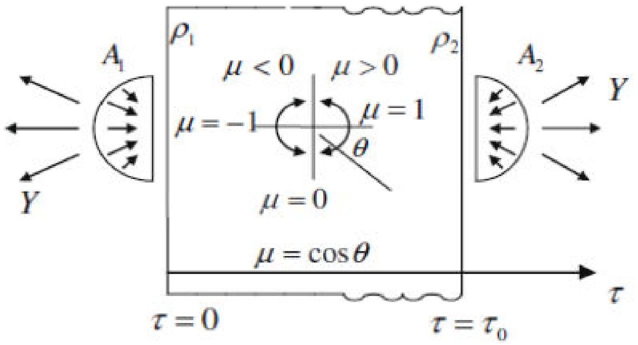

Figure 1: Schematic representation of a participating media exposed to external radiation sources.

Consider a one-dimensional, gray, heterogeneous, isotropically scattering participating medium of optical thickness τ_0 and transparent boundary surfaces as shown in Fig. 1. These boundaries at $\tau = 0$ and $\tau = \tau_0$ reflect diffusely the radiation that comes from the interior of the medium and are subjected to the incidence of radiation originated at external sources with intensities A_1 and A_2, respectively. The mathematical model for the interaction of the radiation with the participating medium is given by the linear version of the Boltzmann equation, which for the case of azymuthal symmetry and a space-dependent albedo is written in the dimensionless form as:

$$\mu \frac{\partial I(\tau,\mu)}{\partial \tau} + I(\tau,\mu) = \frac{\omega(\tau)}{2} \int_{-1}^{1} I(\tau,\mu')d\mu' \ , \ 0 < \tau < \tau_0, \ -1 \leq \mu \leq 1 \quad (1a)$$

$$I(0,\mu) = A_1, \mu > 0, \ I(\tau_0,\mu) = A_2, \mu < 0 \quad (1b)$$

where τ is the optical variable, I is the radiation intensity, μ is the polar angle cosine and $\omega(\tau)$ is the scattering albedo, which is represented as the following expansion:

$$\omega(\tau) = \sum_{k=0}^{K} D_k \tau^k \quad (2)$$

When the geometry, the radiative properties and the boundary conditions are known, problem (1) may be solved and the radiative intensity I is determined for all discrete points in the spatial and angular domains, that is, $0 \leq \tau \leq \tau_0$ and $-1 \leq \mu \leq 1$. In order to solve the direct problem we have used Chandrasekhar´s discrete ordinates method

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(Chandrasekhar, 1960), in which the polar angle domain is discretized, and the integral term on the right hand side of Eq. (1a) is replaced by a Gaussian quadrature. We then used a finite-difference approximation for the terms on the left hand side of Eq. (1a), and by performing forward and backward sweeps, from $\tau = 0$ to $\tau = \tau_0$ and from $\tau = \tau_0$ to $\tau = 0$, respectively. $I(\tau, \mu)$ is determined for all spatial and angular nodes of the discretized computational domain. This is the direct problem. When the radiative properties or the boundary conditions are unknown, but experimental data may be obtained, the unknown parameters can be estimated through an inverse problem approach.

3. MATHEMATICAL FORMULATION AND SOLUTION OF THE INVERSE PROBLEM

For the problem under consideration, given by Eqs. (1) and schematically represented in Fig. 1, the optical thickness, τ_0 , and the space-dependent single scattering albedo, $\omega(\tau)$, are considered unknown, and the inverse problem approach is employed to estimates for τ_0 and the $\omega(\tau)$. In order to estimate the latter, the space-dependent function is considered to be well described by the polynomial expansion given by Eq. (2), and the coefficients, D_k, $k = 0,1,2,..., K$, are estimated in order to recover the original function. Therefore, we have the following parameters to be estimated:

$$\vec{Z} = (\tau_0, D_0, D_1, \dots, D_K) \tag{3}$$

Consider that experimental data are available at both boundaries of the spatial domain, acquired by external detectors, at different polar angles. Therefore, there are N_d available experimental data, Y_i, $i = 1, 2, 3,..., N_d$. This inverse problem can be formulated as an optimization problem, where the main task becomes the minimization of an objective function given by the sum of the squared residues between the experimental data and the predicted values from the direct problem solution.

$$Q(\vec{Z}) = \sum_{k=1}^{N_d} \left[I_i(\vec{Z}) - Y_i\right]^2 = \vec{R}^T \vec{R} \tag{4}$$

In the present work, real experimental data are not available and the experimental intensities, Y_i , are simulated with the direct problem solution using the exact values for the parameters, which are obviously not known in a real application. In order to simulate the experimental fluctuations, random numbers from a normal distribution with zero mean and unitary standard deviation, s, are added to those values:

$$Y_i = I_i(\vec{Z}_{exact}) + \sigma_e \cdot s \tag{5}$$

where σ_e simulates the standard deviation of the experimental error.

To minimize the objective function given by Eq. (4), it is employed the Firefly algorithm, which is described below in details. In order to verify its performance for this inverse problem solution, other variations are developed from the canonical algorithm, using the technique known as the Opposition-Based Learning (OBL). The other variations implemented regard the insertion of a fuzzy concept together with the definition of the center of mass of the firefly swarm.

4. FIREFLY ALGORITHM - FA

There are two characteristics that must be pointed out in order to a better understanding of the Firefly algorithm (FA) (Yang, 2008; Luz et al, 2009):

a) how the variation of light intensity is perceived by the firefly;
b) how the attractiveness between the fireflies is modeled.

The firefly light intensity emission is proportional to the encoded objective function, but the light intensity perceived by the firefly decreases as the distance between the fireflies increases. Therefore, the perceived intensity of a firefly is given by $J(r) = J_0 e^{-\gamma r^2}$, where J_0 is the original light intensity, r is the Euclidean distance between the fireflies i and j, where the j firefly is brighter than the i firefly. γ is a fixed light absorption coefficient. Therefore, the attractiveness β of a firefly can be formulated as:

$$\beta = \beta_0 e^{-\gamma r^2} \tag{6}$$

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where β_0 is the attractiveness for the distance *r* = *0*, and can be fixed as $\beta_0 = 1$. So, the movement of a firefly *i* in the direction of a brighter firefly *j* is determined by:

$$x_i^t = x_i^{t-1} + \beta\left(x_j^{t-1} - x_i^{t-1}\right) + \alpha\left(rand - \frac{1}{2}\right) \tag{7}$$

where the second term is due to the attraction while the third term is randomization with α being the randomization parameter $rand$ a random number simulated from the uniform distribution in the interval [0, 1]. The pseudo code of the Firefly algorithm is given in Fig. 2.

```
Begin
 Objective function f(x),   x = (x_1, ..., x_d)^T
Generate initial population of fireflies x_i   (i = 1, 2, ... n)
Light intensity I_i  at  x_i is determined by  f(x_i)
Define light absorption coefficient  γ
while (t < MaxGeneration)
for I = 1 : n       all n fireflies
  for j =1 : d     loop over all d dimensions
    if (I_j > I_i  ),  Move firefly i towards j; end if
   Attractiveness varies with distance r via  e^(-γr)
   Evaluate new solutions and update light intensity
 End for j
End for i
Rank the fireflies and find the current best
End while
Postprocess results and visualization
End
```

Figure 2: Pseudo code of the Firefly algorithm (FA) (Yang, 2008)

5. OPPOSITION-BASED LEARNING - OBL

This technique was first proposed by Tizhoosh (2005). In that work, it is presented an idea of the creation of a new particle which is positioned at the opposite location of the original one. In their work, the authors believe that it may be better to choose a new particle from an existing one than choosing it from a random process. Therefore, from an particle *x* in the domain $[a_1, b_1]$, the new particle choice process, must follow the expression: *xo*= a_1+ b_1-*x*. The opposite *xo* is always calculated from the generated *x*. Based on the proximity of the particle and its opposite, the search range can be reduced recursively until the estimate or its opposite be very close to the solution (Fig. 3).

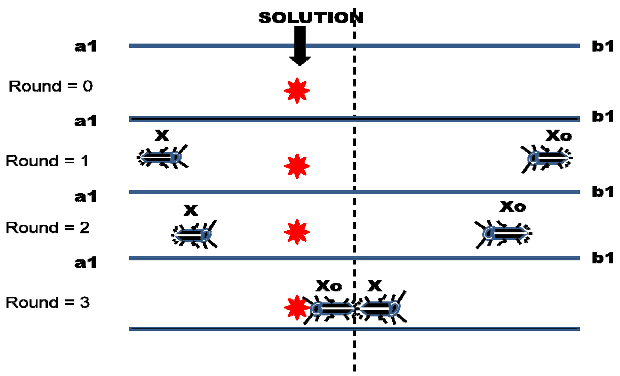

Figure 3: OBL application for solving a one-dimensional equation and optimality estimating x and opposite-estimate xo (Tizhoosh, 2005)

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6. FIREFLY FUZZY (FAz)

The main idea behind this approach was introduced by Becceneri *et al.* (2008). Working with the Ant Colony Optimization algorithm the authors implemented a process where the best ant in a run receives more pheromones than the others. The pheromone delivery decreases as the ant distance from the best path increases. Following the same principle, we established circular zones where the brightest firefly of a run is in the center. Then, the movement of the fireflies are calculated as usual. We established three circular zones where the light intensity of a firefly is compared with the light intensity of the brightest one in the run. Depending of that zone, the firefly will receive a different light absorption coefficient, γ and a randomization parameter α. The chosen zone values are: 0.2, 0.5, 0.8 and 1.0. We also vary the randomization parameter α as follows: 0.02, 0.05, 0,1 and 0.2. This will result in different attractiveness β as in Eq. (6). These ranges were chosen based on Yang (2008).

7. FIREFLY FUZZY WITH CENTER OF MASS (FAcom)

This approach was initially developed by Gandomi (2012), and is based on the idea that some animal species have the ability to form large swarms. This ability seems to be linked with enhanced reproduction, protection from predators, and environmental conditions. The authors demonstrated that a density-dependent attraction of the individuals and location of food are used as objectives which finally lead the population to achieve the global minimum. Following the above strategy, it has been implemented the concept of center of mass, and the firefly swarm moves are based on the position of that center. The center of mass is calculated based on the objective function evaluation, and is calculated as follows:

$$x_d = \frac{\sum_{l=1}^{N_p} \frac{1}{Q(\vec{x_l})} x_{d,l}}{\sum_{l=1}^{N_p} \frac{1}{Q(\vec{x_l})}} \tag{8}$$

where d is the dimension of the function, N_p is the population and Q the objective function.

In this variation, for the fuzzy intensification, instead of using the best firefly of each run as the center for pheromones distribution, a dummy firefly located at the center of mass is used. The light absorption coefficient γ and the randomization parameter α change as the distance of a firefly from the center of mass increases, as shown above in the fuzzy variant.

8. RESULTS AND DISCUSSION

In the results presented for the selected test case below, it has been used σ_e =0.002 in order to simulate the measuring error in the simulation of the experimental data, resulting in errors up to 3.5%. The chosen problem for the test case has an unitary optical thickness, that is, $\tau_0 = 1$, and a quadratic polynomial has been used to describe the spatial variation of the single scattering albedo. The external sources are considered to be $A_1 = 1$ and $A_2 = 0$.

For the inverse problem solution, it is considered three terms in the expansion shown in Eq. (2), that is, K = 3. Therefore, for the case under investigation, we have the following exact values of the parameters to be estimated: τ_0=1; D_0=1; D_1=-1.4; and D_2=0.6. On the optimization problem solution, it´s considered the following parameters for all tests: population number (population of fireflies) = 40, number of generations (MaxGeneration) = 200. The light absorption coefficient γ may, originally, vary from 0.1 to 10.The randomization parameter α may vary from 0.1 to 1.0 as well.

For each optimization method 30 runs were performed with random initial populations, always employing the same experimental data set. Since the methods are stochastic, it is expected that different solutions are obtained at each run. Therefore the main objective of performing several runs is to calculate the mean estimate and their dispersion within the different runs. For the random numbers generation, it was used a set with 30 different seeds which was the same for all methods investigated. The algorithms were executed on a PC with an Intel Core2 Duo CPU T6670 @2.20GHz, 2.96GB RAM, with Window 7.0, 32 bits. Each run, for each method took about 50 minutes.

In Tabs. 1("a" and "b") and 2 below it is presented the solution summary for the studied methods. In the tables, $\vec{Z}_{best}$ and $\vec{Z}_{worst}$ refer to the best and the worst estimates obtained after 30 rounds. Here, the best estimates are considered as those leading to the lowest value in the evaluation of the objective function.

It is noteworthy that the coefficients D_0, D_1 and D_2 are used to recover the $\omega(\tau)$ function using the expansion given by Eq. (2). Therefore, Fig. 4 shows the curves drawn from the best estimates for each method and compared with the exact curve. It´s clear that all methods were able to identify curves that are very exact one. Nevertheless, it is clear from Fig. 4 that the curve estimated by FAcom is the best result in our tests.

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The implementation of the fuzzy concept involves variations in both the light absorption coefficient γ and the randomization parameter α, which resulted in different attractiveness β for fireflies with different light intensities compared with the brighter firefly. At the beginning, we tried just to change de absorption coefficient. However, the results were not encouraging. So, we decided to change the randomization parameter as well. That changed the influence of the random factor in the calculus of a firefly new position during a run.

The FAcom was implemented with the fuzzy concept together as well. That is, the firefly swarm moves around the center of mass and depending of the distance of a firefly to the center of mass, the absorption coefficient γ and the randomization parameter α change. In this implementation, we varied the parameters as we did in the plain fuzzy version.

The FA-OBL population was halved on the generation process in order to keep the same number of accesses to the objective function in comparison with the other methods. As the heuristic creates an opposite, the final number of fireflies evaluated were the same as in the Firefly algorithm.

In Tab. 2 it is shown that the FAcom has the best objective function value among all methods. The original FA implementation stood as the second best. The FA-OBL had the poorest result in our tests. The FAz had a good result but its combination with the center of mass concept outperformed the original algorithm. Although presenting the best individual result, FAcom did not presented the best mean and standard deviation. The FA mean of all rounds showed to be better when compared with the FAcom. The analysis of several other cases must be carried out in order to yield a clearer observation on the performance of the methods.

Finally, Fig. 5 shows the objective function evolution of the best runs based on the number of accesses to the function. The results confirm that the best FAcom run showed the best results when compared to the FA, FAz and FA-OBL. The FA result showed that it went very rapidly to its best value but got stuck. The FA-OBL showed the poorest result. After 8000 accesses it did not came even close to the other methods tested. The FAz was quite near to the FA performance.

Table 1a: Results of all parameters of all methods tested after 30 runs (FA and FAz).

		FA			FA FUZZY (FAz)			
RUN	τ_0	D0	D1	D2	τ_0	D0	D1	D2
1	0.99915	1.0026E+00	-1.4589E+00	6.9494E-01	0.99990	1.0001E+00	-1.3348E+00	5.1505E-01
2	1.00873	9.7787E-01	-1.2227E+00	4.5551E-01	0.99958	9.6184E-01	-1.0824E+00	2.4720E-01
3	1.00148	9.5025E-01	-9.1157E-01	4.3470E-02	0.99554	9.7621E-01	-1.1684E+00	3.3046E-01
4	1.00325	9.4708E-01	-8.8985E-01	4.3470E-02	0.99388	9.5409E-01	-9.5680E-01	8.2551E-02
5	0.99468	9.6123E-01	-1.0350E+00	1.6631E-01	1.00201	9.4537E-01	-9.6171E-01	1.4464E-01
6	0.99602	1.0123E+00	-1.5064E+00	7.3513E-01	0.99454	1.0124E+00	-1.4839E+00	6.9201E-01
7	0.99944	9.5766E-01	-1.0293E+00	1.8465E-01	1.00825	9.6684E-01	-1.1166E+00	3.0471E-01
8	0.99139	9.6930E-01	-1.1020E+00	2.3742E-01	1.00353	9.6535E-01	-1.1111E+00	2.9960E-01
9	0.99885	9.7592E-01	-1.2057E+00	3.9105E-01	0.99865	9.8582E-01	-1.2890E+00	4.7618E-01
10	0.98882	9.5334E-01	-9.4372E-01	3.7807E-02	1.00514	9.7749E-01	-1.1595E+00	3.4020E-01
11	0.99862	1.0021E+00	-1.4219E+00	6.4246E-01	1.01008	9.2728E-01	-7.0062E-01	-2.3137E-01
12	0.99868	9.9896E-01	-1.3729E+00	5.5968E-01	1.00058	1.0230E+00	-1.6104E+00	8.5713E-01
13	1.00380	1.0091E+00	-1.4934E+00	7.3457E-01	0.99932	9.2249E-01	-6.8476E-01	-2.5433E-01
14	0.99677	1.0286E+00	-1.6345E+00	8.5943E-01	1.00044	9.7874E-01	-1.2045E+00	3.7789E-01
15	0.99776	9.5987E-01	-1.0218E+00	1.6297E-01	0.99910	1.0100E+00	-1.4766E+00	6.9415E-01
16	1.00627	9.6361E-01	-1.0948E+00	2.7541E-01	0.98956	9.4093E-01	-7.7485E-01	-2.2415E-01
17	0.99948	9.5077E-01	-9.3671E-01	6.8313E-02	0.99948	9.5077E-01	-9.3671E-01	6.8313E-02
18	1.00263	9.3554E-01	-8.2894E-01	-4.2057E-02	1.00142	9.5996E-01	-1.0283E+00	1.9325E-01
19	0.99848	9.3978E-01	-7.8490E-01	-1.3920E-01	0.99152	9.2821E-01	-7.3603E-01	-1.9668E-01
20	0.99891	9.6234E-01	-1.0596E+00	2.2445E-01	0.99697	1.0009E+00	-1.3657E+00	5.4604E-01
21	1.00139	1.0333E+00	-1.6423E+00	8.5953E-01	0.99186	9.8561E-01	-1.2335E+00	3.8945E-01
22	1.00741	9.4841E-01	-9.3933E-01	1.1426E-01	0.99910	9.8669E-01	-1.2093E+00	3.6572E-01
23	0.99405	9.6613E-01	-1.0452E+00	1.6714E-01	0.99529	9.2944E-01	-7.8481E-01	-1.3258E-01
24	0.99759	9.8093E-01	-1.2464E+00	4.4485E-01	0.99854	9.6888E-01	-1.1035E+00	2.2728E-01

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25	0.99275	9.7796E-01	-1.1176E+00	2.2496E-01	1.00111	9.2888E-01	-7.9594E-01	-8.5772E-02
26	1.00363	9.2971E-01	-7.0510E-01	-2.3719E-01	0.99882	9.1360E-01	-6.2055E-01	-3.0194E-01
27	1.00164	1.0139E+00	-1.5202E+00	7.4907E-01	0.99403	9.3588E-01	-7.8179E-01	-1.1301E-01
28	0.99985	9.5687E-01	-9.7783E-01	9.1097E-02	1.00523	9.5674E-01	-1.0620E+00	2.5463E-01
29	0.99838	9.9144E-01	-1.2914E+00	4.5430E-01	0.99940	9.9710E-01	-1.3939E+00	6.1617E-01
30	1.00240	9.7935E-01	-1.2406E+00	4.4397E-01	0.99163	9.5010E-01	-9.3536E-01	3.6430E-02

Table 1b: Results of all parameters of all methods tested after 30 runs (FAcom and FA-OBL).

	FA Center of Mass (FAcom)				FA- OBL			
RUN	τ_0	D0	D1	D2	τ_0	D0	D1	D2
1	1.00240	9.7935E-01	-1.2406E+00	4.4397E-01	0.94826	1.0303E+00	-1.6582E+00	6.3992E-01
2	1.00045	9.9077E-01	-1.3071E+00	5.0201E-01	0.99261	1.0053E+00	-1.2908E+00	4.5421E-01
3	1.00178	9.7243E-01	-1.1377E+00	2.9839E-01	1.10078	8.9416E-01	-8.9606E-01	3.0449E-01
4	0.99982	9.8348E-01	-1.2313E+00	4.0311E-01	0.99807	9.1334E-01	-6.6520E-01	-2.3883E-01
5	1.00044	9.9577E-01	-1.3497E+00	5.5086E-01	0.99872	8.9837E-01	-5.1385E-01	-4.0080E-01
6	0.99966	1.0093E+00	-1.4793E+00	7.0168E-01	0.93279	1.1175E+00	-1.6759E+00	6.4975E-01
7	1.00228	9.7294E-01	-1.1519E+00	3.2773E-01	1.00681	9.1616E-01	-6.8225E-01	-1.8166E-01
8	0.99868	9.7126E-01	-1.1173E+00	2.7134E-01	1.01736	9.3598E-01	-1.0505E+00	1.7700E-01
9	0.99929	9.8068E-01	-1.2201E+00	3.9867E-01	0.96916	1.0508E+00	-1.3935E+00	1.4038E-01
10	0.99958	9.3736E-01	-8.3263E-01	-5.3068E-02	0.91052	1.0930E+00	-1.6754E+00	3.9066E-01
11	0.99850	9.1878E-01	-6.7397E-01	-2.4074E-01	0.97615	9.9849E-01	-1.1630E+00	5.4477E-02
12	0.99985	9.7997E-01	-1.2014E+00	3.6789E-01	1.10574	9.7148E-01	-1.1731E+00	3.4903E-01
13	1.00050	9.2204E-01	-6.7553E-01	-2.4298E-01	0.99911	9.1407E-01	-6.4862E-01	-2.4626E-01
14	0.99836	9.6714E-01	-1.0791E+00	2.2006E-01	0.99911	9.1407E-01	-6.4862E-01	-2.4626E-01
15	0.99631	9.0963E-01	-5.7515E-01	-3.7393E-01	0.98609	9.5343E-01	-8.8124E-01	-6.5679E-02
16	1.00279	9.3349E-01	-7.7968E-01	-1.2268E-01	0.97046	8.9183E-01	-8.8351E-01	1.5048E-01
17	1.00015	9.6336E-01	-1.0676E+00	2.2871E-01	1.00287	8.8997E-01	-3.9998E-01	-5.6336E-01
18	0.99590	8.8243E-01	-3.1660E-01	-6.8599E-01	0.95991	9.3418E-01	-8.6936E-01	6.6052E-03
19	0.99731	9.1843E-01	-6.4510E-01	-2.9300E-01	1.00499	9.0869E-01	-6.6559E-01	-1.8602E-01
20	0.99933	9.4691E-01	-8.9568E-01	7.2548E-03	0.97713	9.3242E-01	-6.8257E-01	-4.0152E-01
21	0.99758	1.0109E+00	-1.4795E+00	6.9485E-01	1.00142	9.1367E-01	-5.9910E-01	-3.7229E-01
22	1.00021	9.4469E-01	-8.7344E-01	-1.1913E-02	1.00064	9.4015E-01	-8.2888E-01	-1.0749E-01
23	0.99363	8.7545E-01	-2.3970E-01	-7.8220E-01	0.98798	9.2107E-01	-6.2302E-01	-3.4100E-01
24	0.99886	9.4277E-01	-8.8526E-01	5.7979E-03	0.99730	9.3050E-01	-7.8656E-01	-1.3851E-01
25	0.99731	8.6565E-01	-1.2208E-01	-9.3975E-01	0.99819	9.2800E-01	-7.7452E-01	-1.4357E-01
26	1.00149	9.5046E-01	-9.7187E-01	1.2749E-01	0.99545	9.2743E-01	-8.0611E-01	-9.3229E-02
27	0.99936	9.9227E-01	-1.3193E+00	5.1029E-01	0.99009	9.2968E-01	-7.5237E-01	-1.5085E-01
28	1.00120	9.8014E-01	-1.2127E+00	3.8710E-01	0.98534	9.9516E-01	-1.2527E+00	3.1710E-01
29	0.99953	1.0043 E+00	-1.4275 E+00	6.3970 E-01	0.93242	9.6759E-01	-1.0535E+00	2.0052E-01
30	0.99446	8.7217E-01	-1.7949E-01	-8.7184E-01	0.97881	9.4179E-01	-1.0211E+00	3.6087E-01

Table 2: Summary of the best and the worst experimental data results after all runs with all.

		τ_0	D_0	D_1	D_2	$Q(\vec{z})$
$\vec{Z}_{exact}$		1.00	1.00	-1.40	0.60	
FA	$\vec{Z}_{worst}$	1.0036	0.9297	-0.7051	-0.2372	9.1974E-04
	$\vec{Z}_{best}$	0.9986	1.0021	-1.4219	0.6425	9.6559E-06
	μ_z	0.9994	0.9745	-1.1560	0.3216	0,000235
	σ_z	0.0045	0.0277	0.2569	0.3020	0,000215
FAz	$\vec{Z}_{worst}$	0.9988	0.9136	-0.6205	-0.3019	1.0342E-03
	$\vec{Z}_{best}$	0.9994	0.9971	-1.3939	0.6162	1.6528E-05
	μ_z	0.9988	0.9647	-1.0701	0.2173	0,000354
	σ_z	0.0048	0.0295	0.2635	0.3145	0,000344
FAcom	$\vec{Z}_{worst}$	0.9973	0.8657	-0.1221	-0.9398	2.9486E-03
	$\vec{Z}_{best}$	0.9995	1.0043	-1.4276	0.6397	2.8180E-06
	μ_z	0.9992	0.9525	-0.9563	0.0823	0,000608
	σ_z	0.0022	0.0417	0.3869	0.4651	0,000841
FA-OBL	$\vec{Z}_{worst}$	0.9328	1.1175	-1.6759	0.6498	1.2012E-02
	$\vec{Z}_{best}$	0.9324	0.9676	-1.0535	0.2005	2.6818E-04
	μ_z	0.2549	0.0096	-0.4616	0.0106	0,002776
	σ_z	0.8279	0.8200	0.5791	0.3204	0,003116

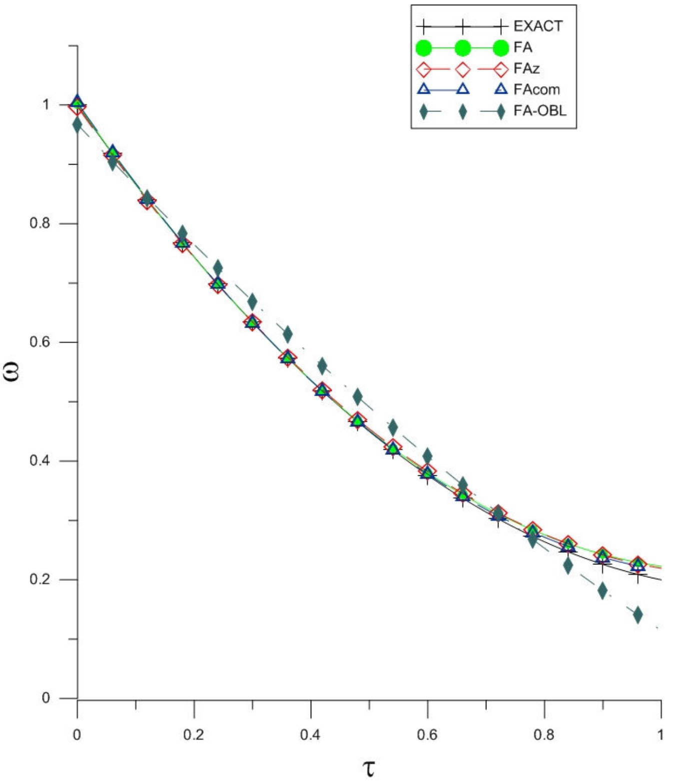

Figure 4: Estimates of $\omega(\tau)$ compared with the exact curve for a noise up to 3.5%.

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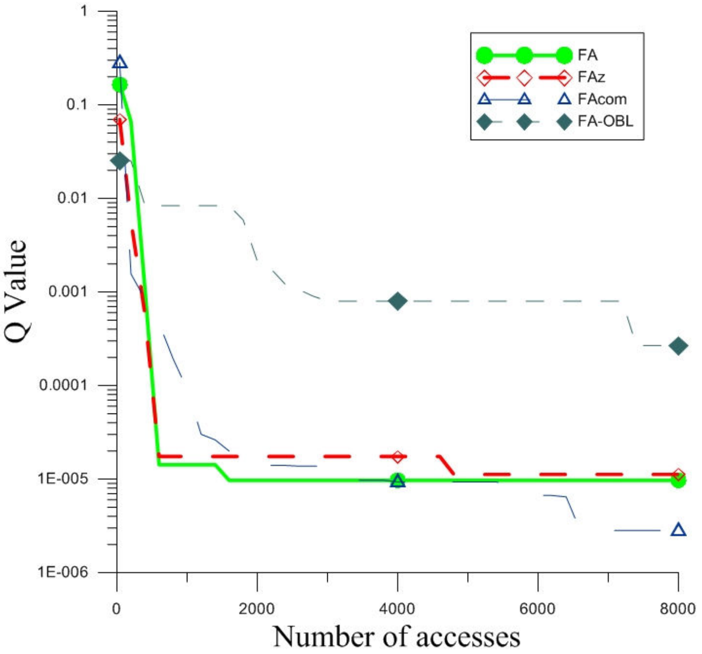

Figure 5: Evolution of the objective function value for a noise up to 3.5%.

9. CONCLUSIONS

In this work, motivated by the inverse problem in radiative transfer, it has been employed a recent developed heuristic algorithm known as the Firefly algorithm (FA) to the minimization of the objective function for the solution of the inverse problem of simultaneous estimation of optical thickness and space-dependent scattering albedo, formulated as a problem of parameter estimation. The results are critically compared to some hybridizations of the Firefly algorithm developed in this work: Firefly fuzzy, Firefly center of mass and Firefly opposed based learning. The final results showed that the hybridization with the center of mass achieved the best individual performance in the tests, indicating good perspective on the use of this heuristic in this class of problems. In the average of the 30 runs performed, the canonical FA algorithm still showed the best performance. The research proceeds in the further investigation of the mechanisms of artificial intelligence of the Firefly Algorithm in order to develop variations and specific hybridizations to ensure its robustness and performance characteristics desirable in solving computationally intensive problems. Several other test cases are going to be tested in order to yield a better observation on the performance of the mechanisms.

10. ACKNOWLEDGMENTS

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