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NONLINEAR DYNAMICS OF A VIBRATION-BASED DUFFING-TYPE ENERGY HARVESTING SYSTEM USING PIEZOELECTRIC MATERIALS

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Abstract. *Energy harvesting is the process by which energy from external sources, such as solar, seismic vibrations, electrical noise and wind energy can be captured and stored. Vibration-based energy harvesting is becoming popular employing piezoelectric materials. The design of energy harvesting systems must be optimized to produce the maximum output for given environmental conditions. This work investigates the influence of nonlinear effects in vibration-based energy harvesting. Periodic and chaotic behaviors are of concern evaluating the system performance in both situations. Nonlinear effects are considered in mechanical system, as stiffness, and also in piezoelectric element. Numerical simulations establish a comparison of the generated power highlighting the influence of nonlinear aspects.*

Keywords: *Numerical simulation, energy harvesting, nonlinear dynamics, piezoelectric.*

1. INTRODUCTION

Smart materials are usually used as sensors and actuators in smart structures. The choice of material suitable for each application depends on many factors and two requirements should be highlighted (Lagoudas, 2008): energy density and frequency. One of the motivations to the use of smart material systems and structures are the adaptive behavior that mimics biological systems allowing special characteristics that include self-repair of damages.

Vibration-based energy harvesting is an example of smart materials application being useful to reduce noise also to avoid critical vibration situations (Erturk and Inman, 2011). Piezoelectric materials are used to convert mechanical into electrical signals. Basically, two different aspects are related to piezoelectric property: direct, when applied mechanical stress causes electric field; or reverse, when applied electric field produces a mechanical stress. Du Toit (2005) discussed nonlinear aspects related to the constitutive behavior of the piezoelectric materials. It was shown that the values of the piezoelectric constants present a significant dependence of strains. Inman and Cudney (2000) investigated the properties of piezoelectric to ensure optimal electromechanical coupling coefficient and which mode use in their essays.

Concerning energy harvesting system, it is important to investigate the power of mechanical energy input regarding the optimization of output power. Shu and Lien (2006) analyzed the power of the piezoelectric energy harvesting in a system of energy generation for AC-DC rectified piezoelectric device. It is shown that the power output depends on the characteristics of the input vibration (frequency and acceleration), the mass of the generator, the electric charge, of the natural frequency, the damping ratio of the mechanical and electromechanical coupling coefficient of the system. Triplett and Quinn (2009) investigated the nonlinear coupling behavior of piezoelectric materials and also some aspects related to the mechanical nonlinearities in a vibration-based energy harvesting. Silva *et al.* (2013) investigated the influence of the hysteretic behavior of the piezoelectric material in a vibration-based energy harvesting.

This article deals with a vibration-based energy harvesting system, investigating the nonlinear dynamics details of the system, showing the influence of different qualitative behaviors in the energy harvesting performance. Basically, a archetypal model is employed, considering a Duffing-type mechanical system connected to an electric circuit with linear piezoelectric element. This work reports periodic and chaotic responses and their influence on the power energy harvesting.

2. ENERGY HARVESTING SYSTEM

Vibration-based energy harvesting system is composed of mechanical system connected to an electrical circuit by means of a piezoelectric element. Figure 1 presents an archetypal model represented by a simple mass-spring-dashpot system connected to an electrical resistance by an piezoelectric element.

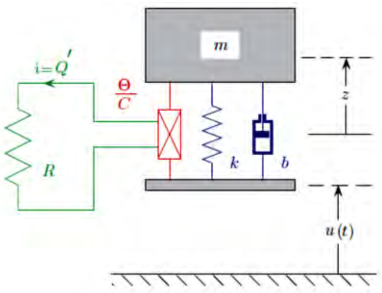

Figure 1. Vibration-based energy harvesting system (Triplett and Quinn, 2009).

A general mathematical model of this system with mass m, may be presented by considering a damper force, F_A, restitution force, F_R, piezoelectric force, F_P. Therefore,

$$m\ddot{x} + F_A + F_R + F_P = 0 \tag{1}$$

By assuming a linear viscous damping with coefficient, $F_A = b\dot{x}$, a restitution force of a Duffing-type, $F_R = k\left(\alpha + \beta x^2\right)x$, a harmonic excitation of the basis, $u(t) = U\sin(\omega t)$, the equation of motion is given by:

$$m\ddot{z} + b\dot{z} + k(\alpha + \beta z^2)z + F_P = F_0 \sin(\omega t) \tag{2}$$

where $F_0 = m\omega^2 U$. (3)

At this point, it is necessary to express the electro-mechanical coupling:

$$F_P = -\frac{\delta Q}{C} \tag{4}$$

where $\delta = \delta(z)$ is the piezoelectric coupling coefficient.
Therefore:

$$m\ddot{z} + b\dot{z} + k(\alpha + \beta z^2)z - \frac{\delta Q}{C} = F_0 \sin(\omega t) \tag{5}$$

The voltage V across the piezoelectric material is described by an electro-mechanical constitutive relation of the form:

$$V = R\dot{Q} = -\frac{\delta z}{C} + \frac{Q}{C} \tag{6}$$

where C is the piezoelectric capacitance and Q is the electrical charge developed in the coupled circuit.

By assuming dimensionless variables defined as follows,

$$\omega_n^{\ 2} = \frac{k}{m},\quad \zeta = \frac{b}{\sqrt{km}},\quad z = c_x x,\quad \rho = RC\sqrt{\frac{k}{m}},\quad Q = c_q q,\quad \Theta = \delta\frac{c_x}{c_q},\quad \tau = \sqrt{\frac{k}{m}}t,\quad \psi = \frac{\omega}{\sqrt{k/m}},$$

$$\gamma = \frac{F_0}{m c_x \omega_n^{\ 2}},\quad \overline{\beta} = \beta c_x^{\ 2},\quad \overline{\varepsilon} = \frac{\left(c_q / c_x\right)^2}{Ck\omega_n^{\ 2}},\quad (*)' = \frac{d(\bullet)}{dx}$$

The equations of motion is then rewritten:

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$$x'' + 2\zeta x' + x(\alpha + \bar{\beta} x^2) - \bar{\varepsilon}\Theta q = \gamma \sin(\psi\tau) \tag{7}$$

$$\rho q' - \Theta x + q = 0 \tag{8}$$

Where, Θ represents the piezoelectric coupling. Crawley & Anderson (1990) proposed a general nonlinear form: $\Theta = \Theta(x) = \theta(1+\lambda|x|)$. In this paper, the piezoelectric coupling is assuming to be linear, and therefore, $\lambda = 0$ and $\Theta(x) = \theta$.
The generated power of the energy harvesting system is defined as:

$$P = \rho \dot{q}^2 \tag{9}$$

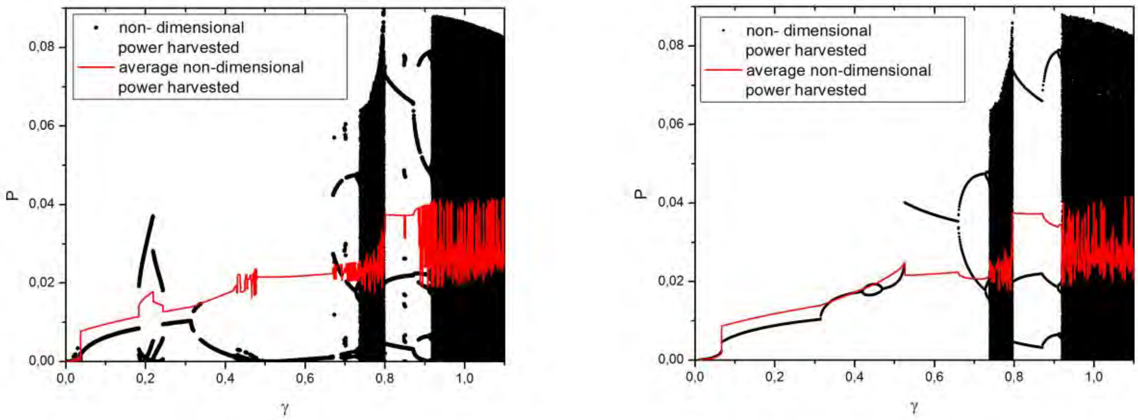
And the average of this potential is represented by:

$$P_{avg} = \frac{1}{T}\int_0^T P(t)dt \tag{10}$$

3. NUMERICAL SIMULATIONS

Numerical procedure considers the fourth-order Runge-Kutta method. Time steps less than 10^{-4} are adopted for all simulations. Numerical simulations are carried out by assuming the following parameters: $\zeta = 0.025, \alpha = 0.01, \bar{\beta} = 50, \bar{\theta} = \bar{\varepsilon}\theta = 0.1,\ \psi = 1.0, \rho = 1.0, \theta = 1.0$.

Different conditions of excitation are now investigated in order to understand the energy output. Figure 2 presents bifurcation diagrams considering power energy for different values of the excitation forcing amplitude. Besides, average power is also plotted. Two different procedures are employed to build the bifurcation diagrams. The first one considers always the same initial conditions, restarting the values to null conditions. On the other hand, the second procedure adopts as initial conditions the value of the previous simulations, using the previous parameter. The difference between both diagrams allows one to identify multistability related to the coexistence of stable solutions. Note that average power increases for some kinds of response. It should be highlighted the good performance associated with the period-3 behavior inside the periodic window.

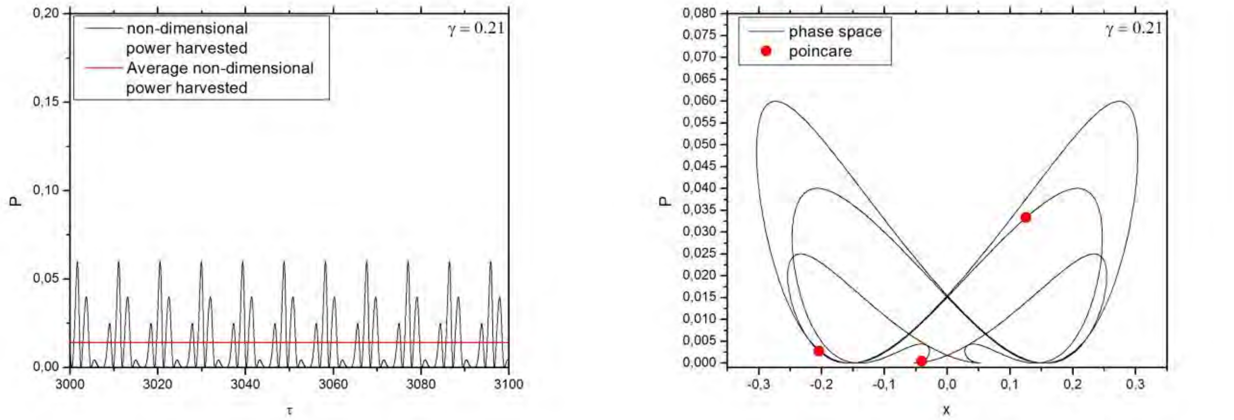


(a) (b)

Figure 2. Bifurcation diagrams showing amplitude power energy for different values of forcing amplitudes (γ).
(a) Same initial conditions for each parameter, always null; (b) Different initial conditions for each parameter.

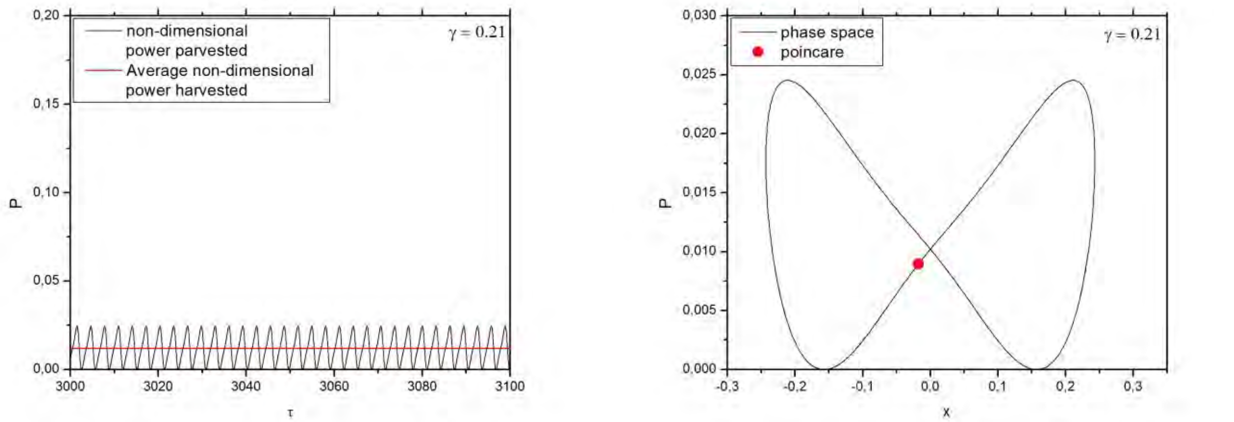
By observing the bifurcation diagrams, it is possible to promote a deeper investigation related to the system performance. Let us start by considering the multistability. Initially, consider ω=1 and γ = 0.21. Figure 3 presents results related to null initial conditions, showing a period-3 response. By assuming different initial conditions, Figure 4 presents a period-1 response. Note that power time histories are different for both cases, resulting in a greater average power related to period-3 compared with the period-1 response. The same analysis is performed for ω=1 and γ = 0.442.

Under this condition, multistability is related to symmetric solutions. Figure 5-6 present these responses showing equivalents period-2 behaviors, with similar power output.



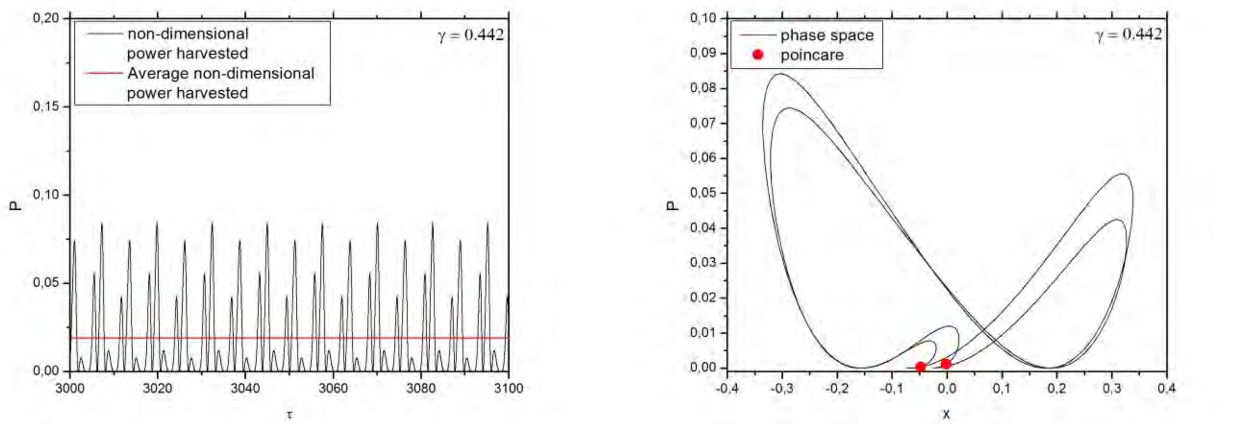
(a) (b)

Figure 3. Energy harvesting for ω=1 and γ = 0.21, under null initial conditions.



(a) (b)

Figure 4. Energy harvesting for ω=1.0 and γ = 0.21 with initial (x, x', q, P) = (-0.01773255, 0.15473466, -0.11230974, 0.00894484).



(a) (b)

Figure 5. Energy harvesting for ω=1 and γ = 0.442, under null initial conditions.

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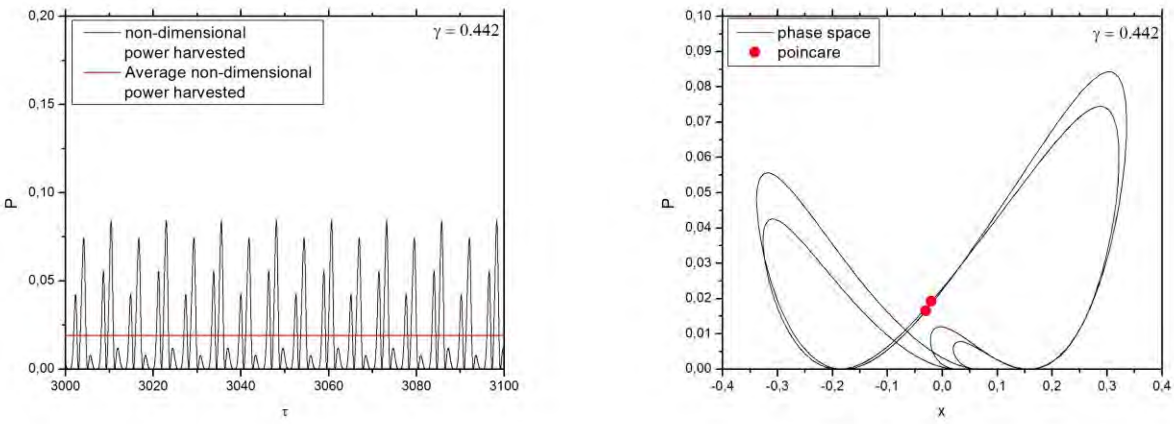
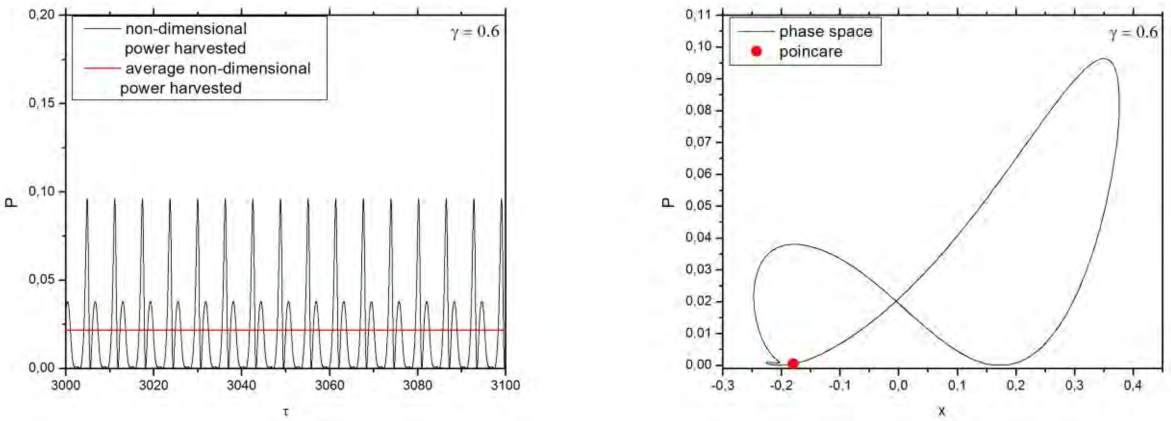


(a) (b)

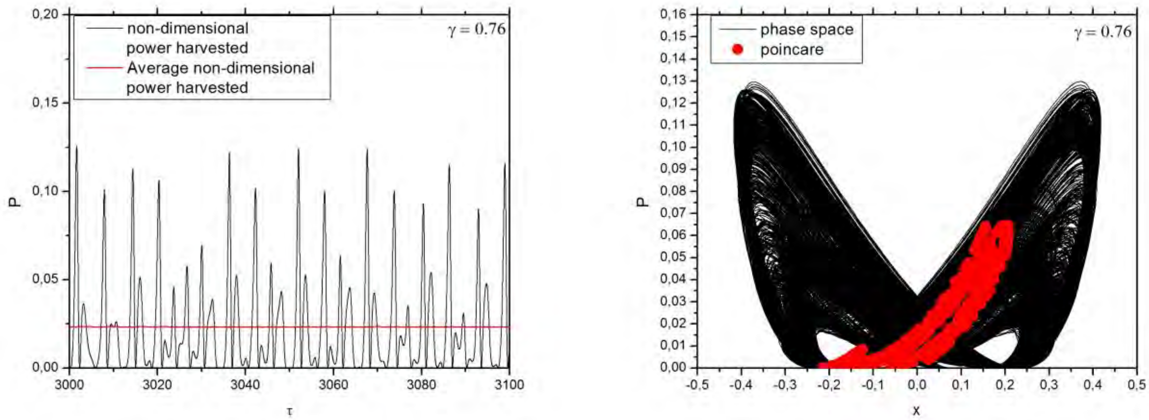
Figure 6. Energy harvesting for ω=1.0 and γ = 0.442 with initial (x, x', q, P) = (-0.01992139, 0.40595125, -0.15860578, 0.01923336).

At this moment, different parameters associated with distinct behaviors are focused on. When γ = 0.6 the system presents a period-1 response, shown in Figure 7. By increasing the value for γ = 0.76, a chaotic behaviors appears (Figure 8). When and γ = 0.82 the system is inside the periodic window, presenting a period-3 response (Figure 9). A different chaotic behavior is reached when γ = 1.046157 (Figure 10). Lyapunov exponents are employed to assure the chaotic behavior of the system. It should be highlighted that the power harvested may alter from one situation to another. Table 1 summarizes the average power to each situation. A power index is defined as the relation between the average power and the maximum value, defined in the periodic window. Figure 11 also shows this tendency. This analysis allows one to define an optimum situation for the energy harvesting system. Based on these results, it is possible to conclude that the best performance occurs inside the periodic window.



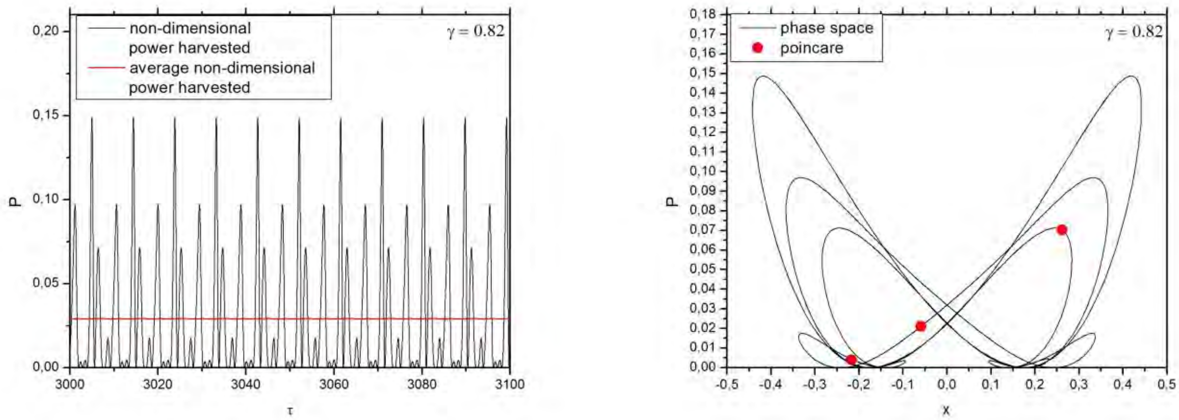
(a) (b)

Figure 7. Energy harvesting for ω=1 and γ = 0.6, under null initial conditions.



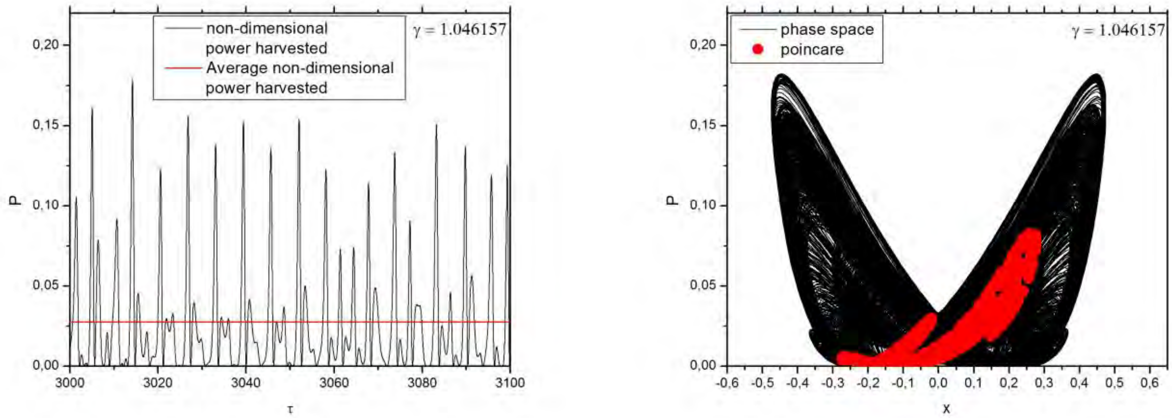
(a) (b)

Figure 8. Energy harvesting for ω=1 and γ = 0.76, under null initial conditions.



(a) (b)

Figure 9. Energy harvesting for ω=1 and γ = 0.82, under null initial conditions.



(a) (b)

Figure 10. Energy harvesting for ω=1 and γ = 1.046157, under null initial conditions.

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Table 1. Power harvested from different kinds of behaviors.

Behavior	Excitation, γ	Average non-dimensional power harvested	Power index
Period-3	0.21 (condition 1)	0.014	0.48
Period-1	0.21 (condition 2)	0.012	0.41
Period-2	0.442 (condition 1)	0.019	0.65
Period-2	0.442 (condition 2)	0.019	0.65
Period-1	0.60	0.022	0.75
Chaotic	0.76	0.023	0.80
Period-3 (periodic window)	0.82	0.029	1
Chaotic	1.046157	0.027	0.94

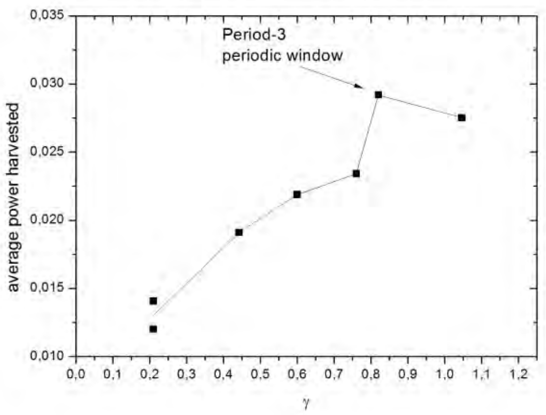

Figure 11. Comparison of the average power harvested for different excitation values, γ.

4. CONCLUSIONS

This paper deals with the nonlinear effects associated with energy harvesting systems. An archetypal system is adopted considering a mechanical system with Duffing-type nonlinearity. The connection with electrical circuit is promoted by a linear piezoelectric element. Different conditions of the harmonic excitation of the basis are analyzed. Numerical simulations shows the existence of multistability related to different coexisting stable solutions. An investigation of the nonlinear dynamics of the energy harvesting system shows that qualitative changes on system response alter the power harvested. In general, it is possible to observe that chaotic behavior tends to increase the maximum generated power, however, the average value is greater under for a period-3 behavior, inside the periodic window.

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