ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

EVALUATION OF FLEXIBILITY FACTOR AND STRESS INDICES FOR CURVED PIPES WITH LARGE DIAMETER TO THICKNESS RATIO

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Abstract. *Industrial pipes are commonly subjected to temperature variations. The presence of constrains due pipe supports or equipment nozzles promote the development of thermal stresses resulting from thermal expansion or contraction. The thermal stresses values are often assessed by engineers using simple solid mechanics models based on beam theory. However, for curved pipe regions the use of curved beam theory results in different stiffness values, consequently affecting the stresses assessment obtained by beam theory models. The ASME III Class 1 code for nuclear piping provides flexibility factors and stress indices for curved pipes to adjust the results that would be reached with the beam theory. This code does not guarantee good accuracy on these factors for diameter to thickness ratios larger than 100. This work presents a methodology to assess flexibility factors and stress indices for curved pipes using a parameterized model based on finite element method. Numerical results for different diameter to thickness ratio are presented and compared to results obtained with other studies and the ASME design code. Flexibility factors and stress indices for curved pipes with diameter to thickness ratios larger than 100 are presented.*

Keywords: *Finite Element Method, Curved Pipe, Flexibility Factor, Stress Indices*

1. INTRODUCTION

Supports and equipments nozzles promote constrains to thermal expansion or contraction in pipes subjected to temperature variations. This condition results in the presence of thermal stresses that must be considered in the design phase in order to ensure the structural integrity of pipes throughout its service life. The thermal stresses and stiffness values are often assessed by engineers using simple solid mechanics models based on beam theory. However, for curved pipe regions the use of curved beam theory results in different stiffness values, consequently affecting the stresses assessment obtained by beam theory models. The ASME III Class 1 code for nuclear pipe provides flexibility factors (k) and stress indices for piping components to adjust the results that would be reached with the beam theory. The methodology considers cross section ovalization effects that promote circumferential stresses, stress amplifications and flexibility reduction.

Pressure vessel and piping codes usually considers two types of stress: primary and secondary stresses. The first one is considered a load-controlled stress as with load rise stress magnitude continues to increase until failure occurs. The second one is termed self-limiting and is a localized stresses that experiments stress accommodation once the yield limit has been reached. Therefore different allowable stresses are used and these two stress types are calculated separately. Stresses promoted by prescribed displacements are a self-limiting stress type and depends directly on the piping stiffness.

The ASME III Class 1presents a methodology where stresses are calculated by equations provided by the code and comparisons with allowable limits for primary, primary-plus-secondary and peak stresses are realized. For bending stress, S_b, in a piping component, the following equation is indicated by the code:

$$S_b = C_2 \frac{M_i}{Z} \tag{1}$$

where M_i is the resultant bending moment, Z is the section modulus, and C_2 is the primary-plus-secondary stress index.

In the structural integrity analysis of pipes, elbows are components that require a special attention, as they are responsible to absorb the major part of thermal deformations of piping systems. Therefore several authors have developed works to describe this behavior considering different loading conditions. The effect of ovalization on structural stiffness of a curved pipe was noted experimentally by Bantlin (1910). Von Karman (1911) published a theoretical work where proposed a flexibility factor (k) to correct stiffness obtained by the beam theory in a curved pipe subjected to a bending moment applied in the plane of the component:

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$$k = \frac{12.h^2 + 10}{12.h^2 + 1} \tag{2}$$

$$h = \frac{t.R_1}{r_2^2} \tag{3}$$

where h is the flexibility characteristic, t is the pipe thickness, r_2 is the radius of pipe cross-section and R_1 is the bend radius.

Kafka and Dunn (1956) developed theory for loading in the plane and consider the effect of internal pressure; Rodabaugh and George (1957) developed theory for in plane and out of plane moments. Fonseca *et al.* (2002) calculated flexibility factors and stress fields using an analytical model applied to curved pipes subjected to bending moments. Lubis and Boyle (2004) used finite element analysis to study the effect of internal pressure on the curved pipes and proposed simple equations to calculate flexibility factor and a stress intensification factor (similar to stress index C_2) in the absence of pressure:

$$k = \frac{1.692}{h}\left[\frac{\exp\left(-0.69\dfrac{r}{R}\right)}{h^m}\right] \tag{4}$$

$$C_2 = \frac{2.19}{h^{2/3}}\left[\frac{\exp\left(-1.12\dfrac{r_2}{R}\right)}{h^{1/6}}\right] \tag{5}$$

$$m = 0.195.\exp\left[-0.09\frac{R}{r_2}\right] \tag{6}$$

Tan *et al.* (2002) used finite element model to compare numerical results with experimental data for out of plane loadings in curved pipes and observed a good agreement. The influence of curved pipe adjacent elements, as flanges, in the structural pipe behavior was considered in several studies (Fonseca *et al.*, 2006).

Recently, the availability of finite element codes has permitted the development of studies that include the influence of damage in the structural integrity of curved pipes. Li and Aggarwal (2010) used numerical models based on the finite element method to study the effect of non-uniform corrosion on the structural integrity of pipe and proposed a methodology combined with the ASME equations.

ASME III class 1 establishes the following general equations for calculating the flexibility factor (k) and the stress index C_2 of curved pipes subjected to bending moments, respectively:

$$k = \frac{1.65}{h} \tag{7}$$

$$C_2 = \frac{1.95}{h^{2/3}} \tag{8}$$

The code establishes that both equations are valid for piping where the diameter to thickness ratio is lower than or equal to 100 and suggests that a more detailed assessment should be made for diameter to thickness ratios larger than 100.

In this paper a parameterized numerical model based on the finite element method is used to obtain flexibility and stress index C_2 factors for curved pipes. Simulations were performed for different diameters and thickness pipe combinations aiming the study of components with diameter to thickness ratios larger than 100. Numerical simulations considers in plane bending loads and the material elastic behavior.

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2. NUMERICAL MODEL

A parameterized numerical model based on the finite element method (FEM) was developed using commercial code ANSYS to study the flexibility and stress distribution in curved pipe regions. The model geometry is composed by a curved pipe region connected at each end to straight regions with a length equal to 5 nominal diameters. Figure 1 presents the mesh (obtained after a convergence analysis), the loading and the boundary conditions. The horizontal straight region end is clamped (blue and orange triangles symbols) and the vertical straight region end is subjected to a bending moment (double blue arrows symbol). The especial beam element ELBOW290 was used in the model. This element considers ovalization of the cross section.

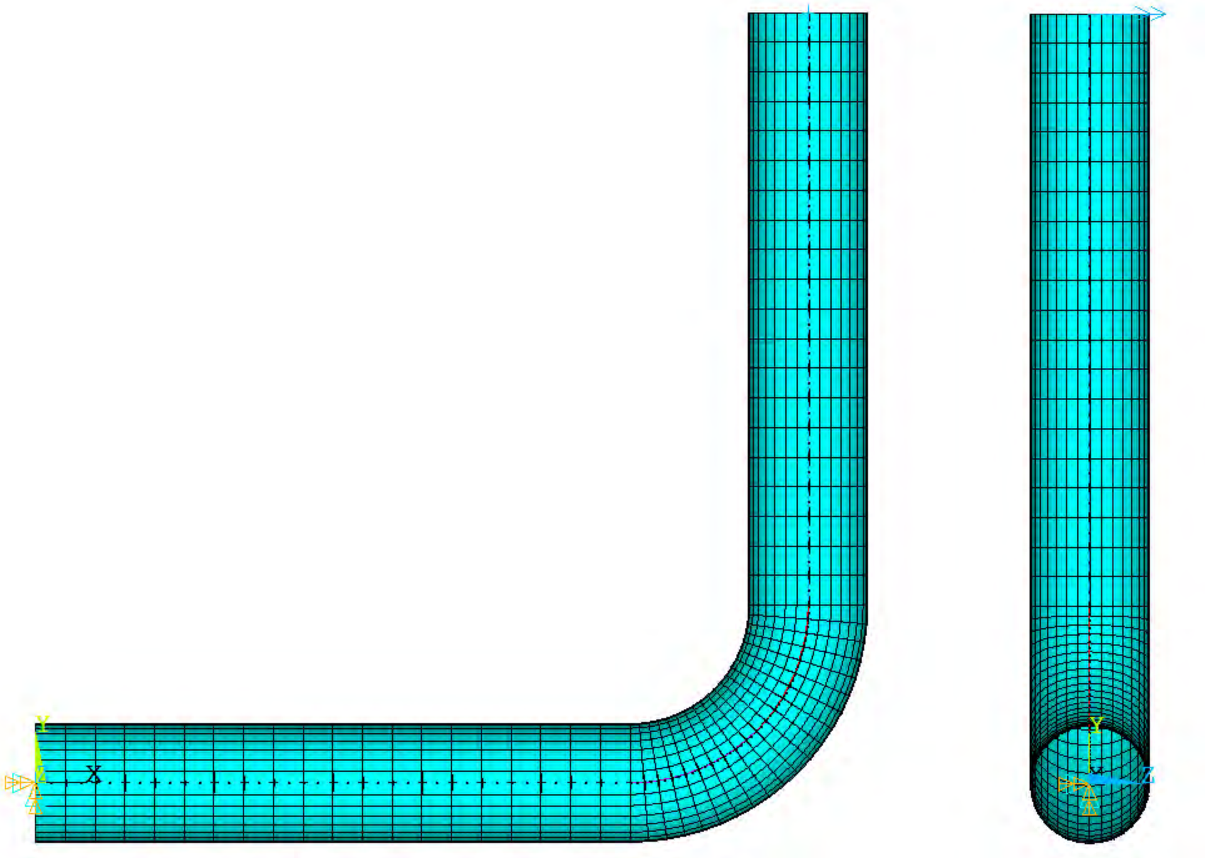

Figure 1. Mesh, boundary conditions and loading for the numerical model

The analysis developed considers a combination of three outside diameter values and nine thicknesses for each curved pipe diameter. Table 1 presents the geometric data for the studied cases considering the geometry of commercial pipes and corroded pipes. For each variation of diameter studied it was realized a convergence analysis in order to obtain results with good accuracy and with minimal computational effort. For all simulations a moment of 1kN.m in plane is applied at the pipe free end. At the other end, all degrees of freedom were restricted.

The flexibility factor (k) is defined as the ratio between the difference of the rotation angles between the curved pipe ends obtained by numerical simulations and the one obtain from analytical model based on beam theory for a beam of circular cross section (without ovalization). Likewise, the stress index C_2 is obtained from the ratio of the maximum circumferential stress obtained from numerical simulations and the longitudinal stress obtained from analytical model based on beam theory. These factors were calculated using the procedure described in the next section.

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Table 1. Geometric data for the studied cases

Nominal Pipe Size (NPS)	Outside Diameter (mm)	R(mm)	Wall Thickness (mm)	D/T
22	559	838	3.28$^{(1)}$	170.43
			4.78	116.95
			5.56	100.54
			7.92	70.58
			10.31	54.22
			14.27	39.17
			20.62	27.11
			23.83	23.46
			38.1	14.67
34	864	1295	4.85$^{(1)}$	178.14
			6.35	136.06
			7.14	121.01
			8.74	98.86
			10.31	83.80
			14.27	60.55
			19.05	45.35
			22.23	38.87
			31.75	27.21
48	1219	1829	6.53$^{(1)}$	186.68
			8.03	151.81
			9.53	127.91
			10.31	118.23
			11.13	109.52
			11.91	102.35
			14.27	85.42
			19.05	63.99
			23.83	51.15

$^{(1)}$ Lower commercial thickness minus 1.5mm (due corrosion).

3. ANALYTICAL MODELS

Analytical models using simple solid mechanics models based on beam theory are used to calculate the rotation angles between the curved pipe ends and the nominal stresses.

3.1 Calculation of flexibility factor (*k*)

Aiming to calculate the difference of rotation between the ends of a curved pipe, consider a curved beam element anchored at one end (point *A*) and with a moment applied to the other end (point *B*). From Castiglano's Energy Theorem:

$$\Delta\theta_{BA} = \frac{\partial U}{\partial M_B} - \theta_A \tag{9}$$

where U is the elastic energy stored in the structure, θ_A is the rotation of point A (zero in this case), $\Delta\theta_{BA}$ is the rotation difference between the two ends and M_B is the in plane bending moment applied. The elastic energy in the structure is given by:

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$$U = \int_A^B \frac{M^2}{2EI} dl \tag{10}$$

where E is the Young's modulus, I is the moment of inertia and $dl = R\, d\varphi$ is the infinitesimal length; R is the radius of component and φ is the angle of curvature. Inserting the Eq. (10) in Eq. (9):

$$\Delta\theta_{BA} = \int_A^B \frac{\partial}{\partial M_B}\left(\frac{M^2}{2EI}\right) dl = \frac{M_B}{EI}\int_A^B dl = \frac{M_B}{EI}\int_0^{\pi/2} R d\varphi = \frac{M_B \pi R}{2EI} \tag{11}$$

Finally, considering $\Delta\theta_{NS}$ the difference of rotation angle between the two ends of a curved pipe obtained from numerical simulations, the flexibility factor (k) can be estimated from:

$$k = \frac{\Delta\theta_{NS}}{\Delta\theta_{BA}} \tag{12}$$

3.2 Calculation of stress index C_2

From beam theory, the nominal maximum longitudinal stress (σ_{nom}) caused by a moment applied into a straight beam is:

$$\sigma_b = \frac{M}{Z} \tag{13}$$

Thus, the stress index C_2 is given by:

$$C_2 = \frac{\sigma_{NS}}{\sigma_b} \tag{14}$$

where σ_{NS} is the maximum circumferential stress obtained from the numerical simulation. It is important to point that beam theory only furnish longitudinal stresses. Equation (14) furnish a methodology provided by ASME of estimate the circumferential stress on curved beams using simple calculations for longitudinal stress obtained for simple straights beams using the beam theory.

4. RESULTS AND DISCUSSION

Figures 2 and 3 show the profile for circumferential and longitudinal stresses distribution observed in the numerical simulations developed for model with NPS 22. It can be seen that ovalization promotes maximum values of the circumferential stresses near the beam neutral line.

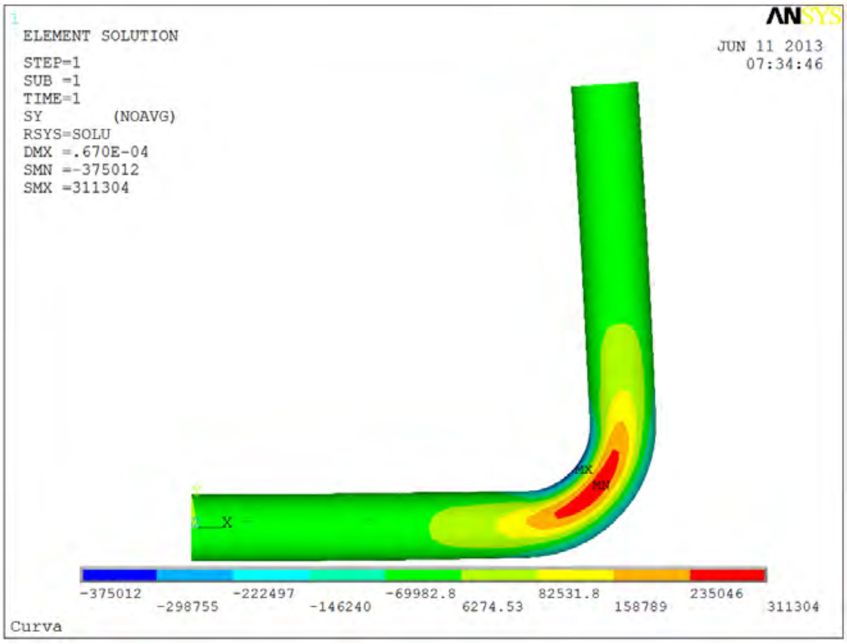

Figure 2. Circumferential stress distribution for a pipe with external diameter of 559 mm (NPS 22)

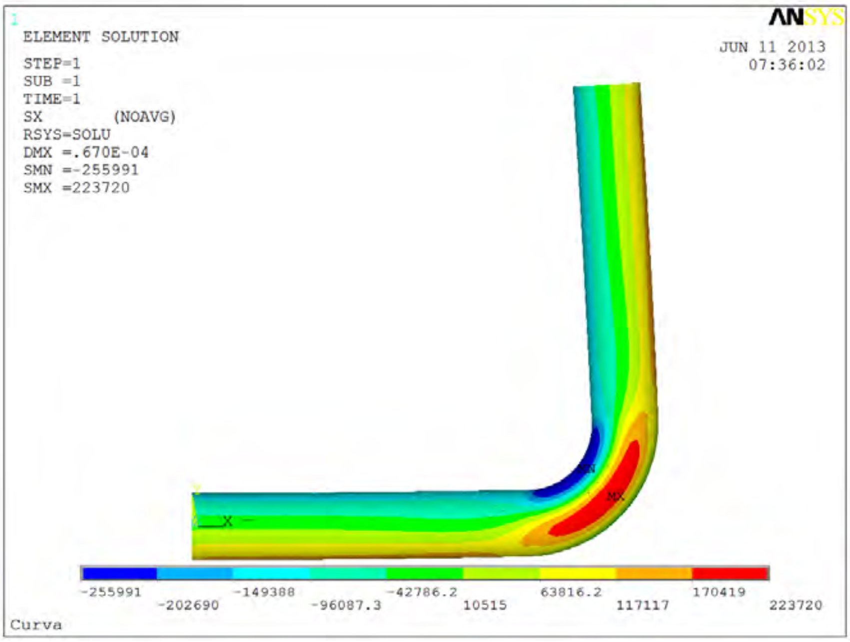

Figure 3. Longitudinal stress distribution for a pipe with external diameter of 559 mm (NPS 22)

Figure 4 shows flexibility factors (k) obtained from the proposed FEM model and equations given by Karman (1911), Lubis (2004) and ASME considering a pipe with external diameter of 559 mm (NPS 22). The factor is plotted as a function of the ratio D/t.

Figure 4 shows that for large diameter to thickness ratios, the Karman (1911) model presents large difference in the flexibility factor values in comparison with others models. The Karman's equation was obtained from analytical model that use relevant simplifications: Poisson's ratio is not considered and values were extracted from numerical series truncated in the first term.

Also the results shows that the proposed FEM model tends to present a better agreement with ASME results as D/t ratio increases.

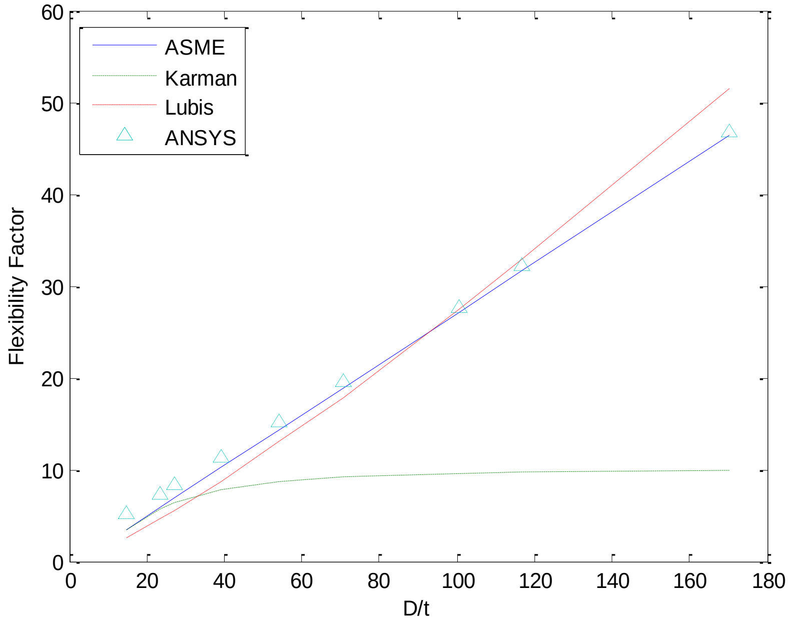

Figure 4. Flexibility factor as a function of D/t ratio for a pipe with an external diameter of 559mm (NPS 22)

Figure 5 shows results obtain for the stress index C_2 as a function of diameter to thickness ratio for a curved pipe with an external diameter of 559 mm (NPS 22), considering data obtained from the proposed FEM model and by ASME and Lubis (2004) equations.

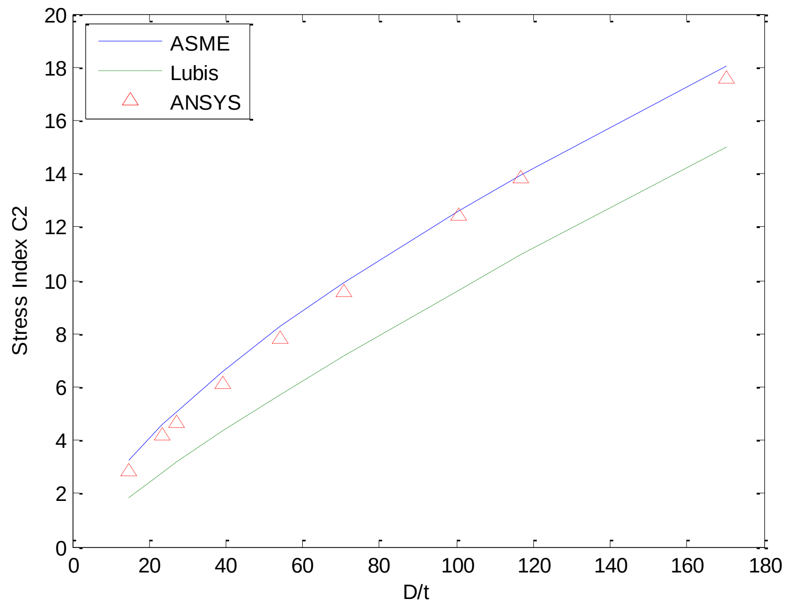

Figure 5. Stress index C_2 as a function of D/t ratio for a pipe with an external diameter of 559mm (NPS 22)

Figure 5 shows that a good agreement is observed for stress index C_2 obtained by ASME and the proposed FEM model. Results obtained from Lubis (2004) present slightly lower values.

For a better comparison between the results obtained with the proposed FEM model and the ASME factors, Figures 6 and 7 show the percentage difference for the flexibility and stress index C_2 considering three diameters.

Figure 6 shows that the difference in flexibility factor tends to be larger for lower D/t ratios. For NPS 22 pipe with D/t=14.6, the difference is about 45%. The difference falls to less than 2% for D/t larger than 100. Flexibility factors values obtained with the proposed FEM model furnish smaller values than the ones obtained from ASME indicating

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that ASME methodology predicts more rigid structures than the proposed FEM model. Therefore, flexibility factors predicted by ASME methodology can be considered conservative.

Figure 7 shows that ASME methodology overestimates stress values promoted by bending moments for all the studied cases (a negative difference is observed). The minimal difference of about 1% occurs for a $D/t \cong 120$, a region where the ASME is less conservative. These data suggests that the ASME methodology could be applied to estimate circumferential stress in curved pipe for D/t larger than 100, but is important to note that the safety margins of the code would be considerably lower.

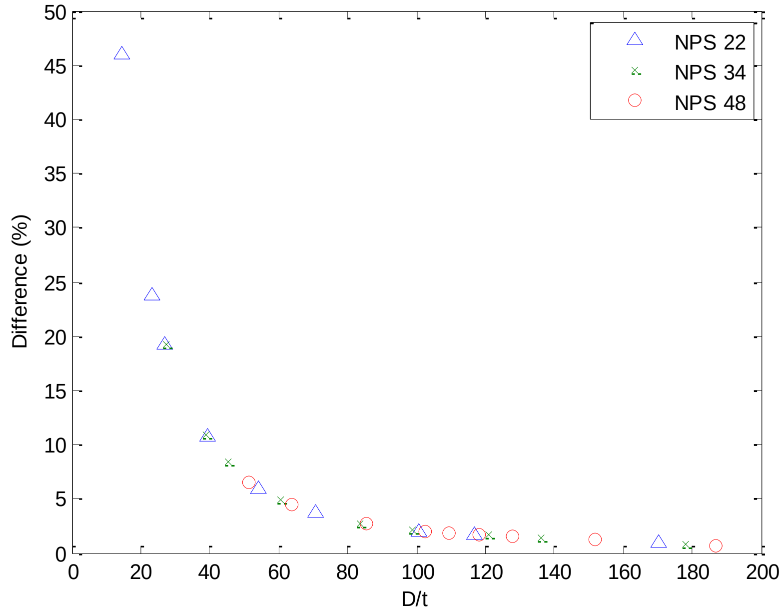

Figure 6. Percentage difference for the predicted flexibility factors using the proposed FEM model and the ASME methodology

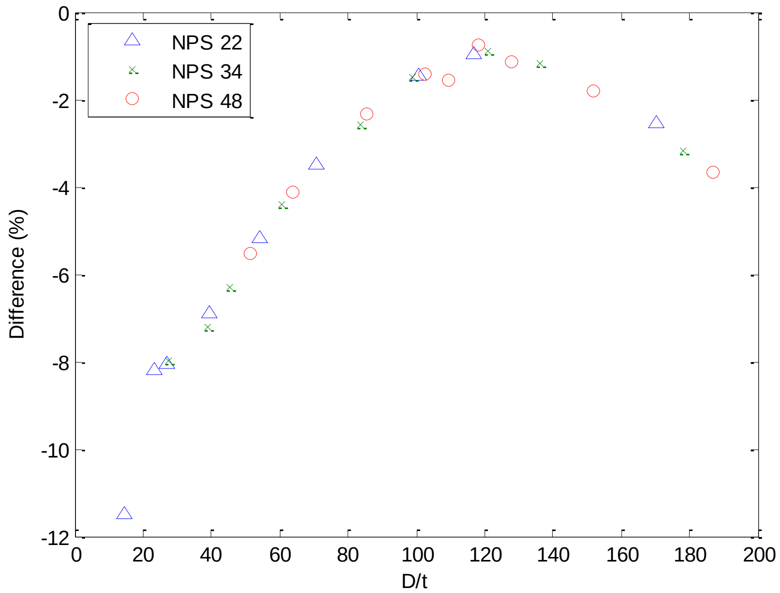

Figure 7. Percentage difference for the predicted stress index C_2 using the proposed FEM model and the ASME methodology

ISSN 2176-5480

5. CONCLUSIONS

The proposed finite element model was used to assess stress index C_2 and flexibility factor (k) in curved pipes for a large range of diameter to thickness ratio. Obtained results shows a consistent agreement with those obtained from literature and ASME III Class 1.

A comparison of the flexibility factor and stress index *C2* given by ASME with the ones obtained by the proposed FEM model indicates that the results tends to similar values as the *D*/*t* ratio increases. However, factors obtained by ASME methodology are less conservative for pipe with *D/t* larger than 100.

6. ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of the Brazilian Research Agencies CNPq and CAPES.

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