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STRESS CONCENTRATION FACTOR CALCULATION FOR A NOTCHED SPECIMEN UNDER ELASTO-PLASTIC LOADING

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Abstract. *For mechanical design purposes, the stress concentration factors are of major importance in fatigue analysis calculation. The general use of these factors requires analytical solutions feasible for several load and geometry cases from the basis of elastic structural range of materials. However, at a notch field tip, the onset of plastic zones and failure processes demands a more comprehensive view of a material behavior. In this sense, the notch sensitivity factors play an important role for fatigue analyses, which is more emphasized when considering these plasticity effects. This paper presents analyses by Finite Element (FEA) and algorithm calculations in order to predict the elasto-plastic stress concentration factor for a 1020 steel alloy with its material behavior mathematically modeled for analysis purposes. Thus, these relations are clarified by considering a particular case of a notched specimen under axial load. The main goal is aimed at validating a methodology for calculation of this generalized factor through a simple procedure, to be applied for the elasto-plastic stress-strain range.*

Keywords: *Stress Concentration Factor, Notch Sensitivity, Elasto-Plastic Analysis, Finite Element Analysis, Monotonic Loading.*

1. INTRODUCTION

In the design of engineering components it is often necessary the use of notches like keyways, grooves, holes or shoulders in order to achieve the desired performance of the mechanical assembly. Such notches induce local stress and strain concentrations which are critical to fatigue-life analysis. Under linear-elastic stress behavior, stress concentration factors for several loading and geometry combination cases were presented in references like Pilkey and Pilkey (2008) and Young and Budynas (2002) allowing the solution of several engineering problems. However, when large concentration factors are present, even small loads (elastic loads) can cause local stresses that produce the onset of plastic zones at the notch root inducing a premature failure process. The stresses exceeding the yield limit cause a change in the material response leading to a decrease in the local stress concentration when compared with linear-elastic case. On the other hand, the strain concentration is larger than in the linear-elastic case. In this sense, the notch sensitivity factors play an important role for fatigue analyses, with a more emphasized relation with these elasto-plastic stress concentration factors.

Pinho-da-Cruz et al (2002) performed finite element analyses (FEA) in order to obtain stress concentration factors in two different configurations of flat specimens. The first configuration was with opposite semicircular edge notches and the second with shoulder fillets. The FEA analyses were carried out by considering both isotropic and kinematic hardening rules, which addressed the limitations of simulation results for monotonic, proportional and reversal loading. Zeng and Fateni (2001) investigated the elasto-plastic stress and strain at notch root in round bar specimens and flat plates analyzing results with two different concentration factors. For round bar specimens a circumferentially notched configuration was adopted and for the flat plates it was adopted a double-edge U-notch configuration. They carried out FEA analysis by adopting a multi-linear kinematic hardening model to describe the steel material behavior. Notch stresses were also calculated through Neuber's rule and Galinka's rule and compared with FEA results, with good agreement for nominal stresses smaller than 80% of yield stress. Miranda et al. (2004) carried out FEA analyses in order to evaluate the elasto-plastic stress and strain distribution considering large deformations at the notch root under several load levels since the usual models do not account for geometrical changes at the notch root. Neuber's approach is verified as able to predict reasonable estimates of concentration factors. Miranda et al (2012) report the required steps to obtain robust finite element triangular meshes to properly evaluate stress intensity factors calculation to determine crack propagation path with consideration for predictions of fatigue life.

Simulation of cyclic loaded parts considering local yielding can be still costly in computational view because the nonlinear material behavior. Trautwein (2005) implemented an approximation technique based on Neuber's rule for

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plastic stress-strain state estimate from linear FEA runs in order to reduce the computational effort needed in traditional fatigue and life endurance analysis for different load levels.

Torabi and Taherkhani (2011) investigated the numerical analysis of a different configuration specimen, the V-notched Brazilian disk applied to the study of crack propagation on brittle material under mixed mode I/II loadings. Torabi and Jafarinezhad (2012) used a variation of the notched Brazilian disk, but with a U-notch tip to obtain design parameters of notch stress intensity factors under mixed loading in a rapid and convenient way by FEA. This specimen configuration is shown to be suitable for evaluation of stress intensity factors in a large range of mixed loading combination from pure mode I (tensile stress) to pure mode II (shear stress).

This paper presents analyses by FEA and algorithm calculations in order to predict the elasto-plastic stress concentration factor for a 1020 steel alloy with its material behavior mathematically modeled for analysis purposes. Thus, these relations are clarified by considering a particular case of a notched specimen under axial load. The main goal is aimed at validating a methodology for calculation of this generalized factor through a simple procedure, to be applied for the elasto-plastic stress-strain range. The main results are focused on predictions for a plastic zone or front evolution, indexed as a function of a load or stress parameter. Also, is calculated the elasto-plastic stress concentration factor as a function of this external load parameter. The results aim at offering subsides for mechanical design purposes, despite the fact that the study was performed over a particular case of geometry and material. However, the described methodology paves a way for a fast and practical calculations form a more complete, parametric design given the input data as notch geometry and material properties.

2. THEORETICAL BACKGROUND

The present work is based on the study of a monotonic, uniaxial loading of a flat plate with a round hole notch. The stress concentration factor is calculated in the extended scope for the elasto-plastic stress states of an AISI 1020 steel material. By assuming a specific set of geometrical parameters, indicated below in the “Fig. 1”, it is worth mentioning that the sections around the notch comprise different stress gradients, according to a chosen direction of analysis taken from the notch center. In the case of the geometry of the specimen indicated in the “Fig. 1”, it is possible to choose a special section named “Analysis Section” passing through the notch center and being perpendicular to the external axial loading direction. For this study, the geometrical constraints are indicated in this figure.

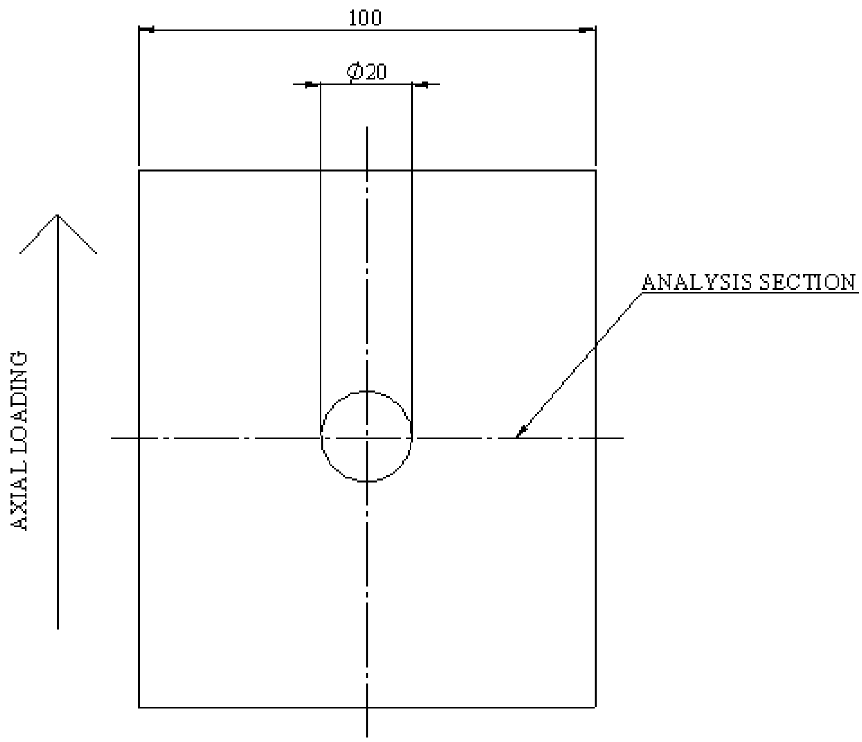

Figure 1. Finite width notched plate with a centered round hole (dimensions in [mm]).

In the basis of the elastic range of the solid materials, the Airy stress functions are recalled for presenting the stress components in polar coordinates for a plane stress problem. However, care must be taken by considering this plate with a finite width, which refers to a set of linear algebraic equations to be solved (Timoshenko and Goodier, 1970):

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$$\begin{bmatrix} 2 & 0 & 6/b^4 & 4/b^2 \\ 2 & 0 & 6/a^4 & 4/a^2 \\ 2 & 6b^2 & -6/b^4 & -2/b^2 \\ 2 & 6a^2 & -6/a^4 & -2/a^2 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} = \begin{Bmatrix} -\frac{S}{2} \\ 0 \\ -\frac{S}{2} \\ 0 \end{Bmatrix} \tag{1}$$

This solution provides a set of values to be entered in the formulas for the polar system stress components:

$$\sigma_r = -\left(2A + \frac{6C}{r^4} + \frac{4D}{r^2}\right)\cos 2\theta \tag{2}$$

$$\sigma_\theta = +\left(2A + 12Br^2 + \frac{6C}{r^4}\right)\cos 2\theta \tag{3}$$

$$\tau_{r\theta} = +\left(2A + 6Br^2 - \frac{6C}{r^4} - \frac{2D}{r^2}\right)\sin 2\theta \tag{4}$$

The polar coordinate system origin is located at the hole center. The angular position of the analysis section compared to the axial loading direction gives:

$$\theta = \pm\frac{\pi}{2} \tag{5}$$

This fact leads to:

$$\tau_{r\theta} = 0 \tag{6}$$

These aspects confirm that "Eq. (2)" and "Eq. (3)" represent the principal stress gradients at the analysis section with angular position depicted in the "Eq. (5)". With the above expressions, "*a*" is the notch radius, "*b*" is the plate semi-width, "*r*" is the coordinate value taken from the notch center to the plate border and through the analysis section line. Moreover, "*S*" refers to the nominal stress in the plate, given by:

$$S = \sigma_{nom} = \frac{F}{2bt} \tag{7}$$

In this case, "*F*" is the axial applied load and "*t*" is the constant thickness of the specimen.

Thus, the stress concentration factor can be calculated for the maximum stresses in the elastic regime at any point in the specimen near to the notch tip:

$$K_{T-e} = \frac{\sigma_{\max-e}}{\sigma_{nom}} \tag{8}$$

In the continuous monotonic loading beyond the yield stress level, the nominal stress σ_{nom} can represent also a reference for describing a stress concentration factor for the material elasto-plastic regime as:

$$K_{T-ep} = \frac{\sigma_{\max-ep}}{\sigma_{nom}} \tag{9}$$

These definitions request for an elasto-plastic study in the analysis section, as the loading expressed by σ_{nom} raises such that the notch neighborhood reaches the yielding. By modeling the load equilibrium across the section, it is possible to obtain the total applied load on the specimen as:

$$F_{total} = \begin{cases} 2\int_{r=a}^{r=b} \sigma_{\theta-e}.dr \quad if \quad \sigma_{\max-e} \le \sigma_y \\ 2\int_{r=a}^{r=b} \sigma_{\theta-ep}.dr \quad if \quad \sigma_{\max-ep} > \sigma_y \end{cases} \tag{10}$$

Thus, the equilibrium equation must be satisfied at each load step represented by the nominal stress level. In the event of yielding, a neighbor material portion of the notch is expressed by a geometric region inside the:

$$a \le r \le r_{front} \tag{11}$$

The evolution of the plastic front is given by " r_{front} " and is dependent of " σ_{nom} ".

For each material regime, it is necessary to keep the stress and strain relations. For example, for an isotropic elastic behavior represented by the Hooke law (Timoshenko and Goodier, 1970) and according to the previous definitions, the principal components for strains are derived as:

$$\begin{aligned} \varepsilon_1 = \varepsilon_\theta &= \frac{1}{E}\left(\sigma_\theta - \upsilon\sigma_r\right) \\ \varepsilon_2 = \varepsilon_r &= \frac{1}{E}\left(\sigma_r - \upsilon\sigma_\theta\right) \end{aligned} \tag{12}$$

The stress components were already defined in the principal stress space. For this guideline, "E" is the isotropic Young modulus and "ν" is the Poisson modulus. In the event of yielding, the total equivalent strain is given by the sum of its component elastic and its component plastic (Kobayashi, Oh and Altan, 1989)

$$\varepsilon = \varepsilon^e + \varepsilon^p \tag{13}$$

The stress-strain relations of the classic plasticity theory are represented by a plastic flow rule for an isotropic hardening behavior:

$$d\varepsilon_{ij}^p = d\lambda \frac{\partial f}{\partial \sigma_{ij}} \tag{14}$$

In this case, "f" is the yield function to be derived against the stress components in order to obtain an incremental solution for the strain components. The remaining parameter is:

$$d\lambda = \frac{3}{2}\frac{d\varepsilon^p}{\sigma_{eqv}} \tag{15}$$

The equivalent expressions are recalled for stress and strain, respectively:

$$\sigma_{eqv} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} \tag{16}$$

$$d\varepsilon^p = \sqrt{\frac{2}{3}}.\left[\left(d\varepsilon_1^p - d\varepsilon_2^p\right)^2 + \left(d\varepsilon_2^p - d\varepsilon_3^p\right)^2 + \left(d\varepsilon_3^p - d\varepsilon_1^p\right)^2\right]^{1/2} \tag{17}$$

In order to obey the isotropic hardening behavior, the material constitutive relation is modeled as:

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$$\sigma_{eqv} = \begin{cases} E\varepsilon & \Rightarrow elastic \\ \sigma_Y + H(\varepsilon - \varepsilon_Y) \Rightarrow plastic \end{cases} \tag{18}$$

It is worth noting that "$\left(\sigma_{Y,}\varepsilon_Y\right)$" is the yield data and "H" is the plastic tangent modulus for the material. The complete solution is obtained through integration of these expressions and by keeping them strictly bond to the compatibility conditions and equilibrium conditions, which permits a complete description of the problem.

3. ANALYSIS ALGORITHM

An analysis algorithm was developed to study the monotonic stress loading of the specified specimen. Despite the fact that the same analysis may be obtained through any FEA commercial software, it is of great interest a tool for fast calculations of the stress-strain gradients on the specific regions of a notched specimen. For this, the previous equations were organized in such a manner to provide these gradients for the analysis section showed in the "Fig. 1". As the problem of notched specimens demands the calculation of several geometrical cases represented by proper parameters and focused on quick parametric runs, the present algorithm emerges as a good option to this type of work.

This algorithm is depicted on the "Fig. 2".

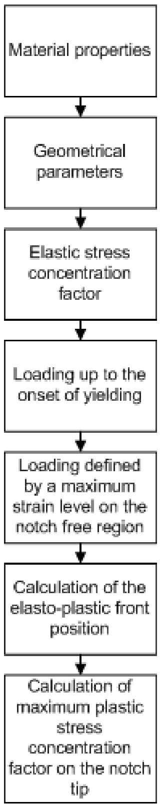

Figure 2. Algorithm structure for numerical analyses.

For obtaining the desired evaluation and according to the presented hypotheses and restrictions for analysis purposes, it is necessary to impose some material properties. Thus, it was chosen the AISI 1020 for this study, whose main property values are indicated in the "Table 1" acquired in a generic form from technical literature:

Table 1. Material data properties for AISI 1020 Steel.

Material Properties	Value
Young Modulus (GPa)	210.0
Poisson Modulus	0.29
Plastic Tangent Modulus (MPa)	460.0
Yield Stress (MPa)	210.0
Yield Strain (%)	0.10

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On the other hand, by considering the current test specimen, the geometrical data parameters for this case were showed in the "Fig. 1".

By specifying the stress load level up to the yielding point, it is necessary to obey the yield strain at the notch-free region. Thus, by raising the stress level in the nominal region in a stepped manner, it is possible to calculate the stress gradient over the geometrical section of the specimen. The integration of the strain increments is followed by considering an assumed fixed relation through the increments in the principal directions 1 and 2. This relation is obtained by FEA analyses. The stress concentration near to the notch is calculated according to the elastic or the elasto-plastic behavior equations presented before, depending on the critical stress level. For each case, the approach was presented in the previous pages of this paper. As defined before, the elasto-plastic front transition is calculated through its coordinate position "r_{front}".

Finally, a complete definition of the stress state in the analysis permits the extraction of its maximum value and the calculation of the plastic stress concentration factor, as described by "Eq. (9)".

4. RESULTS AND DISCUSSION

4.1 On the validity of the present modeling

For the sake of verification for validity, a set of results was calculated by this algorithm implemented in the Matlab ™ program and compared to FEA analysis performed in the Ansys™ program.

The FEA values were calculated based on a meshed model with the same conditions for geometry and material properties indicated before. In this case, a 3D mesh with 3180 Solid45 elements was prepared (plate thickness of 5 mm), with a more detailed refinement near to the notch tip. The applied load was divided in 70 sub cases, with a progressive, rising displacements over its borders. A nonlinear solution routine was available when demanded, referring to the yielding of the material. "Figure 3" shows a von Mises equivalent stress gradient around the notch for a detached part of the specimen and for the maximum displacement imposed, with emphasis to the plastic region of the material.

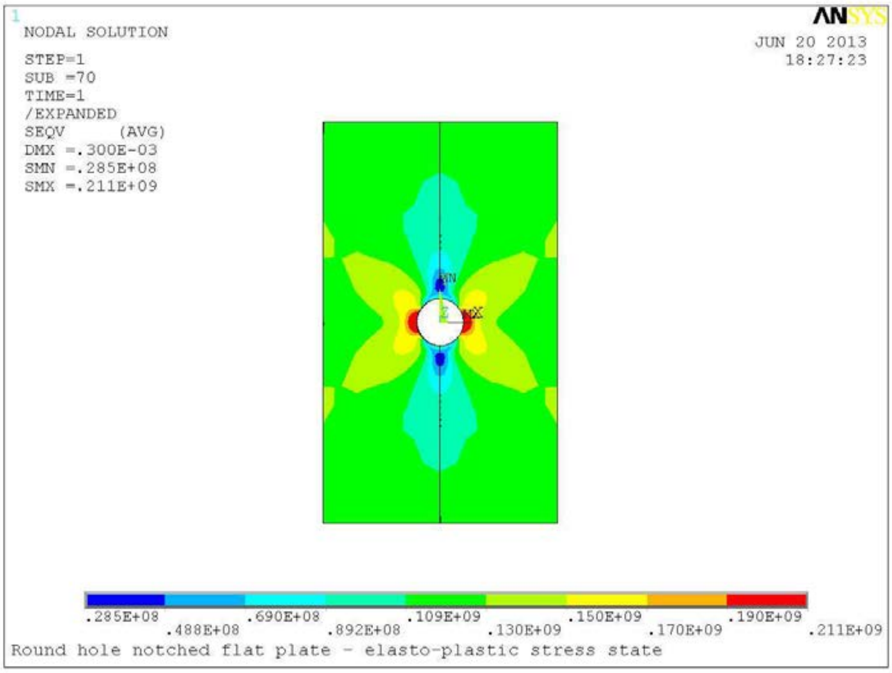

Figure 3. von Mises stress gradients on the notch region for the last load case.

In order that both analyses have the same basis for comparisons purposes, a nominal stress level necessary to attain the same or approximate stresses on the notch region was specified for both analyses. The curves of principal stress distribution over the analysis section were gathered and plotted together in the next figure, considering a nominal stress level of 115.0 MPa in the FEA analysis and 124.5 MPa with the analysis algorithm. Thus, it is demonstrated in the next figure a level of maximum error of 8.5% between the two analysis processes, which was taken as acceptable for this study.

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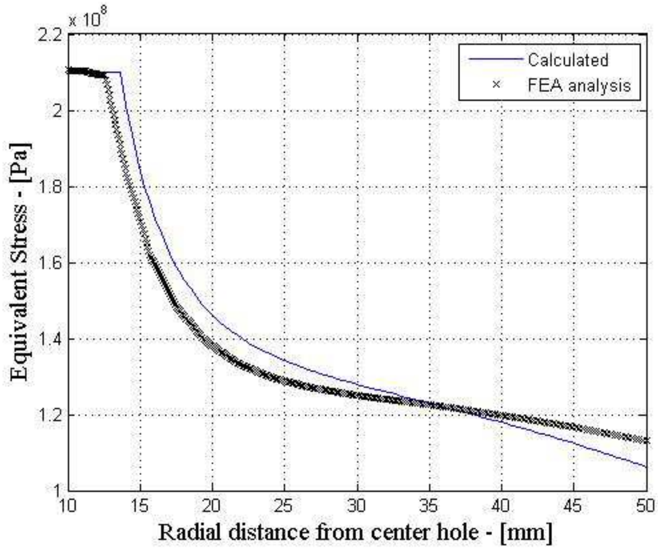

Figure 4. von Mises stress gradients for an average 120.0 MPa axial stress level on the nominal region.

4.2 Main results

As the algorithm presented a good correlation to FEA analyses, all the calculations may be performed through the use of it. As an initial result, "Fig. 5" shows a clear evolution of the plastic front radius against the nominal stress level, obtained from a set of calculations for the current geometry and material properties. This problem is solved for several loadings applied to the notch free region, starting from 20% up to 120% of the material yield stress. It must be remembered that the notch radius is 10mm. So, the maximum width for this plastic region is the numeric difference between the plastic front radius and the notch radius.

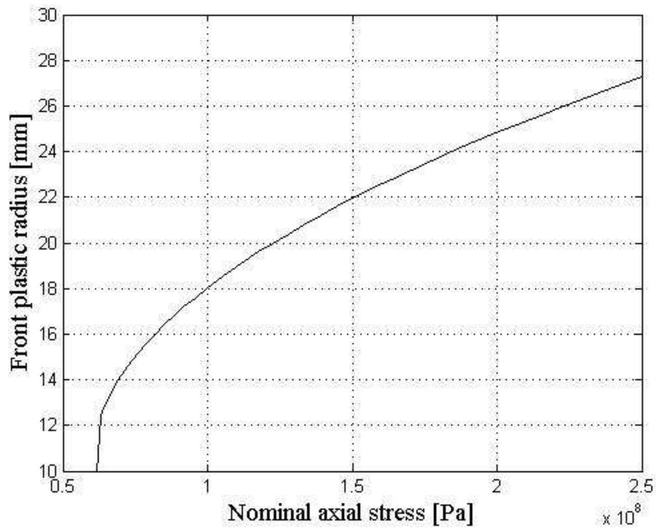

Figure 5. Plastic front radial position versus nominal stress level.

It is observed that, by considering the nominal stress ranged from 25% to 60% of the chosen material yield stress, the equivalent strains span between 0.025% and 0.060%, a level below the yield strain of the material. This fact is of interest in experimental tensile tests, where one can put the right instrumentation according to the standards to evaluate the highly local hardening on the notch root, while the nominal region is in the elastic, linear range.

Moreover, for this plastic front evolution and proposed geometry an exponential mathematical function was adjusted to express this aspect. It is expressed by:

$$r_{front} = -0.9206.\sigma_{nom}{}^{-0.04438} + 0.4281 \tag{19}$$

The least squares adjustments were obtained with a correlation value of:

$$r^2 = 0.9923 \tag{20}$$

In the "Eq. (19)", it must be observed that the values for nominal stress are in "*Pa*" units for obtaining the values of plastic front in "millimeters" units. Also, it is suitable only for the geometry restricted to this study. Therefore, it is possible to develop similar equations for other geometries, which is left for a future work.

The plastic front bounds the concentration stresses near the notch tip. As a measure to evaluate these stresses, it is considered the first principal stress to be compared to the nominal stress in the notch free region. "Equation (9)" fixed this relation, plotted in the next figure as:

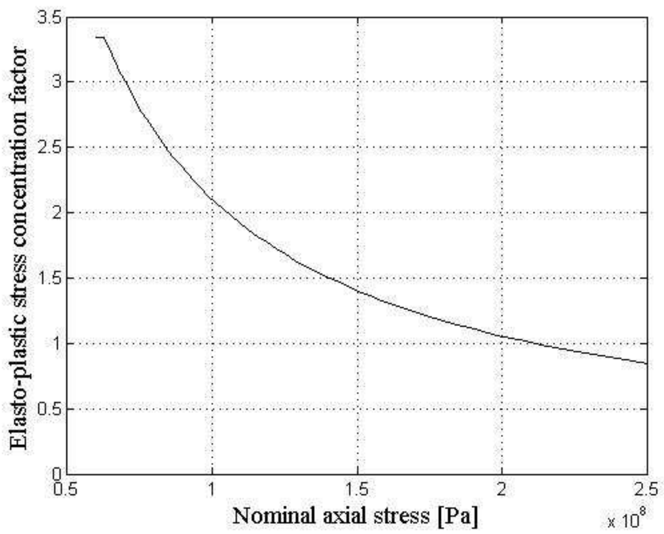

Figure 6. Elasto-plastic stress concentration factor versus nominal stress.

This figure illustrates the reduction of the stress concentration factor after yielding while the external load is raised. For the present geometry, the yield started with a nominal stress of 62.8 MPa and for an elastic stress concentration factor of 3.344. After the onset of yielding, the decay was modeled by least squares method, which led to:

$$K_{T-ep} = \frac{71.77e6}{\sigma_{nom}^{0.94}} - 0.06445 \tag{21}$$

with a correlation of:

$$r^2 = 0.9993 \tag{22}$$

Thus, it was developed a methodology for fast calculation of the elasto-plastic stress concentration factors and the plastic front evolution. Through the use of these simple adjusted mathematic expressions, it is possible to obtain predictions for general studies in mechanical design such as in the case of fatigue-life analyses. The same methodology may be repeated for other geometry and material properties, by considering this type of notched specimen and loading. Moreover, a similar procedure may be prepared for other test specimens.

5. CONCLUSION

It was demonstrated the feasibility of this methodology to be used in parametric studies for general material and geometry combinations applied to the presented notched specimen and type of loading. As mentioned before, a similar procedure is possible for other type of specimens and loadings. It must be observed that a broad scope to develop an analytical expression is possible, but it must be remembered that some information from FEA shall be obtained to feed up these general models. On the other side, a more complete mathematical expression is necessary to put more light on these elasto-plastic studies, despite the fact that the non-linear aspects are the major drawbacks for this sort of study. As a guide used in this work and to be considered as form to cope with these non-linear aspects, it was used in this work an elastic stress gradient based on Airy´s stress function model, a bilinear constitutive material model, an isotropic hardening, a monotonic loading and other aspects from the classical theory of plasticity. It must be cited also that new models are necessary to ponder the FEA analyses against these non-linear problems, even by considering other phenomena such as the cinematic hardening and the residual stresses. These suggestions are left also for a future work as an additional contribution to guide the implementation of non-linear models in FEA analyses and to guide the studies focused on the problem of mesh sensitivity.

6. ACKNOWLEDGEMENTS

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