ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

APPLICATION OF WAVELETS TO CHARACTERIZE DYNAMIC BEHAVIOR OF ENERGY HARVESTING SYSTEMS

Itamar Iliuk
Escola Politécnica da Universidade de São Paulo, Av. Prof. Luciano Gualberto, travessa 3, n. 158, 05508-900 São Paulo, SP, Brazil
itamar.iliuk@usp.br
José Manoel Balthazar
UNESP- Rio Claro, Department of Statistics, Applied Mathematics and Computation (DEMAC), Av. 24 A, 1515, Bela Vista, CEP: 13506-700, Rio Claro, SP, Brazil
jmbaltha@rc.unesp.br
Angelo Marcelo Tusset
UTFPR- Ponta Grossa, PR, Department of Engineering Science, Av. Monteiro Lobato, Km 04, s/n. CEP: 20 - 84016-210 - Ponta Grossa-PR, Brazil
a.m.tusset@gmail.com
Átila Madureira Bueno
UNESP – Campus experimental de Sorocaba Av. Três de Março 511, Alto da Boa Vista, CEP: 18087-180, Sorocaba, SP, Brazil
atila@sorocaba.unesp.br
José Roberto Castilho Piqueira
Escola Politécnica da Universidade de São Paulo, Av. Prof. Luciano Gualberto, travessa 3, n. 158, 05508-900 São Paulo, SP, Brazil
piqueira@lac.usp.br

Abstract. *Wavelet analysis has been applied to a variety of pertinent problems in engineering. This paper describes the use of wavelet-based methods to find and analyze energy harvesting systems behavior. Energy harvesting systems based in single and double-well potential presents both periodic or chaotic behavior. High and low energy orbits are contained in the steady state of this systems. The large amplitude periodic response of the energy harvester is preferred to chaos not just because of its larger gain compared to that of the chaotic response, but also because the periodic response is preferred to chaotic response for processing the voltage output using a nonlinear energy harvesting circuit for charging a battery or a capacitor efficiently. The analysis of the oscillatory movements of the systems were identified with good accuracy by means of wavelet-based methods. Looking for the scalograms and global energy spectrum, it were very useful tools to validate the type of motion found, periodic, quasi-periodic or chaotic. We can found with relative accuracy the amplitude and frequency of operation that generates more energy for each model. This technique can be used as a measurement tool to assist the validation of chaos in dynamic systems together with the calculation of Lyapunov exponents and Poincare maps.*

Keywords: *Energy Harvesting, Piezoelectricity, Chaos, Wavelets.*

1. INTRODUCTION

Piezoelectric energy harvesters has been studied for several researchers, and many non-linear devices has been modeled as a forced Duffing oscillator. In this paper some of these models were demonstrated. Among the different models can be mention the works of (Triplett and Quinn, 2009; Erturk, 2009; Iliuk *et al.*, 2012, 2013), without undeserved others. Energy harvesting systems based in single and double-well potential oscillators can be presents both periodic or chaotic behavior. High and low energy orbits are contained in the oscillatory regime of this systems. In order to identify this behaviors an analysis using continuous wavelet transform (CWT) can be performed alternatively to classical methods used in dynamical systems theory. Wavelet analysis has been applied to a variety of pertinent problems in engineering in accordance with (Addison, 2002). A practical guide to wavelet analysis was developed by (Torrence and Compo, 1998). In this work the goal of the authors was the decomposition of a time series in space or time-frequency domain. They used the data series of the El Nino, because the statistics of the data change in addition, the authors also includes a statistical significance test. Reboita (2004) used the wavelet transform toolkit proposed by (Torrence and Compo, 1998) on two data sets, with the objective of determining the atmospheric systems that cause greater climate variability in southern Brazil, in order to provide information for the future development of a regional climate model. An analysis of the dynamics of the Duffing oscillator was performed using the wavelet cross-spectrum by (Kyprianou and Staszewski, 1999) as an alternative to classical analysis of input and output. The work of (Permann and Hamilton, 1992), emphasize the robustness of wavelet analysis even when the system presents a chaotic behavior as a Duffing oscillator in short time series. The study of the dynamic behavior of the mechanical oscillations of a non-ideal system, which describes a physical model that deals with the motion of a block and an electric motor DC was studied by (Chierice Jr, 2007). This analysis was performed using the complex Morlet wavelet, following the technique proposed by (Torrence and Compo, 1998).

This paper describes the application of continuous wavelet transform (CWT) using the complex wavelet of Morlet contained in the toolkit developed by (Torrence and Compo, 1998) to identification and analyze of energy harvesting systems behavior.

2. MATHEMATICAL MODELS

The schematic model and the equations as well as the dynamic responses of the systems were presented in two sections, separated by the type of excitation in models with ideal energy source and models with non-ideal energy source. The phase portraits and history of displacement in time were obtained by numerical integration of the equations of motion to demonstrate the behavior of each system. This results were used as basis to comparison with the results founded by the application of wavelet technique. Two models are nonlinear and excited by a ideal energy source. Other two models are nonlinear and excited by a non-ideal energy source, where the model of non-ideal portal frame were displayed responses using a passive control system, Nonlinear Energy Sink (NES), and without the passive control.

2.1 Ideal energy harvester models

The ideal models, are those excited by a well defined ideal energy source as the form of Eq. (1).

$$A\cos(\omega t) \tag{1}$$

The first model presented in Fig. 1 is defined by Eq. 2 and Eq. 2 it has been studied by (Triplett and Quinn, 2009) and looking for the dynamic of the system presented in the phase portrait and history of displacement in time showed in Fig. 2 can be see a periodic behavior.

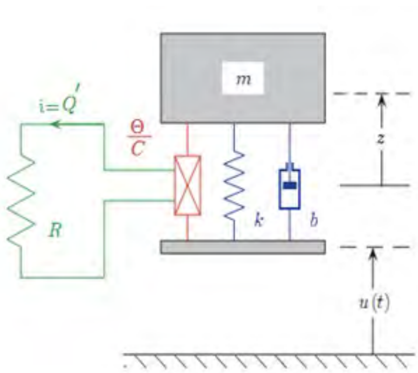

Figure 1. Ideal energy harvester by (Triplett and Quinn, 2009)

$$\ddot{x} + 2\epsilon\zeta\dot{x} + x(1 + \epsilon\alpha x^2) - \epsilon\theta(1 + \beta|x|)q = \epsilon\gamma\sin(\omega t) \tag{2}$$

$$p\dot{q} - \theta(1 + \beta|x|)x + q = 0 \tag{3}$$

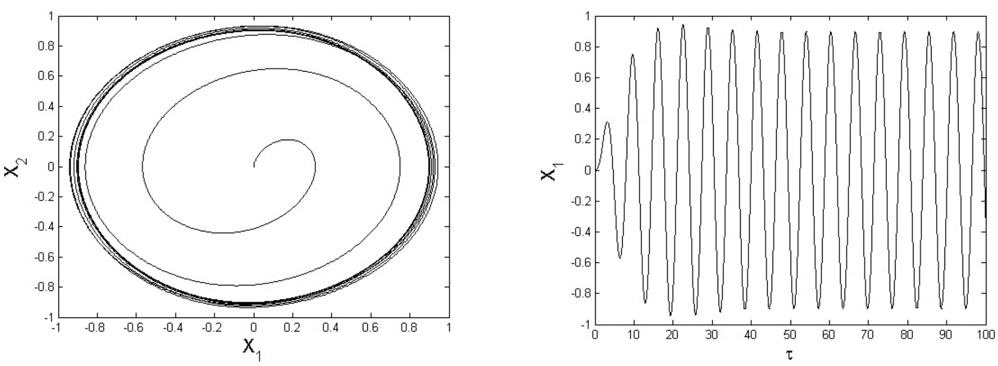

Figure 2. Phase portrait on the left and time history of displacement on the right, system response presents a periodic behavior

ISSN 2176-5480

The second model was proposed by Erturk(2009) as the forced Duffing oscillator showed in Fig. 3. This model represented by Eq. 4 and Eq. 5 shows a chaotic behavior. The responses to numerical integration of the equations that governing the movement can be seen in Fig. 4 where was depicted the phase portrait and the time history of displacement.

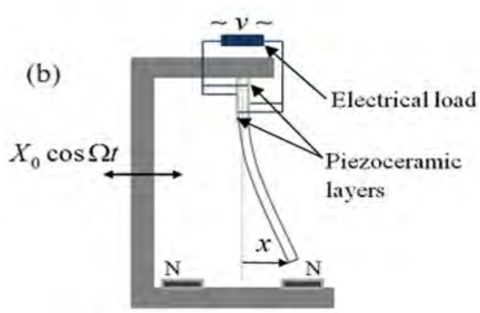

Figure 3. Ideal energy harvester by (Erturk, 2009)

$$\ddot{x} + 2\zeta\dot{x} - \frac{1}{2}x(1 - x^2) - \chi v = f \cos \Omega t \quad (4)$$

$$\dot{v} + \lambda v + \kappa \dot{x} = 0 \quad (5)$$

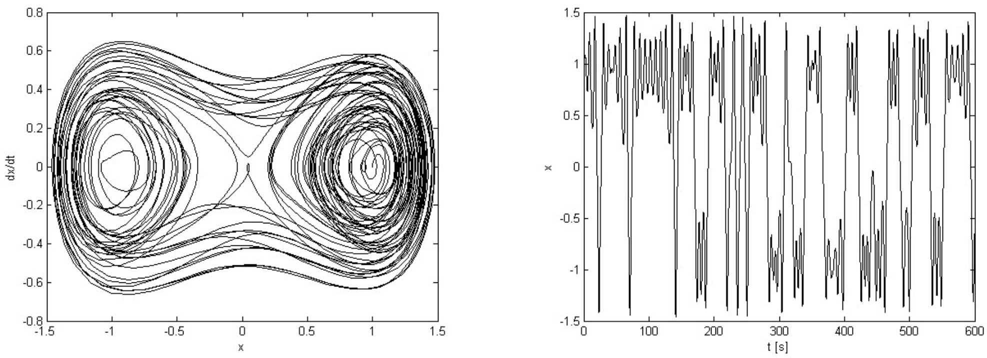

Figure 4. Phase portrait on the left and time history of displacement on the right, system response presents a chaotic behavior

2.2 Non-ideal energy harvester models

The models of energy harvesting presented in this section are non-ideal because of the type of excitation font is a DC motor with limited power. An extensive analyses of non-ideal theory can be found in (Balthazar *et al.*, 2003).

The resistive torque applied to the motor is represented by the function $H(\dot{\varphi})$ and the driving torque of the energy source (motor) is represented by $L(\dot{\varphi})$. In this work, the function which defines the energy source is a linear function that represents the curve of torque versus the velocity of the DC motor, Eq. 6, in accordance with (Balthazar *et al.*, 2003; Tusset *et al.*, 2013).

$$H\dot{\varphi} - L\dot{\varphi} = V_1 - V_2\dot{\varphi} \quad (6)$$

Where V_1 are related to the voltage applied to the armature of the D.C. motor, that is, a possible control parameter of the problem, and V_2 is a constant for each model of the DC motor considered.

The model presented in Fig. 5 was studied by (Iliuk *et al.*, 2012). The coupled equations of motion Eq. 7 to Eq. 9, showed a quasi-periodic behavior. It can be seen in the phase portrait and time history of displacement depicted in Fig. 6.

$$M\ddot{x} + c\dot{x} + k_1x + k_2x^3 = m_0r(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi) + \frac{d(x)}{C}q \quad (7)$$

$$I\ddot{\varphi} = m_0r\ddot{x}\cos\varphi + L(\dot{\varphi}) - H(\dot{\varphi}) \quad (8)$$

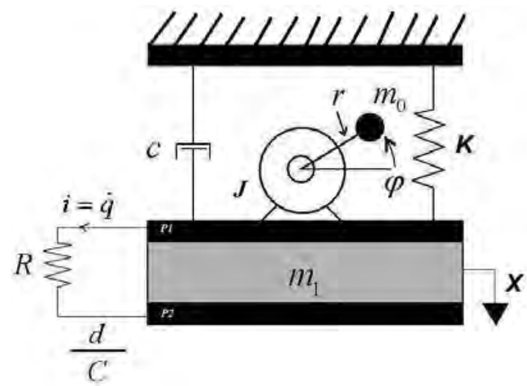

Figure 5. Non-ideal energy harvester by (Iliuk *et al.*, 2012)

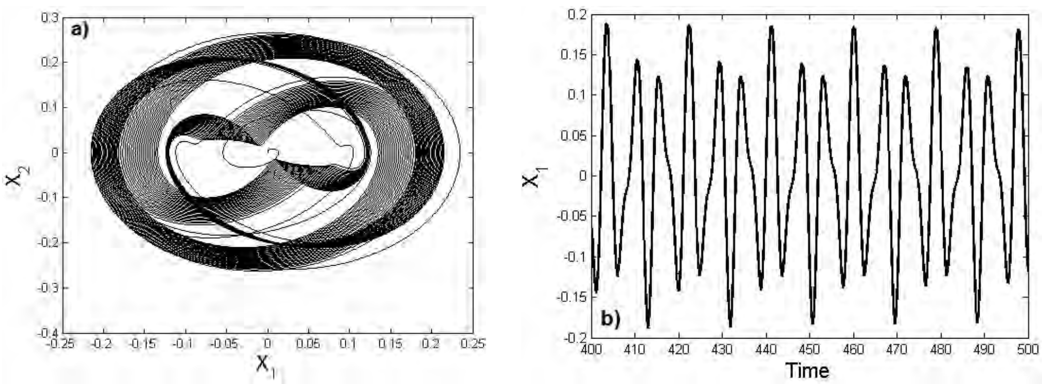

Figure 6. Phase portrait on the left and time history of displacement on the right, system response presents an aperiodic behavior

$$R\dot{q} - \frac{d(x)}{C}x + \frac{q}{C} = 0 \tag{9}$$

The next model showed in Fig. 7 is a simple portal frame under non-ideal excitation considered as a Non-Ideal System (NIS), coupled to a Nonlinear Energy Sink (NES) passive controller. The coupled equations are Eq. 10 to Eq. 13. The phase portrait and time history of displacement in Fig. 8 shows the chaotic behavior of the system without (NES) passive controller attached. In Fig. 9 the system with (NES) demonstrates a periodic behavior.

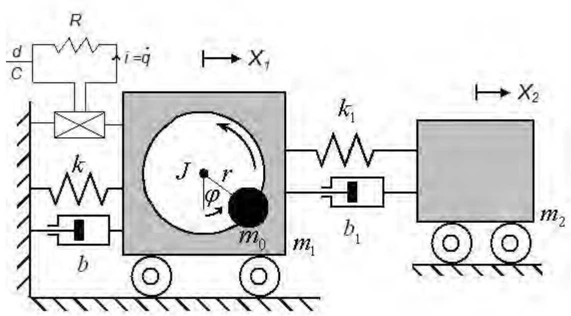

Figure 7. Non-ideal energy harvester by (Iliuk *et al.*, 2013)

$$(m_1 + m_0)\ddot{x_1} + b\dot{x_1} + b_1(\dot{x_1} - \dot{x_2}) - k_l x_1 + k_n l x_1^3 + k_1(x_1 - x_2)^3 = m_0 r(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi) + \frac{d(x_1)}{C}q \tag{10}$$

$$m_2\ddot{x_2} - b_1(\dot{x_1} - \dot{x_2}) - k_1(x_1 - x_2)^3 = 0 \tag{11}$$

$$(J + r^2 m_0)\ddot{\varphi} - m_0 r\ddot{x_1}\sin\varphi = V_1 - V_2\dot{\varphi} \tag{12}$$

ISSN 2176-5480

$$R\dot{q} - \frac{d(x_1)}{C}x_1 + \frac{q}{C} = 0 \qquad (13)$$

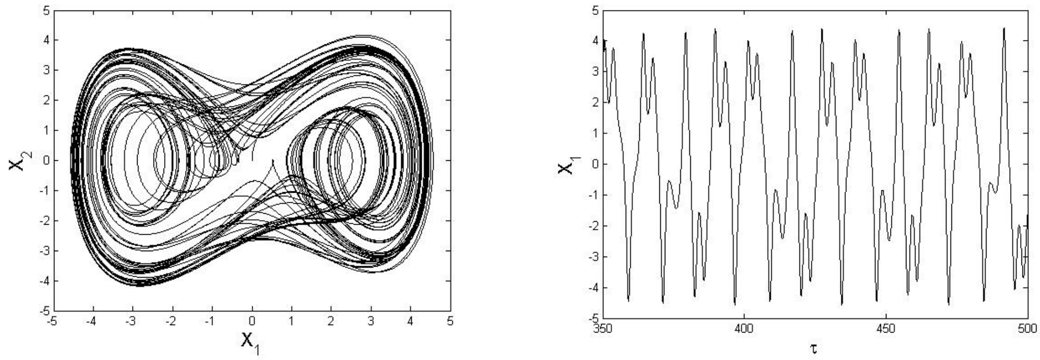

Figure 8. Phase portrait on the left and time history of displacement on the right, system response presents a chaotic behavior

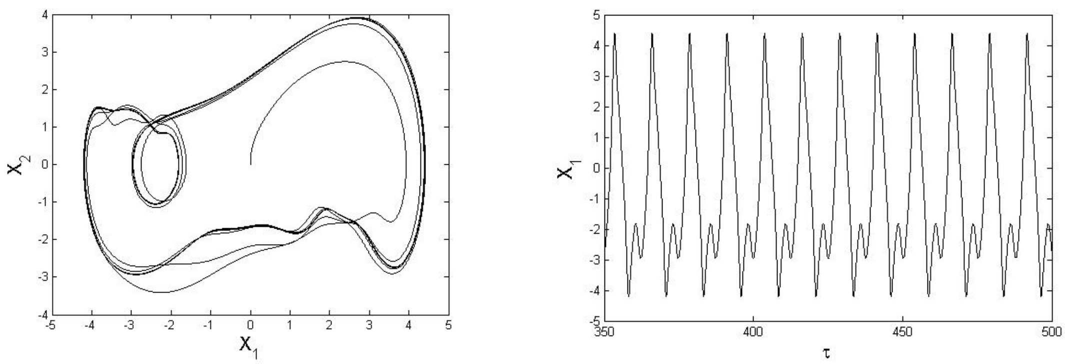

Figure 9. Phase portrait on the left and time history of displacement on the right, system response presents a periodic behavior

3. APPLICATION OF CONTINUOUS WAVELET TRANSFORM

In this section are presented the results of the application of continuous wavelet transform on time series data for each model studied. All figures presents a partial representation of displacement in time of the system. In order to localize of phenomena at time-frequency domain, the wavelet power spectrum (scalogram) was depicted using the command contour of Matlab with the colormap jet in ranges from blue to red, and passes through the colors cyan, yellow, and orange where the blue color represents the lower value of wavelet coefficients and the red color the large value of wavelet coefficients. The thick black contour encloses regions of greater than 95% confidence for a red-noise process with a lag-1 coefficient of 0.72 in accordance with (Torrence and Compo, 1998). The cone of influence also is depicted to indicate the region where edge effects become importat. As reported by (Torrence and Compo, 1998) the global wavelet spectrum provides an unbiased and consistent estimation of the true power spectrum of a time series, i.e. represents the equivalent Fourier power spectrum of the signal. Wong and Chen (2001) in this work stated that the contours of the modulus are represented by a series of turbulence-like cells in the time-frequency domain, which indicates that the impulse responses consist of a set of harmonic modes with different but very close frequencies. The amplitudes of these harmonic modes are constants for the linear case since the turbulence cells are regularly arranged along the time axis. But, for nonlinear system, the amplitudes of these harmonic modes are not constants as shown by the random distribution of turbulence cells. This statement is used to classification of the system behavior in periodic or chaotic.

Figure 10 shows a periodic behavior in the time history of displacement with a wavelet power spectrum indicating a concentration of energy in a specific scale characterizing the movement of one period. The global wavelet spectrum shows a well defined peak of frequency. This is the frequency of excitation true applied to the system.

In Fig. 11 is depicted the irregular oscillatory movement in the time history of displacement and the turbulence cells presented a random distribution in the wavelet power spectrum characterizing the chaotic motion of the system. In

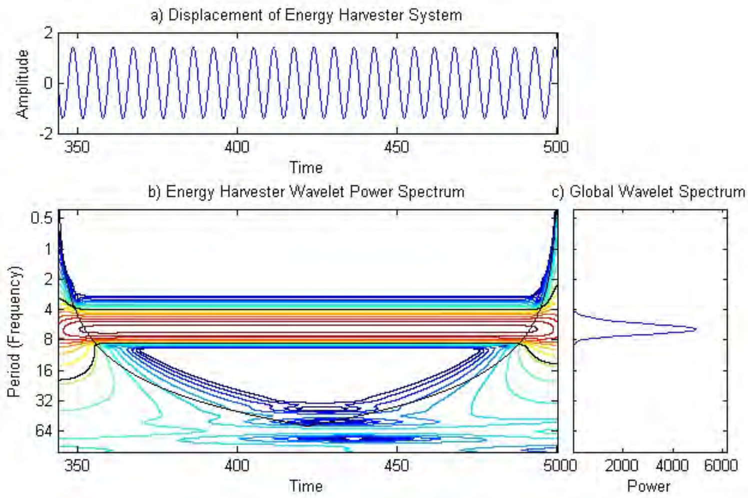

Figure 10. Energy Harvesting model by (Triplett and Quinn, 2009): a)time history of displacement, b) wavelet power spectrum - (scalogram), c) global wavelet spectrum

the global wavelet spectrum the energy has spread among various scales as a result of different oscillation frequencies, multiple harmonic modes are characteristic of the chaotic behavior in the nonlinear system.

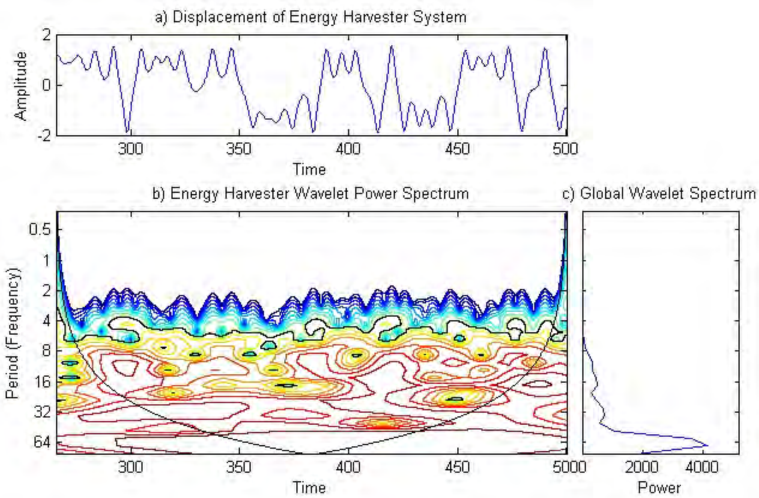

Figure 11. Energy Harvesting model by (Erturk, 2009): a)time history of displacement, b) wavelet power spectrum - (scalogram), c) global wavelet spectrum

The cantilever beam system showed in Fig. 12, has a quasi-periodic oscillatory behavior in the time history of displacement. Analysis of the wavelet power spectrum shows the distribution of the energy in more than one scale. The global wavelet spectrum demonstrate that two principal frequencies are present, this is a characteristic of this type of motion.

The next model presented in Fig. 13 is the non-ideal system of portal frame and it was simulated without the addition of passive absorber (NES). As was expected the system modeled as a forced Duffing oscillator, presented a chaotic motion as can be verified by analyzing in the time history of displacement. The wavelet power spectrum also presents the turbulence cells with a random distribution characterizing the chaotic motion of the system. Global wavelet spectrum shows many frequencies presents in the system.

In Fig. 14 the non-ideal system of portal frame was simulated with passive absorber (NES) coupled. As was to be expected for the value of the control parameter determined by (Iliuk *et al.*, 2013) in the bifurcation diagram of the system, a periodic behavior was achieved. This demonstrates the efficiency of passive control and can be seen in the time history of displacement. In the wavelet power spectrum shows the turbulence cells with a regular distribution along the time axis characterizing a periodic behavior. The global wavelet spectrum perfectly characterize the two oscillation periods of the model.

ISSN 2176-5480

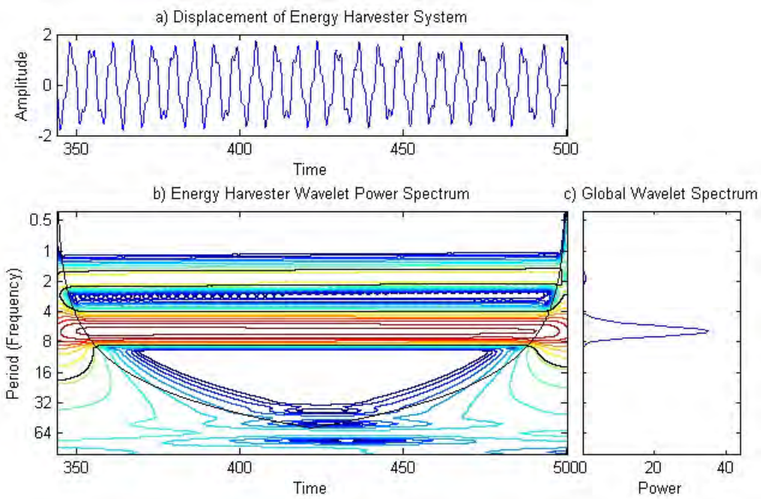

Figure 12. Energy Harvesting Cantilever Beam model by (Iliuk *et al.*, 2012): a)time history of displacement, b) wavelet power spectrum - (scalogram), c) global wavelet spectrum

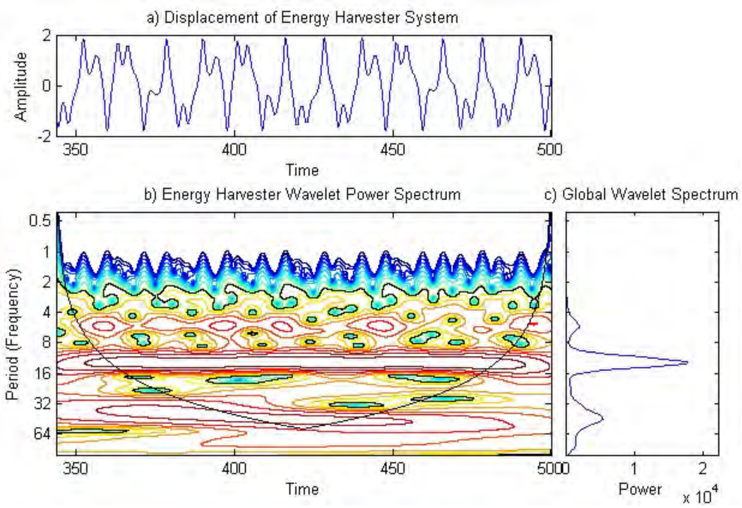

Figure 13. Energy Harvesting Portal Frame model without NES by (Iliuk *et al.*, 2013): a)time history of displacement, b) wavelet power spectrum - (scalogram), c) global wavelet spectrum

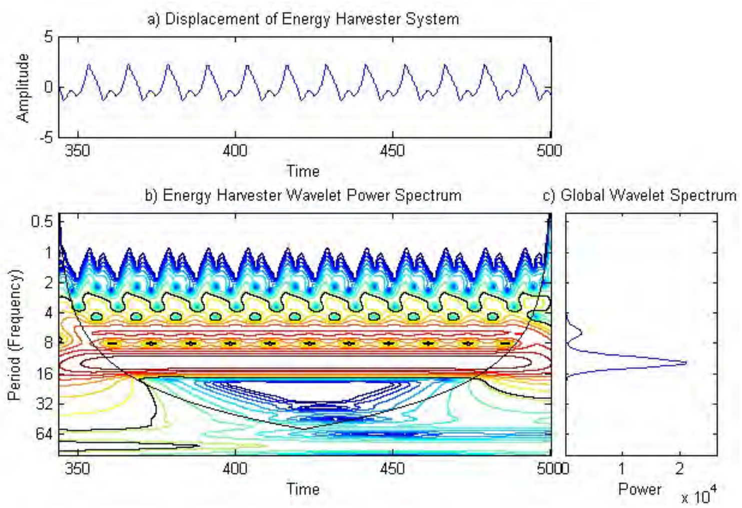

Figure 14. Energy Harvesting Portal Frame model with NES by (Iliuk *et al.*, 2013): a)time history of displacement, b) wavelet power spectrum - (scalogram), c) global wavelet spectrum

ISSN 2176-5480

4. CONCLUSIONS

The analysis of the oscillatory movements of the energy harvester systems can be identified with reasonable accuracy using the continuous wavelet transform (CWT). The toolbox developed by (Torrence and Compo, 1998) is a practical set of tools to analysis of scalogram and global energy spectrum. It can be very useful tool in some cases to identification of type of motion found, e.g. Periodic, Quasi-Periodic and/or Chaotic. Many toolboxes has been developed to analyses of stationary and non-stationary data sets. The results of the application of wavelet transform can be used as an auxiliary measure for the validation of chaos in dynamical systems together with the traditional measurements make by calculation of Lyapunov exponents and Poincaré maps.

5. ACKNOWLEDGMENTS

The authors thank FAPESP, CNPq and CAPES, Brazilian financial agencies.

6. REFERENCES

Addison, P.S., 2002. *The Illustrated Wavelet Transform Handbook*. Institute of Physics Publishing, London, 1st edition.

Balthazar, J.M., Mook, D.T., Weber, H.I., Brasil, R.M.L.R.F., Fenili, A., Belato, D. and Felix, J.L.P., 2003. "An overview on non-ideal vibrations". *Meccanica*, Vol. 38, pp. 613–621.

Chierice Jr, N., 2007. *O uso da análise de Fourier, de wavelets e dos expoentes de Lyapunov no estudo de um sistema dinâmico não-ideal com atrito seco e excitação externa*. Master's thesis, Universidade Estadual Paulista, Rio Claro.

Erturk, A., 2009. *Electromechanical Modeling of Piezoelectric Energy Harvesters*. Ph.D. thesis, Virginia Polytechnic Institute and State University, Blacksburg, VA.

Iliuk, I., Balthazar, J.M., Tusset, A.M., Pontes, B.R. and Felix, J.L.P., 2012. "On non-ideal and chaotic energy harvester behavior". *Journal of Differential Equations and Dynamical Systems*, Vol. 21, pp. 91–105.

Iliuk, I., Balthazar, J.M., Tusset, A.M., Pontes, B.R., Felix, J.L.P., Bueno, A.M. and Piqueira, J.R.C., 2013. "Application of passive control to energy harvester efficiency using a non-ideal portal frame structural support system".

Kyprianou, A. and Staszewski, W.J., 1999. "On the cross wavelet analysis of duffing oscillator". *Journal of Sound and Vibration*, Vol. 228, pp. 199–210.

Permann, D. and Hamilton, I., 1992. "Wavelet analysis of time series for the duffing oscillator: The detection of order within chaos". *Physical Review Letters*, Vol. 69, pp. 2607–2610.

Reboita, M.S., 2004. *Elementos da Variabilidade Climatica no Extremo Sul do Brasil, no Periodo de 1990 a 2001*. Master's thesis, Universidade Federal do Rio Grande, Rio Grande.

Torrence, C. and Compo, G.P., 1998. "A practical guide to wavelet analysis". *Bulletin of the American Meteorological Society*, Vol. 79, pp. 61–78.

Triplett, A. and Quinn, D.D., 2009. "The effect of non-linear piezoelectric coupling on vibration-based energy harvesting". *Journal of Intelligent Material Systems and Structures*, Vol. 20, pp. 1959–1967.

Tusset, A.M., Balthazar, J.M., and Felix, J.L.P., 2013. "On elimination of chaotic behavior in a non-ideal portal frame structural system, using both passive and active controls". *Journal of Vibration and Control*, Vol. 19, pp. 803–813.

Wong, L.A. and Chen, J.C., 2001. "Nonlinear and chaotic behavior of structural system investigated by wavelet transform techniques". *International Journal of Non-linear Mechanics*, Vol. 36, pp. 221–235.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.