APPLICATION OF DESIGN DIAGRAM FOR UNMANNED AERIAL VEHICLE SIZING

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Abstract. During the conceptual design of an airplane, the designers have to find a suitable aircraft configuration to fulfill the mission requirements. Matching weight, wing area and engine thrust to these constraints is a critical step in this process. Among the available design tools, the design diagram is appropriate to solve this task. The aim of this methodology is to represent each performance requirement, such as takeoff distance and climb gradient, as a correlation of wing loading and thrust-to-weight ratio. By superposing the curves of each requirement in a design diagram plot (wing loading vs. thrust-to-weight ratio) it is possible to find a feasible region for the design. Then, a design point is selected within this region, and this result allows the sizing of the wing area and the engine selection. The main challenge of this design tool is the derivation of relationships for each kind of performance constrain. The goal of this work was to derive these relationships for some usual requirements faced by unmanned aerial vehicles, considering an acceptable fidelity level for the conceptual design. A case study was performed for the SAE Aerodesign East 2012 Competition. The team of the Instituto Tecnológico de Aeronáutica chose the aircraft's wing area using the design diagram methodology, considering the competition requirements and engine constrains. Finally, both the reliability of this design tool and the performance of the aircraft were evaluated during the competition, when the airplane reached the desired maximum takeoff weight.

Keywords: Design Diagram, Unmanned Aerial Vehicle, Aircraft Conceptual Design

1. INTRODUCTION

During the conceptual design of an airplane, the designer must match weight, size and engine of the aircraft with the mission requirements. Usually, the weight of the airplane is represented by the Maximum Take-off Weight (W_0), while it's size is represented by the reference area (S) and the engine is represented by the Take-off Thrust (T_0). By combining these parameters, it is possible to establish two ratios that play a major role in aircraft performance: the wing loading (W_0/S) and the thrust-to-weight ratio (T_0/W_0) (Anderson, 1999).

The Design Diagram methodology helps the designer to select appropriate values of wing loading and thrust-to-weight ratio for the aircraft to meet the performance requirements. Then, relationships between W_0/S and T_0/W_0 for each requirement are necessary in order to apply this methodology. Also, a preliminary drag polar and propulsive model must be given.

Several aircraft design books (Roskam (1997) and Raymer (1992)) present wing loading and thrust-to-weight ratio relationships for many requirements, such as cruise speed, climb gradient and take-off distance. When these relationships were applied during the design of a radio-controlled cargo airplane for the SAE Aerodesign East 2012 competition, some problems were found. The main issue of these derivations were the neglection of thrust variations with airspeed and the assumption of small angles of attack, which was not true for this kind of airplane.

The main goal of this work was to derive wing loading and thrust-to-weight ratio equations for performance requirements faced by unmanned aerial vehicles, but without these simplifying assumptions. Secondly, the methodology was evaluated using the aircraft designed by the team of the Instituto Tecnológico de Aeronáutica (ITA) for the SAE Aerodesign East 2012 competition as a case study.

Even though, the relationships presented here are also applicable to manned aircraft designs.

2. DESIGN DIAGRAM METHODOLOGY

It is possible to show that most of the aircraft performance requirements can be translated as relationships between wing loading and thrust-to-weight ratio. These relationships can be plotted in a graph with W_0/S as the horizontal axis and T_0/W_0 as the vertical axis. This graph is called the Design Diagram.

Each requirement represents a curve in the Design Diagram. For points below the curve, the combinations of wing loading and thrust-to-power ratios do not satisfy the requirement, while points above the curve represent feasible designs. So, by superposing curves from all the mission requirements, it is possible to define an acceptable region of the Design Diagram that satisfies all these requirements, and the designer can select a combination of W_0/S and T_0/W_0 from this

area as the Design Point for the aircraft.

These two ratios can be used to select an appropriate engine for the aircraft and also to determine the wing area. Then these parameters allows the designer to size the remaining parts of the airplane with other design tools, such as adimensional coefficients.

The best Design Point usually has the highest wing loading and the lowest thrust-to-power ratio from the acceptable region, as this leads to an aircraft with the lowest weight and the lowest cost (Roskam, 1997).

In the next paragraphs, the relationships for several kinds of performance requirements will be derived considering the point-mass model for the airplane.

2.1 Prerequisites

The Designer must have previous data about the aircraft, the engine and the mission in order to apply the Design Diagram. This demand can be summarized by the following items:

- Aerodynamic data of the aircraft

- Thrust curve of the powerplant

- Mission requirements

The aerodynamic data of the aircraft are the models of lift and drag coefficients (C_L and C_D, respectively) for the trimmed aircraft. If the mission involves several flight regimes, the designer can account for the effects of Reynolds and Mach number over these coefficients. It is also important to define the stall conditions (α_{max}, C_{Lmax} and C_{Dmax}) for the aerodynamic model.

$$
\begin{aligned}
C_L &= C_L(\alpha, \mathrm{Re}, \mathrm{M}) \\
C_D &= C_D(\alpha, \mathrm{Re}, \mathrm{M})
\end{aligned}
\tag{1}
$$

The aerodynamic model also has to consider certain conditions such as flaps deflection, landing gear deployment and windmilling engine, as certain requirements are evaluated at these situations.

The Design Diagram methodology merges performance analysis in different flight regimes. Then, it is important to consider how does the available thrust changes with throttle (π), airspeed (v) and air density (ρ). This is described by the thrust curve. For the Design Diagram methodology, it is advantageous to express the thrust curve as a ratio between the available thrust (T) and the reference thrust (T_0). The reference thrust is usually chosen as the maximum static thrust of the powerplant:

$$
\frac{T}{T_0} = \frac{T}{T_0}(\pi, v, \rho)
\tag{2}
$$

The designer must know which performance requirements guide the aircraft mission. These requirements can be specified by clients or by regulatory agencies. This information helps the designer to select the appropriate curves for the Design Diagram. Cruise speed, take-off distance, landing distance and climb gradients are common type of requirements.

This text will show that it is also possible to evaluate structural requirements with the Design Diagram Methodology, such as the structural efficiency (ratio between the payload and the airplane's empty weight). But this type of analysis demands previous data correlating the size of the aircraft (reference area) and it's structural weight.

2.2 Stall Speed

Many performance requirements are given in function of the stall speed. Then it is important to correlate the design variables (wing loading and thrust-to-weight ratio) with the stall speed.

For this work, the stall speed will be defined as the minimum airspeed such that the aircraft is capable of maintaining level flight. The starting point to calculate this airspeed will be the point-mass equations for the longitudinal flight of an airplane (Hull, 2007):

$$
\begin{cases}
\dot{v} &= g\dfrac{T\cos(\alpha + \alpha_T) - D - W\sin(\gamma)}{W} \\[2ex]
\dot{\gamma} &= g\dfrac{T\sin(\alpha + \alpha_T) + L - W\cos(\gamma)}{W \cdot v}
\end{cases}
\tag{3}
$$

Considering that the airplane is in equilibrium, $\dot{v} = \dot{\gamma} = 0$, then:

$$\begin{cases} T\cos(\alpha + \alpha_T) - D - W\sin(\gamma) = 0 \\ \\ T\sin(\alpha + \alpha_T) + L - W\cos(\gamma) = 0 \end{cases} \tag{4}$$

Eliminating the thrust from the equations, it follows that:

$$\tan(\alpha + \alpha_T) = \frac{W\cos(\gamma) - L}{W\sin(\gamma) + D} \tag{5}$$

Applying the condition for level flight ($\gamma = 0$) and using adimensional coefficients from aerodynamics:

$$\tan(\alpha + \alpha_T) = \frac{W - q_\infty S C_L}{q_\infty S C_D} \tag{6}$$

It is possible to isolate the airspeed from the equation 6. It follows that:

$$v = \sqrt{\frac{2}{\rho(C_L + C_D \tan(\alpha + \alpha_T))} \cdot \frac{W}{S}} \tag{7}$$

In order to minimize the airspeed (v), it is necessary to maximize $C_L + C_D \tan(\alpha + \alpha_T)$, which is done by selecting the condition for maximum lift coefficient. Then, the stall speed (v_s) is given by:

$$v_s = \sqrt{\frac{2}{\rho(C_{Lmax} + C_{Dmax} \tan(\alpha_{max} + \alpha_T))} \cdot \frac{W}{S}} \tag{8}$$

If the stall speed is given as a requirement, it is possible to find the maximum acceptable wing loading with the following equation:

$$\left(\frac{W}{S}\right)_{stall} = \frac{\rho v_s^2}{2}(C_{Lmax} + C_{Dmax} \tan(\alpha_{max} + \alpha_T)) \tag{9}$$

The equation 9 refers to a vertical line in the Design Diagram. It is also possible to find a minimum thrust-to-weight ratio necessary for this condition by substituting equation 8 in 4:

$$\left(\frac{T}{W}\right)_{stall} = \frac{C_{Dmax}}{C_{Lmax}\cos(\alpha_{max} + \alpha_T) + C_{Dmax}\sin(\alpha_{max} + \alpha_T)} \tag{10}$$

This value of thrust-to-weight ratio is usually smaller when compared with other requirements. So it is common to neglect it when plotting the Design Diagram.

2.3 Climb Gradient

Some airworthiness codes (such as FAR 23 (FAA, 1965a), FAR 25 (FAA, 1965b) and STANAG 4671 (NSA, 2009)) specifies airspeeds (v) and climb gradients (γ) that the aircraft must follow during certain flight phases. Usually, the required airspeed is given in function of the stall speed of the aircraft:

$$v = K_s v_s \tag{11}$$

Substituting v_s from equation 8:

$$v = K_s \sqrt{\frac{2}{\rho(C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T))} \cdot \frac{W}{S}} \tag{12}$$

The corresponding dynamic pressure for the climb is:

$$q_\infty = \frac{K_s^2}{C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T)} \cdot \frac{W}{S} \tag{13}$$

Considering that the airplane is climbing in equilibrium, the equations 4 and 5 are valid. From the last one it follows that:

$$\tan(\alpha + \alpha_T) = \frac{W\cos(\gamma) - q_\infty S C_L}{W\sin(\gamma) + q_\infty S C_D} \tag{14}$$

Substituting the dynamic pressure from equation 13:

$$\tan(\alpha + \alpha_T) = \frac{(C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T))\cos(\gamma) - K_s^2 C_L}{(C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T))\sin(\gamma) + K_s^2 C_D} \tag{15}$$

It is necessary to solve equation 15 for α considering C_L and C_D as functions of the angle of attack.

After finding the climb angle of attack, the thrust-to-weight ratio can be found using the first part of equation 4, then:

$$\frac{T}{W} = \frac{1}{\cos(\alpha + \alpha_T)}\left(\sin(\gamma) + \frac{q_\infty S C_D}{W}\right) \tag{16}$$

From equation 13:

$$\left(\frac{T}{W}\right)_{\text{climb}} = \frac{1}{\cos(\alpha + \alpha_T)}\left(\sin(\gamma) + \frac{K_s^2 C_D}{C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T)}\right) \tag{17}$$

As the weight and thrust for the climb condition may differ from the reference weight (W_0) and reference thrust (T_0), it is necessary to use the normalization procedure described in section 2.10 before plotting the results on the Design Diagram. At a first glance, it is reasonable to say that the wing loading has no influence over the climb gradient requirement. This is not true due to the thrust decay with airspeed (T/T_0), as will be shown in section 2.10.

2.4 Take-off Distance

Another type of performance requirement is to take-off in a given distance. For the next analysis, it will be considered only the acceleration of the aircraft until the rotation airspeed (v_{rot}). So, there must be a margin between the rotation distance (s_{rot}) and the desired take-off distance (s_{to}). An option to avoid the estimation of this margin is to actually consider an increment of the angle of attack after the rotation speed was achieved and then calculate when the aircraft takes off, but this will not be exploited in this text.

The rotation airspeed is also defined in function of the stall speed of the aircraft. Using the same formulation from the section 2.3(see equation 12), the rotation airspeed is:

$$v_{rot} = K_s\sqrt{\frac{2}{\rho(C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T))} \cdot \frac{W_0}{S}} \tag{18}$$

The values of K_s used for climb and take-off requirements might not be the same (FAA (1965a) and FAA (1965b)).

Considering the point-mass model for the aircraft, the travelled distance to achieve rotation speed is given by (Hull, 2007):

$$s_{rot} = \int_0^{v_{rot}} \frac{W}{g} \cdot \frac{v}{T\cos(\alpha + \alpha_T) - D - \mu N}dv \tag{19}$$

Where g is the acceleration of gravity, μ is the friction coefficient and N, the normal force, is:

$$N = \max(0, W - L - T\sin(\alpha + \alpha_T)) \tag{20}$$

It is possible to rearrange equations 19 and 20 in terms of wing loading and thrust-to-weight ratio:

$$s_{rot} = \int_0^{v_{rot}} \frac{v}{g} \cdot \frac{1}{\frac{T}{W}\cos(\alpha + \alpha_T) - \frac{\rho v^2}{2}\frac{C_D}{W/S} - \mu\left(1 - \frac{\rho v^2}{2}\frac{C_L}{W/S} - \frac{T}{W}\sin(\alpha + \alpha_T)\right)} \, dv \qquad (21)$$

During the take-off run, it is important to account for thrust variations with airspeed. So the current thrust-to-weight ratio (T/W) can be correlated with the reference thrust-to-weight ratio (T_0/W_0) with the following equation:

$$\frac{T}{W} = \frac{(T/T_0) \cdot (T_0/W_0)}{(W/W_0)} \qquad (22)$$

Where W_0 and T_0 refers to the weight and thrust, respectively, at the beginning of the take-off run, when the aircraft is still. The same idea applies to the wing loading:

$$\frac{W}{S} = \frac{W}{W_0}\frac{W_0}{S} \qquad (23)$$

The fraction T/T_0 comes from the engine model (see section 2.1). The fraction W/W_0 accounts for the weight variations during take-off (like fuel consumption). For preliminary analysis, this weight variation can be neglected, thus $W/W_0 = 1$. Substituting Eq. 22 and 23 into 21:

$$s_{rot} = \int_0^{v_{rot}} \frac{v}{g} \cdot \frac{1}{\frac{T}{T_0}\left(\frac{T_0}{W_0}\right)_{to}\cos(\alpha + \alpha_T) - \frac{\rho v^2}{2}\frac{C_D}{W_0/S} - \mu\left(1 - \frac{\rho v^2}{2}\frac{C_L}{W_0/S} - \frac{T}{T_0}\left(\frac{T_0}{W_0}\right)_{to}\sin(\alpha + \alpha_T)\right)} \, dv \quad (24)$$

Equation 24 represents the take-off requirement at the Design Diagram. It is not possible to isolate the thrust-to-weight ratio from equation 24, so it is necessary to solve it iteratively for a given set of wing loadings.

It is still necessary to define an angle of attack (α) to calculate C_L and C_D at the Eq. 24. The optimum value of angle of attack will minimize the take-off distance. As α influences only the denominator of the integral in Eq. 24, the optimum value of α will be given by:

$$\frac{d}{d\alpha}\left(\frac{T}{T_0}\left(\frac{T_0}{W_0}\right)_{to}\cos(\alpha + \alpha_T) - \frac{\rho v^2}{2}\frac{C_D}{W_0/S} - \mu\left(1 - \frac{\rho v^2}{2}\frac{C_L}{W_0/S} - \frac{T}{T_0}\left(\frac{T_0}{W_0}\right)_{to}\sin(\alpha + \alpha_T)\right)\right) = 0 \quad (25)$$

A numerical optimization can be done in order to find the optimal angle of attack. But as the Eq. 24 will be solved iteratively, adding an optimization at each iteration will slow down the calculations. An alternative is to use simplified analytical results for this problem (Hull, 2007):

$$\alpha_{opt} = \frac{1}{C_{L\alpha}}\left(\frac{\mu - k_1}{2k} - C_{L0}\right) \qquad (26)$$

Equation 26 can be obtained from Eq. 25, considering that the angle of attack is small, the lift curve is given by $C_L = C_{L0} + \alpha C_{L\alpha}$ and the drag polar is $C_D = C_{D0} + k_1 C_L + k C_L^2$.

2.5 Landing Distance

The landing distance analysis is quite similar to the take-off analysis since the forces applied on the aircraft are the same. Considering that the aircraft touches down the ground with an approach speed v_{app}, it is possible to use Eq. 21 as a baseline, adjusting the integration limits:

$$s_{lan} = \int_{v_{app}}^{0} \frac{v}{g} \cdot \frac{1}{\frac{T}{W}\cos(\alpha + \alpha_T) - \frac{\rho v^2}{2}\frac{C_D}{W/S} - \mu\left(1 - \frac{\rho v^2}{2}\frac{C_L}{W/S} - \frac{T}{W}\sin(\alpha + \alpha_T)\right)} \, dv \qquad (27)$$

Equations 22 and 23 can be used to substitute T/W and W/S in Eq. 27. But this time the ratio W/W_0 is different from 1, as the landing weight (W_{lan}) might be different from the take-off weight (W_0). However, it is still plausible to use a constant value for W/W_0 during the integration process. For the next derivations, consider:

$$\frac{W}{W_0} = \frac{W_{\text{lan}}}{W_0} \tag{28}$$

Then Eq. 27 becomes:

$$s_{\text{lan}} = \int_{v_{app}}^{0} \frac{\frac{v}{g}}{\frac{\frac{T}{T_0}\left(\frac{T_0}{W_0}\right)_{\text{lan}}}{\frac{W_{\text{lan}}}{W_0}}\cos(\alpha + \alpha_T) - \frac{\frac{\rho v^2}{2}C_D}{\frac{W_{\text{lan}}}{W_0}\frac{W_0}{S}} - \mu\left(1 - \frac{\frac{\rho v^2}{2}C_L}{\frac{W_{\text{lan}}}{W_0}\frac{W_0}{S}} - \frac{\frac{T}{T_0}\left(\frac{T_0}{W_0}\right)_{\text{lan}}}{\frac{W_{\text{lan}}}{W_0}}\sin(\alpha + \alpha_T)\right)}\, dv \tag{29}$$

The requirements usually define the approach airspeed as a function of the stall speed of the aircraft. Using the derivations from the section 2.3(see equation 12), the approach airspeed is:

$$v_{app} = K_s \sqrt{\frac{2}{\rho(C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T))} \cdot \frac{W_{\text{lan}}}{W_0} \cdot \frac{W_0}{S}} \tag{30}$$

The integrand from Eq. 29 must be negative in the whole integration interval to maintain mathematical consistency. Evaluating the denominator of the integrand for $v = 0$ we have:

$$\frac{T/T_0 \cdot (T_0/W_0)_{\text{lan}}}{W_{\text{lan}}/W_0}\cos(\alpha + \alpha_T) - \mu\left(1 - \frac{T/T_0 \cdot (T_0/W_0)_{\text{lan}}}{W_{\text{lan}}/W_0}\sin(\alpha + \alpha_T)\right) < 0 \tag{31}$$

If the engine is in idle condition during landing, the value of T/T_0 is small, so satisfying Eq. 31 shouldn't be an issue.

2.6 Cruise Speed

The clients may demand a specific cruise speed at a certain altitude for the airplane. This type of requirement can also be displayed in the Design Diagram. The first step to find this curve is to calculate the dynamic pressure at the cruise conditions:

$$q_\infty = \frac{\rho v_{cru}^2}{2} \tag{32}$$

Where v_{cru} is the cruise speed and ρ is the air density at the cruise altitude. Secondly, it is possible to use Eq. 14 with $\gamma = 0$ for level flight:

$$\tan(\alpha + \alpha_T) = \frac{W - q_\infty S C_L}{q_\infty S C_D} \tag{33}$$

Equation 33 can be expressed in terms of wing loading as follows:

$$\tan(\alpha + \alpha_T) = \frac{W/S - q_\infty C_L}{q_\infty C_D} \tag{34}$$

For a given set of wing loading, this equation has to be solved iteratively for α, considering C_L and C_D as functions of the angle of attack. After α is known, the corresponding thrust-to-weight ratio can be found using Eq. 16 for $\gamma = 0$:

$$\left(\frac{T}{W}\right)_{\text{cruise}} = \frac{1}{\cos(\alpha + \alpha_T)}\left(\frac{q_\infty C_D}{W/S}\right) \tag{35}$$

As the weight and thrust for the cruise condition may differ from the reference weight (W_0) and reference thrust (T_0), the use of the normalization procedure described in section 2.10 is also necessary for this type of requirement.

2.7 Level Curve Radius

If there is a minimum curve radius specified in the aircraft mission, the procedure described in this section allows to determine the feasible region of the Design Diagram according to this restriction.

Considering that the aircraft is banked with an angle ϕ in a level turn of radius R with airspeed v_{cur}, the following relationships comes from equilibrium conditions (Hull, 2007):

$$
\begin{cases}
T\cos(\alpha + \alpha_T) - D &= 0 \\[2mm]
\cos(\phi) \cdot (T\sin(\alpha + \alpha_T) + L) - W &= 0 \\[2mm]
\sin(\phi) \cdot (T\sin(\alpha + \alpha_T) + L) - \dfrac{Wv^2}{gR} &= 0
\end{cases}
\tag{36}
$$

Expressing Eq. 36 in terms of wing loading and thrust-to-weight ratio, we have:

$$
\begin{cases}
\dfrac{T}{W}\cos(\alpha + \alpha_T) &= \dfrac{q_\infty C_D}{W/S} \\[3mm]
\cos(\phi) \cdot \left(\dfrac{T}{W}\sin(\alpha + \alpha_T) + \dfrac{q_\infty C_L}{W/S} \right) &= 1 \\[3mm]
\sin(\phi) \cdot \left(\dfrac{T}{W}\sin(\alpha + \alpha_T) + \dfrac{q_\infty C_L}{W/S} \right) &= \dfrac{v^2}{gR}
\end{cases}
\tag{37}
$$

If the curve airspeed is given in terms of the stall airspeed, the dynamic pressure can be rewritten as (see Eq. 13):

$$
q_\infty = \frac{\rho v_{cur}^2}{2} = \frac{K_s^2}{C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T)} \cdot \frac{W}{S}
\tag{38}
$$

Where K_s is the ratio between the desired airspeed and the stall airspeed for the curve requirement.

The angle ϕ can be found using the second and third parts from Eq. 37:

$$
\tan(\phi) = \frac{v_{cur}^2}{gR} = \frac{2K_s^2}{\rho gR} \cdot \frac{W/S}{C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T)}
\tag{39}
$$

Where g is the acceleration of gravity. Substituting the value of T/W from the first part of Eq. 37 into the second part, we have:

$$
C_L + C_D\tan(\alpha + \alpha_T) = \frac{C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T)}{K_s^2\cos(\phi)}
\tag{40}
$$

This equation has to be solved iteratively for α, considering C_L and C_D as functions of the angle of attack. After α is known, the corresponding thrust-to-weight ratio can be found using the first part of Eq. 37:

$$
\left(\frac{T}{W} \right)_{\text{curve}} = \frac{K_s^2}{\cos(\alpha + \alpha_T)} \cdot \frac{C_D}{C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T)}
\tag{41}
$$

Using Eq. 39, 40 and 41, it is possible to correlate a given set of wing loadings to a set of corresponding thrust-to-weight ratios for the curve requirements. As the weight and thrust during the curve phase may differ from the reference weight (W_0) and reference thrust (T_0), it is necessary to use the normalization procedure described in section 2.10 before plotting the results on the Design Diagram.

2.8 Hand launch take-off

Some light Unmanned Aerial Vehicles may be designed to operate without the need of a runway for take-off and landing. An alternative in this case is the hand launch take-off. For this operation to be successful, the aircraft has to be

launched with an airspeed greater than the stall airspeed (v_s). Mathematically speaking, the requirement may set a value of $K_s > 1$ such that the throwing airspeed (v) is:

$$v = K_s v_s \tag{42}$$

Cross (2004) states that the kinetic energy (KE) obtained by an object of mass m thrown by a human arm is given by:

$$KE = \frac{mv^2}{2} = \frac{mA}{m+B} \tag{43}$$

In Eq. 43, A and B are adjustable parameters that depends on the work that the thrower's arm can exert and the mass of the thrower's arm, respectively. In the work presented by Cross (2004), the values of A ranged from 94 J to 179 J and the values of B ranged from 0.3 kg to 0.71 kg.

Using Eq. 8, Eq. 42 and 43, we find that:

$$\left(\frac{W_0}{S}\right)_{\text{handlaunch}} = (C_{Lmax} + C_{Dmax}\tan(\alpha_{max} + \alpha_T))\frac{\rho}{K_s^2}\frac{A}{B + m_0} \tag{44}$$

Unfortunately a guess for the take-off mass (m_0) is needed in order to apply Eq. 44. It is suggested to use a value slightly greater than the maximum mass expected for the airplane.

The wing loading given by Eq. 44 corresponds to a vertical line on the Design Diagram. Only wing loadings smaller than this one will satisfy the hand launch requirement.

2.9 Weight Distribution

Consider the following weight breakdown for the aircraft:

$$W_0 = W_{fix} + W_{var} + W_{prp} + W_{fuel} + W_{pay} \tag{45}$$

Where W_0 is the maximum take-off weight, W_{fix} is the weight of fixed components of the aircraft (which does not depend of the aircraft size), W_{var} represents the weight fraction that varies with the aircraft size, W_{prp} is the weight of the propulsive system, W_{fuel} is the fuel weight and W_{pay} is the payload weight.

The payload weight fraction can be expressed in function of the structural efficiency of the aircraft (SE), which will be defined as follows:

$$\text{SE} = \frac{W_{pay}}{W_0 - W_{pay}} = \frac{W_{pay}}{W_{fix} + W_{var} + W_{prp} + W_{fuel}} \tag{46}$$

Using Eq. 45 and 46:

$$W_0 = (W_{fix} + W_{var} + W_{prp} + W_{fuel}) \cdot (1 + \text{SE}) \tag{47}$$

For the conceptual design phase, it may be interesting to correlate the weight of the variable components (W_{var}) with the aircraft size (represented by the wing area) and the weight of the propulsive system (W_{prp}) with the reference thrust (T_0). So Eq. 47 can be rewritten as:

$$\frac{1}{1 + \text{SE}} = \left(\frac{W_{fix}}{W_0} + \frac{W_{var}/S}{W_0/S} + \frac{T_0/W_0}{T_0/W_{prp}} + \frac{W_{fuel}}{W_0}\right) \tag{48}$$

$$\frac{T_0}{W_0} = \frac{T_0}{W_{prp}}\left(\frac{1}{1 + \text{SE}} - \frac{W_{fix}}{W_0} - \frac{W_{fuel}}{W_0} - \frac{W_{var}/S}{W_0/S}\right) \tag{49}$$

The values of SE and W_{fix}/W_0 may be specified as design targets, while T_0/W_{prp} and W_{var}/S may be obtained from historical data. The fuel fraction (W_{fuel}/W_0) can be calculated using information from the aircraft mission. If W_{end} is the aircraft weight at the end of the mission, the fuel weight is given by:

$$W_{fuel} = W_0 - W_{end} \tag{50}$$

$$\frac{W_{fuel}}{W_0} = 1 - \frac{W_{end}}{W_0} \tag{51}$$

The fraction W_{end}/W_0 from Eq. 51 can be split in fractions for each mission phase. For example, consider the following phases: take-off run, climb, cruise, loiter, descent and landing. Then Eq. 51 becomes:

$$\frac{W_{fuel}}{W_0} = 1 - \frac{W_{end}}{W_{landing}} \frac{W_{landing}}{W_{descent}} \frac{W_{descent}}{W_{loiter}} \frac{W_{loiter}}{W_{cruise}} \frac{W_{cruise}}{W_{climb}} \frac{W_{climb}}{W_{run}} \frac{W_{run}}{W_0} \tag{52}$$

Some aircraft design books gives estimations for most of these weight fractions (see Roskam (1997) and Raymer (1992)), but some fractions actually depends on the current design specifications, such as cruise range, cruise altitude and loiter time. To estimate these fractions, further analysis is needed. Consider the following definition for specific fuel consumption (C) (Hull, 2007):

$$C = -\frac{\dot{W}_{fuel}}{T} = -\frac{\dot{W}}{T} \tag{53}$$

Considering that the aircraft is in level cruise with small angle of attack, we have:

$$\begin{cases} T &= D \\ W &= L \end{cases} \tag{54}$$

Using Eq. 53 and 54, and assuming a parabolic drag polar ($C_D = C_{D0} + k_1 C_L + k C_L^2$), it follows that:

$$\dot{W} = -C q_\infty S C_{D0} - C k_1 W - \frac{C K W^2}{q_\infty S} \tag{55}$$

As the aircraft begins the cruise phase with the weight equals to W_{climb}, this value will be used to adimensionalize Eq. 55:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{W}{W_{climb}}\right) = -\frac{C q_\infty C_{D0}}{W_{climb}/S} - C k_1 \frac{W}{W_{climb}} - \frac{C K}{q_\infty} \frac{W_{climb}}{S}\left(\frac{W}{W_{climb}}\right)^2 \tag{56}$$

The value for W_{climb}/S can be obtained using a reference wing loading (W_0/S) and estimates for the fuel consumption during the previous phases:

$$\frac{W_{climb}}{S} = \frac{W_{climb}}{W_{run}} \frac{W_{run}}{W_0} \frac{W_0}{S} \tag{57}$$

If a constant cruise speed is assumed, then $v_{cru} = \mathrm{d}x/\mathrm{d}t$ and q_∞ is given by Eq. 32. So Eq. 56 becomes:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{W}{W_{climb}}\right) = -\frac{C q_\infty C_{D0}}{v_{cru}(W_{climb}/S)} - \frac{C k_1}{v_{cru}} \frac{W}{W_{climb}} - \frac{C K}{v_{cru} q_\infty} \frac{W_{climb}}{S}\left(\frac{W}{W_{climb}}\right)^2 \tag{58}$$

As Eq. 58 represents a separable differential equation, it can be integrated over the cruise range R:

$$\int_0^R \mathrm{d}x = \int_1^{W_{cruise}/W_{climb}} \frac{1}{-\dfrac{C q_\infty C_{D0}}{v_{cru}(W_{climb}/S)} - \dfrac{C k_1}{v_{cru}} \dfrac{W}{W_{climb}} - \dfrac{C K}{v_{cru} q_\infty} \dfrac{W_{climb}}{S}\left(\dfrac{W}{W_{climb}}\right)^2} \cdot \mathrm{d}\left(\frac{W}{W_{climb}}\right) \tag{59}$$

$$R = \int_{1}^{W_{cruise}/W_{climb}} \frac{1}{-\dfrac{Cq_{\infty}C_{D0}}{v_{cru}(W_{climb}/S)} - \dfrac{Ck_1}{v_{cru}}\dfrac{W}{W_{climb}} - \dfrac{CK}{v_{cru}q_{\infty}}\dfrac{W_{climb}}{S}\left(\dfrac{W}{W_{climb}}\right)^2} \cdot \mathrm{d}\left(\frac{W}{W_{climb}}\right) \qquad (60)$$

The value of W_{cruise}/W_{climb} can be found by solving Eq. 60.

An analogous procedure can be done for the loiter phase, which may require a loiter time E. This yields the following equation:

$$E = \int_{1}^{W_{loiter}/W_{cruise}} \frac{1}{-\dfrac{Cq_{\infty}C_{D0}}{W_{cruise}/S} - Ck_1\dfrac{W}{W_{cruise}} - \dfrac{CK}{q_{\infty}}\dfrac{W_{cruise}}{S}\left(\dfrac{W}{W_{cruise}}\right)^2} \cdot \mathrm{d}\left(\frac{W}{W_{cruise}}\right) \qquad (61)$$

Where:

$$\begin{aligned} q_{\infty} &= \frac{\rho v_{loiter}^2}{2} \\[2mm] \frac{W_{cruise}}{S} &= \frac{W_{cruise}}{W_{climb}}\frac{W_{climb}}{W_{run}}\frac{W_{run}}{W_0}\frac{W_0}{S} \end{aligned} \qquad (62)$$

The value of W_{loiter}/W_{cruise} can be found by solving Eq. 61.

After all weight fractions are known, Eq. 49 and Eq. 52 can be used to correlate a given set of wing loading with corresponding values of thrust-to-weight ratios.

2.10 Normalization

The relationships derived for some requirements correlates wing loadings and thrust-to-weight ratios for a specific phase of the mission. Before joining the constraint curves from several requirements it is necessary to ensure that all these curves refer to the same conditions of weight and thrust, as these parameters change during the flight. Usually, the reference condition is the take-off with the Maximum Take-off Weight (MTOW).

In order to normalize the wing loading and the thrust-to-weight ratio, it is necessary to know the ratio between the current phase weight and the reference weight (W/W_0) and the ratio between the current phase required thrust and the reference thrust (T/T_0). These values can be found using the relationships presented during sections 2.9 and 2.1.

For a given reference wing loading interval, it is possible to calculate the corresponding wing loading at the current phase using Eq. 63:

$$\frac{W}{S} = \frac{W}{W_0}\frac{W_0}{S} \qquad (63)$$

After using the requirement's equations to obtain the current thrust-to-weight ratio (T/W), the thrust-to-weight ratio for the reference conditions (T_0/W_0) can be found using Eq. 64:

$$\frac{T_0}{W_0} = \frac{(W/W_0)(T/W)}{(T/T_0)} \qquad (64)$$

Certain requirements dictates that the aircraft must fly at a certain condition with inoperative engines. This can be taken in account during the normalization procedure. If the aircraft has N engines and N_f engines are non-operative, then 64 becomes:

$$\frac{T_0}{W_0} = \frac{N}{N - N_f} \cdot \frac{(W/W_0)(T/W)}{(T/T_0)} \qquad (65)$$

It is important to note that T/T_0 depends on the aircraft's airspeed during the current mission phase, which in turn may depend on the given wing loading. In these cases it is necessary to use Eq. 12 to obtain the airspeed using the requirements of the phase.

The corresponding values of W_0/S and T_0/W_0 are now ready to be plotted on the Design Diagram.

3. CASE STUDY: SAE AERODESIGN EAST 2012 COMPETITION

The aircraft designed by the team of Instituto Tecnológico de Aeronáutica for the SAE Aerodesign East 2012 Competition will be used as a case study of a Design Diagram Methodology implementation, what was actually done during the conceptual design phase of this aircraft.

The goal of this competition was to design a radio-controlled cargo airplane which would carry as much payload as possible in a complete circuit around the runaway. So, the mission phases were: take-off, climb, curve, cruise, descent, go-around and landing. This aircraft would be controlled by a pilot on the ground, and visual contact would be maintained during the entire flight.

The Maximum Take-off Weight (considering both payload and aircraft) allowed was 55 pounds (24.9 kg) and the engine's combustion chamber volume displacement was restricted to a maximum of 0.65 cubic inches. The take-off and landing distances were limited to 200 ft. Other mission requirements are summarized in Tab. 1.

As the aircraft configuration would be very similar to previous designs made by the team, the aerodynamics models was based on historical data (see Fig. 1 a) and b)). The thrust curve (Fig. 1 c)) was obtained through wind tunnel testing of the engine OS .65 AX with an APC 15x4 propeller and a tuning pipe.
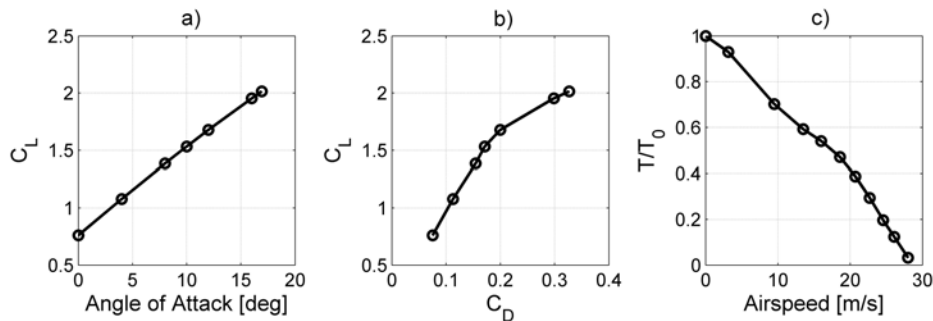

Figure 1. Prerequisites for the Design Diagram Methodology. Pictures a) and b) are the aerodynamic data for the trimmed aircraft ($Re = 3.0 \cdot 10^5$). Picture c) is the thrust curve for the engine OS .65 AX with an APC 15x4 propeller and a tuning pipe ($\rho_{ref} = 1.15 kg/m^3$, $T_0 = 49.1N$).

Table 1. Summary of the mission requirements adopted for the SAE Aerodesign East 2012 Competition.

Mission Phase	Parameter	Value	Mission Phase	Parameter	Value
Take-off	distance (s_{rot})	60 m	Descent	gradient (γ)	-0.052 rad
	stall speed factor (K_s)	1.1		stall speed factor (K_s)	1.3
	friction coefficient	0.03	Go-around	gradient (γ)	0.03 rad
Climb	gradient (γ)	0.03 rad		stall speed factor (K_s)	1.2
	stall speed factor (K_s)	1.1	Landing	distance (s_{lan})	60 m
Curve	radius (R)	30 m		stall speed factor (K_s)	1.3
	stall speed factor (K_s)	1.1		friction coefficient	0.3

For this particular aircraft, the weight breakdown considered only the weight of fixed components (engine, landing gear, electrical system and data acquisition system), the weight of the variable components (wing, tail and fuselages) and the payload. So the Weight Distribution relationship derived from Eq. 49 became:

$$\left(\frac{W_0}{S}\right)_{weight} = \frac{W_{var}/S}{\frac{1}{1+\text{SE}} - \frac{W_{fix}}{W_0} - \frac{W_{fuel}}{W_0}} \tag{66}$$

The weight of the fixed components of the aircraft totalized 2.2 kg, and as the MTOW was limited at 24.9 kg then $W_{fix}/W_0 = 0.088$. Previous aircraft built by the team allowed to estimate the value of W_{var}/S as $12.56N/m^2$. The targeted structural efficiency (SE) was 3. As the fuel corresponded to a small fraction of the airplane's weight ($W_{fuel}/W_0 = 0.006$), weight changes during the mission were neglected, then $W/W_0 = 1$ for all phases.

These consideration resulted in the Design Diagram of Fig. 2. As the engine and the MTOW were already set, the value of T_0/W_0 was known to be 0.201, what led to the "engine " line at the Design Diagram. So the Design Point had to lie in the segment of this line that was bounded by the feasible design region. The selected wing loading was $101N/m^2$, which allowed a 9% margin from the climb requirement. This resulted in a wing area of $2.42m^2$.

Ney Secco, Bento Mattos
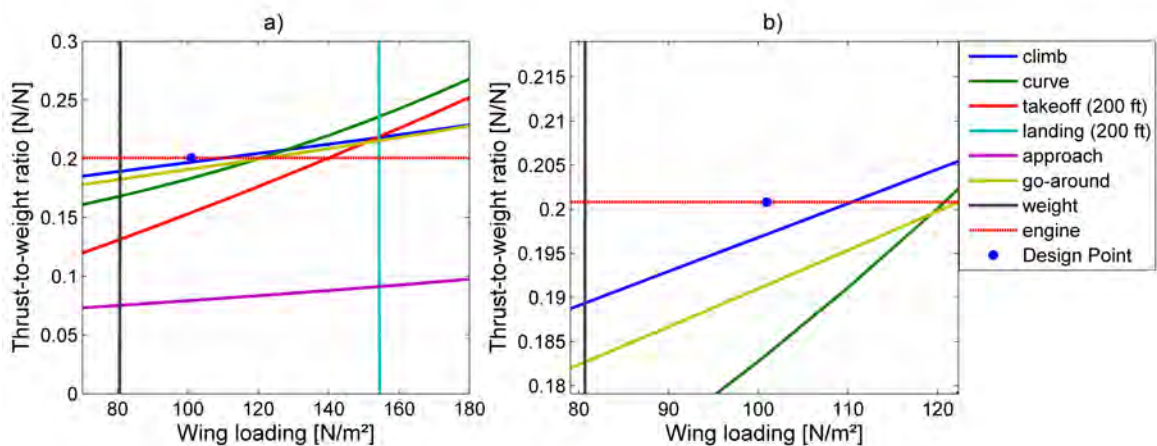Application of Design Diagram for Unmanned Aerial Vehicle Sizing

Figure 2. Design Diagram for SAE Aerodesign East 2012 requirements and the Design Point selected by ITA's team. Picture b) is a crop of Picture a) emphasizing the vicinities of the Design Point. $\rho = 1.18 kg/m^3$.

This design was actually built and tested during the SAE Aerodesign East 2012 competition, when the aircraft accomplished the proposed mission with an MTOW of 24.93 kg.

4. CONCLUSION

This work presented relationships between wing loading and thrust-to-weight ratio for several mission phases that can be faced by unmanned airplanes: Climb gradient, take-off distance, landing distance, cruise speed, level curve and hand launch take-off. The possibility of including aspects of the aircraft's structures and systems in the Design Diagram Methodology was also shown.

It is important to normalize the thrust and the weight in each restriction to a reference condition (usually take-off) before comparing the requirements' curves with the Design Diagram.

This methodology proved to be useful as it helps the designer to make appropriate choices of engine and wing area. Also, the Design Diagram gives an overview of how each requirements impacts the design and which of these requirements are most restrictive.

Finally, the methodology was applied during the conceptual design of a radio-controlled airplane for the SAE Aerodesign East 2012 competition and the aircraft actually accomplished the mission for which it was designed.

5. ACKNOWLEDGEMENTS

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