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# ANALYSIS OF THE BEHAVIOUR OF A RIGID CHASSIS WITH DIFFERENTIATED FRONT AND REAR SUSPENSIONS THROUGH POWER FLOW

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***Abstract***

*This paper presents a mathematical model solved by means of the power flow. It allows that the vehicle model be subdivided into subsystems, which interact through kinematic links, transmitting only forces, linear and angular velocities.*

*The implemented model analyzes the dynamic behavior of a complete vehicle composed by suspensions of overlapped arms in the front set and swing axle in the rear set. The dynamic behavior can be analyzed using data obtained by kinematics, since each subsystem may be analyzed alone, as there is a segregation of the kinematics and dynamics in the subsystem equating.*

*The use of different kinds of suspensions in a same chassis allows to evaluate the difference in the behavior between them and the effects caused on the car body.*

***Keywords:*** *modularity, vehicular dynamics ,swing-axle suspension, double A-arm suspension, Power flow.*

## 1. INTRODUCTION

The classical mathematical models used to study show a satisfactory result only if small displacements are considered for the suspended mass, which represents the chassis, and to the non-suspended masses representative of the wheels. The ¼ vehicle model (Jazar, 2008) is particularized for a system mass-spring-damper with two degrees of freedom, and of ½ vehicle to a similar system having four degrees of freedom, where the suspensions are represented only by their springs and dampers. From a given input signal, it is possible to study, in both cases the displacement of the suspended masses and non-suspended (chassis and tire) and on the model of ½car also the chassis roll. Linear models offer a response of the vehicle dynamics without consider the geometry of the suspensions and the variation of the camber, caster, Kingpin and convergence angles.

This work has the objective of presenting the modular modeling of subsystems of a ground vehicle in its spatial form. The equating based in the power flow allows modeling the subsystems chassis, suspension, wheels and tires in a modular way, thus signifying that these subsystems can be replaced by equivalent, since the causality be maintained.

From the methodology used (Costa Neto, 2008), a vehicle model in built with suspensions of the type overlapped arms on the front set, and swing axle in rear set, thus demonstrating the practicability of the linkage of different suspensions subsystems in the same chassis without it has undergone modifications in it equating, as well as evaluating the difference in the behavior of the two suspensions.

## 2. LITERATURE REVIEW

In reference (Costa Neto, 2008), the vehicle is divided into subsystems, using modular approach through the power flow. This approach facilitates the visualization of relations of causality between the subsystems, making it possible to define clearly who are the input and output variables of each module. The approach is based on the technique of bound graphs (Karnopp, 1990), which is used in multidomain systems in general, and uses some concepts of methodology of kinematic converters applied to multibody systems (Hiller, 1986). The advantage of a modeling based on the power flow is that the modules can be replaced by others since the causality be maintained.

## 3. METHODOLOGY

### 3.1 Chassis

The chassis is considered as a rigid body with six (6) degree of freedom: longitudinal position, lateral position, vertical position, lateral angle of inclination ("Roll"), pitch angle ("Pitch") and steering angle ("Yaw" ), as can be seen in Fig. 1.

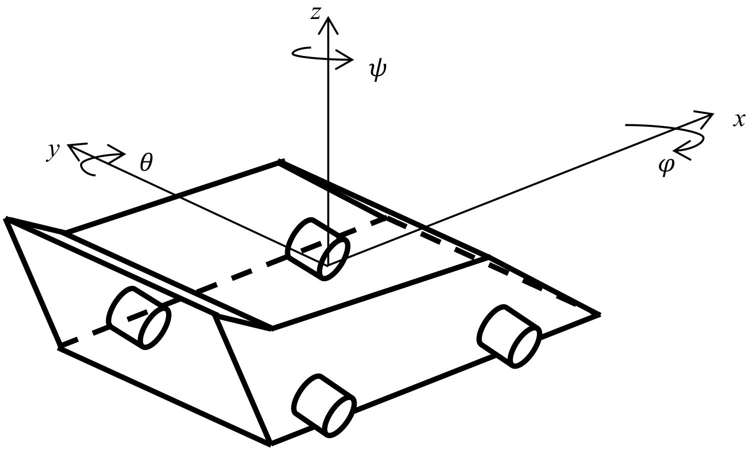


Figure 1. Representation of the vehicle coordinates

Solved by power flow, the chassis is modeled as a Fig. 2 receiving efforts and responding in flow.

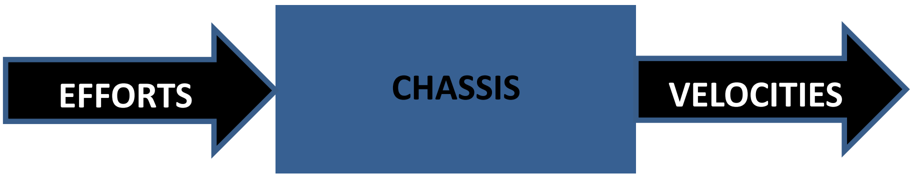


Figure 2. Diagram of power flow of the chassis

The Eq. (1) of the position of any hardpoint in the coordinates of fixed referential, when derived, gives the absolute velocity of these points Eq. (2).

$${}^{f}\underline{r}_{pi} = {}^{f}\underline{r}_{c} + {}^{f}T^{c}\,{}^{c}_{c}\underline{r}_{pi} \tag{1}$$

$${}^{f}\underline{v}_{pi} = {}^{f}\underline{v}_{c} - {}^{f}T^{c}\,{}^{c}_{c}\underline{\tilde{r}}_{pi}\;{}^{c}\underline{\Omega} \tag{2}$$

From the Eq. (2) it is possible to write the relation where the velocities of the center of gravity are transmitted to any point on the chassis. This relationship appears in the Eq. (3) in matrix form, which is defined as the matrix of kinematic linkages (Costa Neto, 2008).

$${}^{f}\underline{v}_{pi} = \Theta_{pi}\begin{bmatrix} {}^{f}\underline{v}_{c} \\ {}^{c}\underline{\Omega} \end{bmatrix} \tag{3}$$

The power conservation allows to define the dynamic behavior from the kinematic relationships established where to calculate the effort transmitted force applied at any point of the structure to the center of gravity from a must be pre-multiplying this vector force applied on a point by the transposed of the kinematic linkages matrix Eq. (4) .

$$\begin{bmatrix} \sum {}^{f}\underline{F}_{c} \\ \sum {}^{c}\underline{M}_{c} \end{bmatrix} = {\Theta_{pn}}^{t}\,{}^{f}\underline{F}_{pi} \tag{4}$$

Where:
${}^{f}\underline{F}_{c}$ – Force transmitted to the center of gravity;
${}^{f}\underline{F}_{pi}$ – Force applied to the point $i$ in the local referential;
$i$ – Named point of the chassis;
${}^{c}_{c}\underline{r}_{pi}$ – Position of the point $i$ in the local referential;
${}^{f}\underline{r}_{pi}$ – Position of the point $i$ in the inertial referential;
${}^{f}\underline{r}_{c}$ – Position of the center of gravity in the inertial referential;
${}^{f}T^{c}$ – Transformation of coordinates matrix;
${}^{f}\underline{v}_{pi}$ – Absolute velocity of the point $i$ in the fixed referential;

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${}^{f}\underline{v}_{c}$ – Velocity of the center of gravity in the fixed referential;
$\Theta_{pi}$ – Kinematics linkage matrix of the chassis; and
${}^{c}\underline{\Omega}$ – Angle variation rate of the local referential.

### 3.2 Suspension

The suspension, as can be seen in Fig. 3 is a system that receives it's hardpoints velocities and the wheels, and responds with efforts produced by its elastic elements which are constituted by the spring and damper.


Figure 3. Power flow diagram of the suspension

The suspension used in the rear set of the swing-axle kind is presented in the Fig. 4.

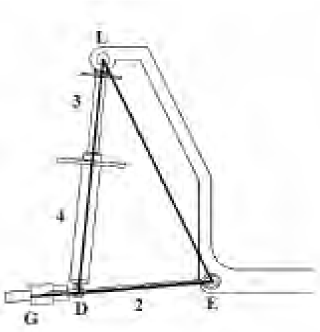


Figure 4. Geometry of the system swing axle suspension (Costa Neto, 2008)

In the Fig. 5, considering that the bodies are in the same plane, we have:

Body 1 – Chassis;
Body 2 – Axle;
Body 3 – Piston rod; and
Body 4 – Damper body.

Modelying in a form that it geometry be considered, the suspension is equating as (Costa Neto, 2008) achieving the Eq. (5).

$$\begin{bmatrix} {}^{f}w_{si} \\ {}^{f}w_{hi} \\ \dot{\beta}_{1i} \\ {}^{f}v_{Gyi} \end{bmatrix} = A_m \begin{bmatrix} {}^{f}v_{Lyi} \\ {}^{f}v_{Lzi} \\ {}^{f}v_{Eyi} \\ {}^{f}v_{Ezi} \\ {}^{f}v_{Gzi} \end{bmatrix} \tag{5}$$

Where:

$$A_m = \begin{bmatrix} \frac{sin\beta_6}{(r_2+\beta_1)} & -\frac{cos\beta_6}{(r_2+\beta_1)} & -\frac{sin\beta_6}{(r_2+\beta_1)} & \left\{\frac{cos\beta_6}{(r_2+\beta_1)} - \frac{r_1 cos(\beta_6-\beta_5)}{(r_2+\beta_1)(r_4 cos\beta_5)}\right\} & \frac{r_1 cos(\beta_6-\beta_5)}{(r_2+\beta_1)(r_4 cos\beta_5)} \\ 0 & 0 & 0 & -\frac{1}{(r_4 cos\beta_5)} & \frac{1}{(r_4 cos\beta_5)} \\ -cos\beta_6 & -sin\beta_6 & cos\beta_6 & sin\beta_6 - \frac{r_1 sin(\beta_6-\beta_5)}{(r_4 cos\beta_6)} & \frac{r_1 sin(\beta_6-\beta_5)}{(r_4 cos\beta_6)} \\ 0 & 0 & 1 & \frac{sin\beta_6}{cos\beta_6} & -\frac{sin\beta_6}{cos\beta_6} \end{bmatrix} \quad (6)$$

$r_1$ – Distance ED in the Fig. 5;
$r_2$ – Distance presented by the number “4” in the Fig. 5;
${}^f v_E$ – Velocity of the hardpoint E in the inertial referential;
${}^f v_G$ – Velocity of the linkage point of the Wheel with the suspension;
${}^f v_L$ – Velocity of the hardpoint L in the inertial referential;
${}^f w_h$ – Angular velocity of the axle;
${}^f w_s$ – Angular velocity of the strut;
$\beta_1$ – Spring lenght;
$\beta_5$ – Swing arm orientation at the fixed referential; and
$\beta_6$ – Strut orientation at the fixed referential.

The employed suspension in the front set, of the kind overlapped arms is presented in the Fig. 6.

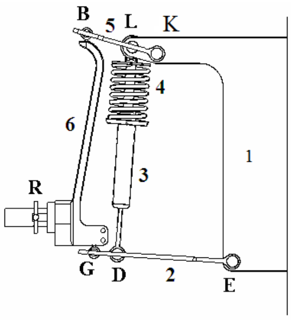


Figure 6. Geometry of the suspension system overlapped arms (Costa Neto, 2008)

In the Fig. 6, the bodies added to the suspension swing axle at the Fig. 4 are:

Body 2 – Lower arm;
Body 5 – Upper arm;
Body 6 – Kingpin.

The relation matrix between the input and output velocities of the double-A arm, is presented in Eq. (7).

$$\begin{bmatrix} {}^f w_{hi} \\ {}^f w_{si} \\ {}^f w_{bi} \\ {}^f w_{ki} \\ \dot{\beta}_{1i} \\ {}^f v_{Gyi} \end{bmatrix} = B_m \begin{bmatrix} {}^f v_{Jyi} \\ {}^f v_{Jzi} \\ {}^f v_{Lyi} \\ {}^f v_{Lzi} \\ {}^f v_{Eyi} \\ {}^f v_{Ezi} \\ {}^f v_{Rzi} \end{bmatrix} \quad (7)$$

Where:
${}^f v_E$ – Velocity of the hardpoint E in the fixed referential;
${}^f v_J$ – Velocity of the hardpoint J in the fixed referential;
${}^f v_L$ – Velocity of the hardpoint L in the fixed referential;
${}^f v_R$ – Velocity of the linkage point of the wheel with the suspension;

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${}^{f}w_b$ – Angular velocity of the upper arm;
${}^{f}w_h$ – Angular velocity of the axle;
${}^{f}w_k$ – Angular velocity of the kingpin;
${}^{f}w_s$ – Angular velocity of the strut;
$\beta_1$ – Spring lenght;
$\beta_5$ – Guidance of the axle in the fixed referential; and
$\beta_6$ – Guidance of the strut in the fixed referential.

### 3.3 Wheel

The Wheel as being considered as a rigid body in modeled the same way as the chassis and it equating based in the kinematics linkage matrix can be seeing in the Eq. (8).

$$\begin{bmatrix} v_{Ry} \\ v_{Rz} \end{bmatrix} = \begin{bmatrix} 1 & 0 & R_w \cos\beta_7 \\ 0 & 1 & R_w sin\beta_7 \end{bmatrix} \begin{bmatrix} v_{Gy} \\ v_{Gz} \\ \dot{\beta}_7 \end{bmatrix} \tag{8}$$

This equating determines the way where the velocities of the hardpoint of the wheel with the suspension are transmitted to the Wheel, considering the camber angle $\beta_7$.

### 3.4 Tire

The relation matrix between the input and output velocities of the tire, modeled as elastic linear system, where it equating is shown in the Eq. (9):

$$\begin{bmatrix} v_{Py} \\ v_{Pz} \end{bmatrix} = \begin{bmatrix} \cos\beta_7 & sin\beta_7 & -\cos\beta_7 & -sin\beta_7 \\ sin\beta_7 & \cos\beta_7 & sin\beta_7 & -\cos\beta_7 \end{bmatrix} \begin{bmatrix} v_{0y} \\ v_{0z} \\ v_{Ry} \\ v_{Ry} \end{bmatrix} \tag{9}$$

## 4. RESULTS

The model presented in this work was subjected to tests that analyses their vertical behavior in order to compare the response of the system due to the influence of different suspension mechanisms and equal complacent elements. The first analysis was the position of the hardpoints of the chassis with the suspension to stabilize after being released due to its own weight, these values are reported in Fig 7.

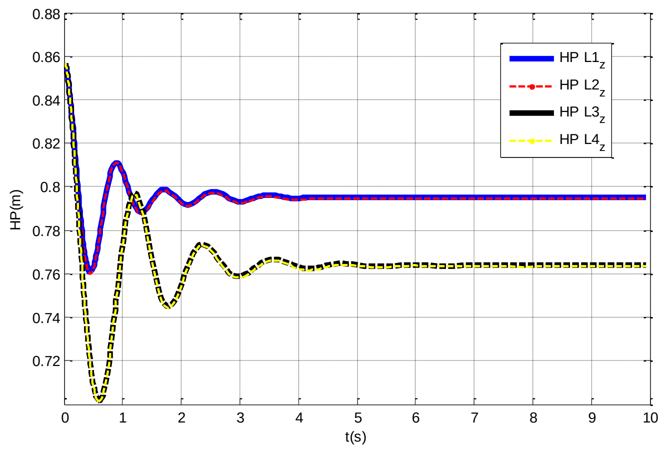


Figure 7. Height of the hardpoints $L_1, L_2, L_3 and L_4$.

The graph of "Figure 7 "shows a height difference of the hardpoints of the chassis after reaching equilibrium, this shows that the mounting assembly of the front axle with overlapping arms suspension kept the chassis higher, which causes the appearance of a constant and negative displacement of the "pitch" angle.

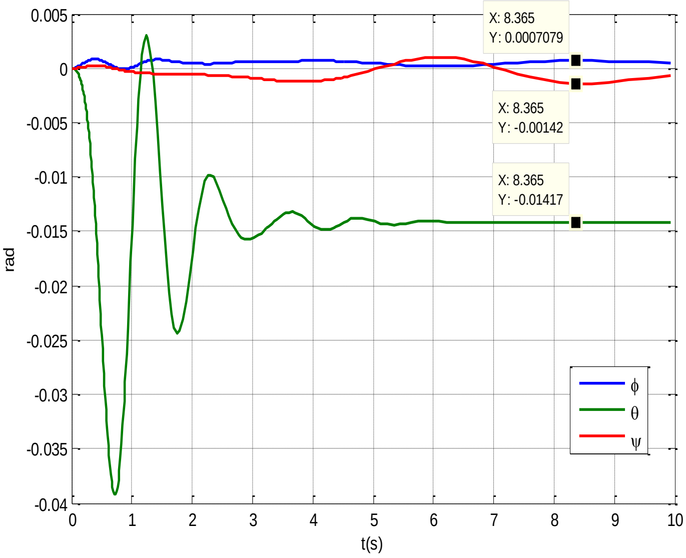


Figure 8.Angular displacement

The Fig. 8 represents the angular displacement of the chassis. The stabilization in a constant and negative value of the angle $\theta$, reports a negative “Pitch” due the lifting of the front part of the chassis.

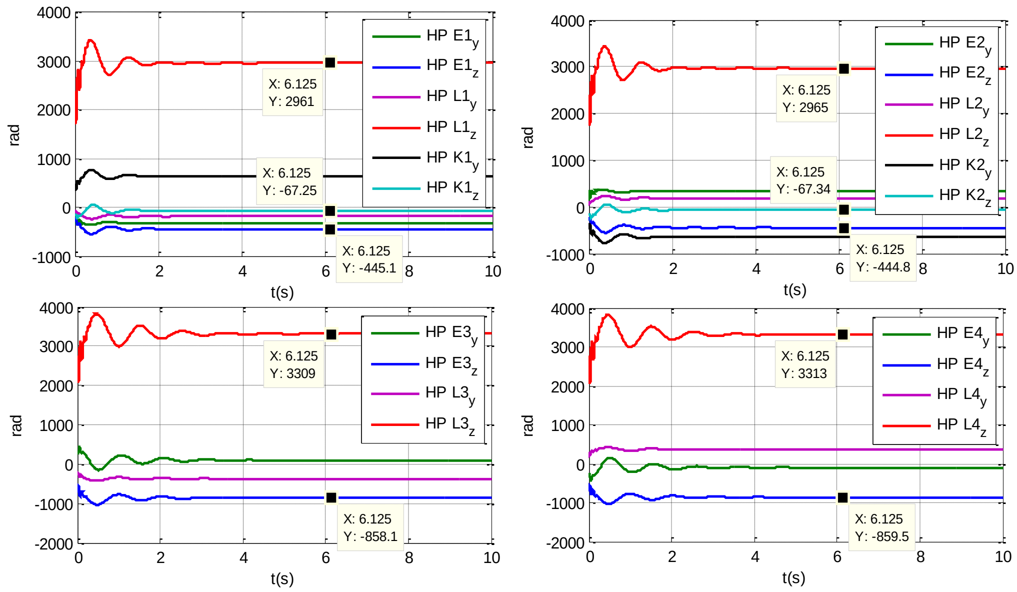


Figure 9. Transmitted forces of the hardpoints

In Fig. 9 it is possible to note that the sum of forces in the vertical direction of each suspension assembly corresponds to the fourth part of the value of the weight of the chassis, which in this case is produced by a mass of 1000Kg, considering the transmission of weight due to slope. Respect to the sum of the horizontal forces, this results in a null value, which indicates that there is no lateral movement.

A second analysis in order to minimize the effects of different mechanisms, the springs of the rear suspension had their elastic constant value increased, measured prior to 16000 N / m by passing the measure 18000 N / m. To evaluate the chassis behavior, in each suspension was chosen a point of the same height relative to the center of gravity and shown its displacement in Fig. 10.

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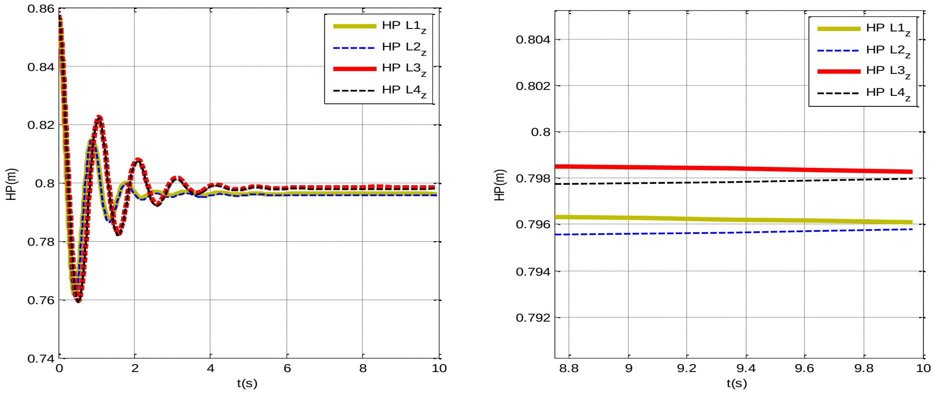


Figure 10. Height of the hardpoints $L_1, L_2, L_3 and L_4$

The height of the hardpoints in the Fig. 10 and the angular displacement of the chassis in Fig. 11, shows that the slop of the chassis to the ground was minimized to the maximum after the change in the parameters of the springs stabilizing with a slop of 0.06º.

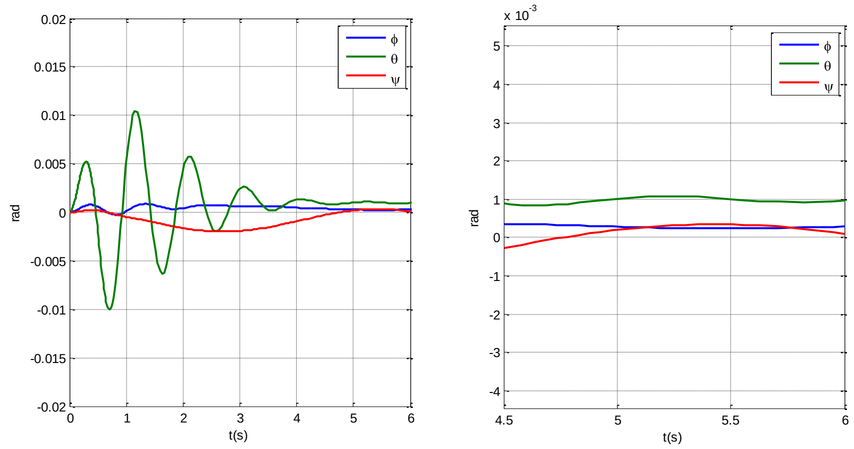


Figure 11. Angular displacement of the chassis

## 5. CONCLUSION

The computational model of the subsystems enable de coupling of different suspensions in the front and rear axles, where the chassis is a tridimensional body equated and the suspensions are bidimesional, working only in the plane YZ. The model enables analyse the influence of the difference of the mechanisms over the kinematic and dynamic behavior of the chassis as the influence due to the range of the parameters. The modularity of the models here represented is reached due to a equating based in the power flow, making possible to obtain the kinematics bounds between the systems.

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