

## RADIATIVE HEAT TRANSFER EVALUATION IN COMPLEX ENCLOSURE USING ZONAL METHOD

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**Abstract.** The zonal method is distinguished by the easy conceptual implementation and by the accuracy of results, when applied to radiative heat transfer problems in practical engineering problems. In this work, we proposed the development of zonal method applied to a two-dimensional symmetric cavity with plane surfaces and complex surfaces using cubic spline to modeling these surfaces. The radiative properties of participating medium were determined by the weighted sum of gray gases method. According to the literature, the proposed method yields satisfactory results for radiative heat transfer both plane and in complex geometries problems.

**Keywords:** Complex Geometries, Gray Gases, Zonal Method

### 1. INTRODUCTION

Steam generations are equipment to steam generating at pressures above atmospheric. The fire tube boiler is a steam generation classes that usually apply a corrugated surface technology in the furnace aiming to improve mechanic strength and heat transfer. The sketch of corrugated surface is shown in Fig (1)

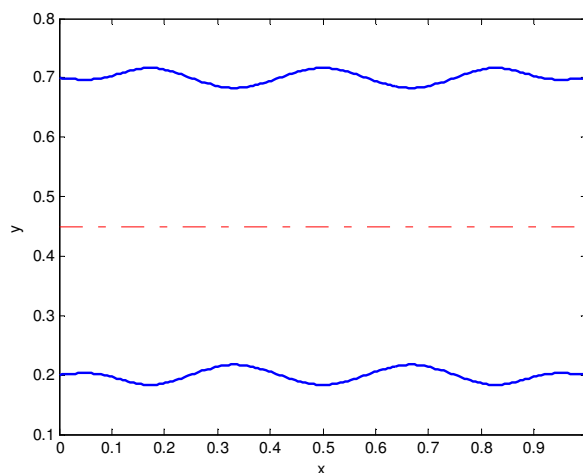


Figure 1- The drafting of two-dimensional corrugated surface

Coelho (1997) developed the radiative transfer in two-dimensional enclosure with obstacles for different geometric configuration, using rectangular cavity and three-dimensional enclosure containing five baffles for simulating a boiler furnace with superheaters using zonal method, discrete transfer method, discrete ordinates method and finite volume method. All the methods predicted similar evolution of radiative heat transfer, but discrete ordinates method and finite volume methods were the fastest in convergence time.

Nunes et al. (2000) developed thermal radiative transport in irregularly-shaped axisymmetric bodies containing a homogenous, anisotropically scattering medium using N-bounce method that approximates total exchanges factors by summing direct and user designated higher order terms representation of multiple reflections/scattering. Five different problems were studied for non-circular enclosures with the results were found to be in excellent agreement with those in literature.

Talukdar (2006) developed discrete transfer method for irregular geometries applied a variety of two-dimensional problems with rectangular geometries and cylindrical enclosures with satisfactory results when compared with literature benchmark.

The zonal method was initially established by Hottel and collaborators, according with was presented in Hottel and Sarofim (1976). This method is based on the division of volume gas and of heat exchange surface in isothermal elements in order to account radiation heat transfer considering the interchanges between all surface to surface, surface to volume and volume to volume, as can be seen in the Fig. 2.

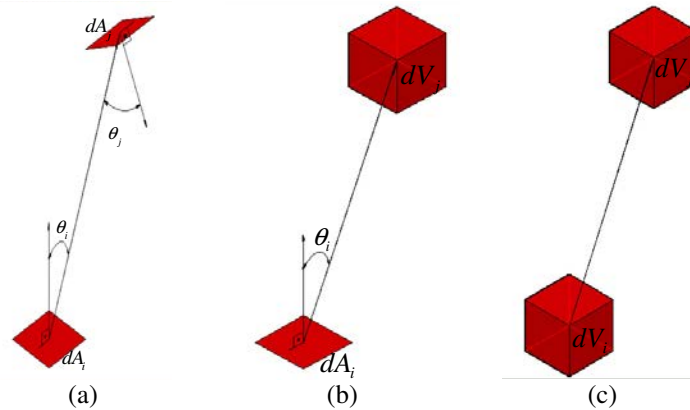


Figure-2 Direct exchange area between surfaces -surfaces (a) between surfaces -volumes (b) and between volumes - volumes (c)

The direct exchange areas, Fig 1, are determined by:

$$\begin{aligned}
 s_i s_j &= \int_{A_j} \int_{A_i} \frac{\cos(\theta_i) \cdot \cos(\theta_j) \cdot e^{(-K \cdot r)}}{\pi \cdot r^2} dA_i dA_j \\
 g_i s_j &= \int_{A_j} \int_{V_i} \frac{K \cdot \cos(\theta_j) \cdot e^{(-K \cdot r)}}{\pi \cdot r^2} dV_i dA_j \\
 g_i g_j &= \int_{V_j} \int_{V_i} \frac{K \cdot K \cdot e^{(-K \cdot r)}}{\pi \cdot r^2} dV_i dV_j
 \end{aligned} \tag{1}$$

Where  $s_i s_j$ ,  $g_i s_j$  and  $g_i g_j$  are the direct exchange area for surface-surface, surface-volume and volume-volume respectively;  $K$  is the gray gas extinction coefficient;  $r$  distance between two zonal elements and  $\theta$  is the polar angle. The zonal elements for volume and area are represented respectively by  $V_i$  and  $A_i$ . To account for the effects of multiple reflections and the emission of gases between the surfaces is used the total exchange area through the matrix representation, as realized to Rhine and Tucker (1991) and Noble (1975)

The spectral bands effect of gas for radiation absorption, especially  $\text{CO}_2$  and  $\text{H}_2\text{O}$ , is considered using a weighted sum of gray gases method. The method parameters are used as proposed by Smith et al (1982) using three gray gases and four polynomials coefficients for temperature fit. Based in this coefficients is calculating the weighted of gray gases, which are associated to direct exchange area to determine the direct flux area.

The heat exchange between the surfaces and volumes are given:

$$\begin{aligned}
 \dot{Q}_{i-j} &= \varepsilon_i \cdot A_i \cdot E_i - \sum_{j=1}^m \overline{S_i S_j} \cdot E_i - \sum_{j=1}^l \overline{S_i G_j} \cdot E_{g,i} \\
 \dot{Q}_{i-j} &= \sum_{j=1}^l \overline{G_i G_j} \cdot E_{g,i} - \sum_{j=1}^m \overline{G_i S_j} \cdot E_i - 4 \cdot K_i \cdot V_i \cdot E_{g,i}
 \end{aligned} \tag{2}$$

Where  $\varepsilon_i$  is the surface emissivity;  $E_i$  and  $E_{g,i}$  are emissive black body power for surface and gases volumes elements respectively;  $\overline{S_i S_j}$ ,  $\overline{S_i G_j}$  and  $\overline{G_i G_j}$  are the direct flux area between surface-surface, surface-volumes and volume-volume respectively and  $K_i$  is gray gas extinction coefficient.

## 2. METHODOLOGY

The integral solution of the direct exchange areas, Eq. (1), represents the largest computational effort of the method. The literature usually applies the Gaussian quadrature method, in this manuscript was applied the integration method by discrete sum developed by Olsommer (1997), given by:

$$d_i d_j = \sum_{ddj} \sum_{ddi} \frac{\cos(\theta_i)^{ctr_i} \cdot \cos(\theta_j)^{ctr_j} \cdot K_i^{1-ctr_i} \cdot K_j^{1-ctr_j} \cdot e^{-K \cdot r_{ij}}}{\pi \cdot r_{ij}^2} \delta d d_i \delta d d_j \quad (3)$$

Where  $d_i$  e  $d_j$  are unified direct exchange area,  $s_i s_j$ ,  $g_i g_j$  e  $g_i s_j$ ; the terms  $dd_i$  e  $dd_j$  are the zonal exchange elements,  $A_i$  or  $V_i$ . The control variable,  $ctr$ , determine if exchange factors is between areas,  $ctr=1$ , or volumes,  $ctr=0$ .

The direct exchange areas are three-dimensional quantities that account the two-dimensional energy balance applied the smoothing method for direct exchange area, as established by Lawson (1995), based on isothermal balance of the Eq.(2), as follow:

$$\begin{aligned} s_i s_j &= s_i s_j \frac{A_i}{\sum_k s_i s_k + \sum_k s_i g_k} \\ s_i g_j &= s_i g_j \frac{A_i}{\sum_k s_i s_k + \sum_k s_i g_k} \\ g_i g_j &= g_i g_j \frac{4K_i V_i}{\sum_k g_i s_k + \sum_k g_i g_k} \end{aligned} \quad (4)$$

These equations are used in iterative process until that maximum discrepancy between the current and previous modified direct exchange factor is less than  $10^{-10}$ .

The corrugated surface was modeled using a natural cubic spline as established by Gerald and Wheatley (1994), as follow:

$$g_i(x) = a_i (x - x_i)^3 + b_i (x - x_i)^2 + c_i (x - x_i) + d_i \quad (5)$$

Where  $a_i$ ,  $b_i$ ,  $c_i$  and  $d_i$  are  $i$ -th polynomial coefficients for the spline  $g_i(x)$ . Firstly was determined the amplitude and the step points for corrugated surface. Based on these points was fitting the cubic spline, by setting the eq (5) coefficients. The zonal method with weighted sums of gray gases was implemented for a cavity with black isothermal walls with length of 1.0m and height 0.5m

Using the two zonal element centers can be calculated a straight line and by the analytical intersection between this line and cubic spline for surface can be found shaded areas. The straight lines are showed in fig (3).

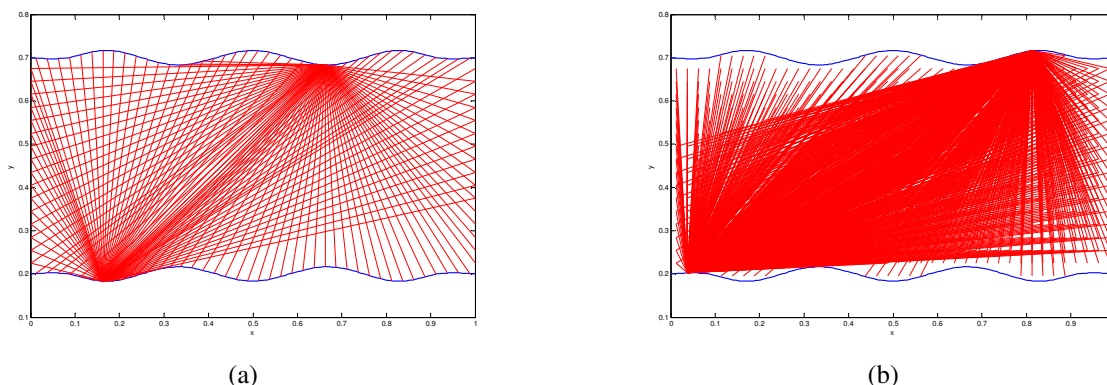


Figure 3 – Direct Exchange area for zonal elements for surface -surface (a) and surface-volume (b)

The plane surface enclosures are determined according to Goutiere et al. (2000), in which the gas concentrations of participating gases are assumed to be uniform and equal to 10%CO<sub>2</sub> and 20% H<sub>2</sub>O; and the gases temperature profile are determined divided the isothermal enclosure in two regions, as showed by Eq. (6)

$$\begin{cases} x \leq 0.1 T(x, y) = (14000x - 400) \cdot (1 - 3 \cdot y_o^2 + 2 \cdot y_o^3) + 800 [K] \\ x > 0.1 T(x, y) = -\frac{10000}{9}(x - 1) \cdot (1 - 3 \cdot y_o^2 + 2 \cdot y_o^3) + 800 [K] \end{cases} \quad (6)$$

Where

$$y_o = \frac{|0.25 - y|}{0.25} \quad (7)$$

To implement the method was developed a FORTRAN code subdivided in subroutines as show in the fig (4)

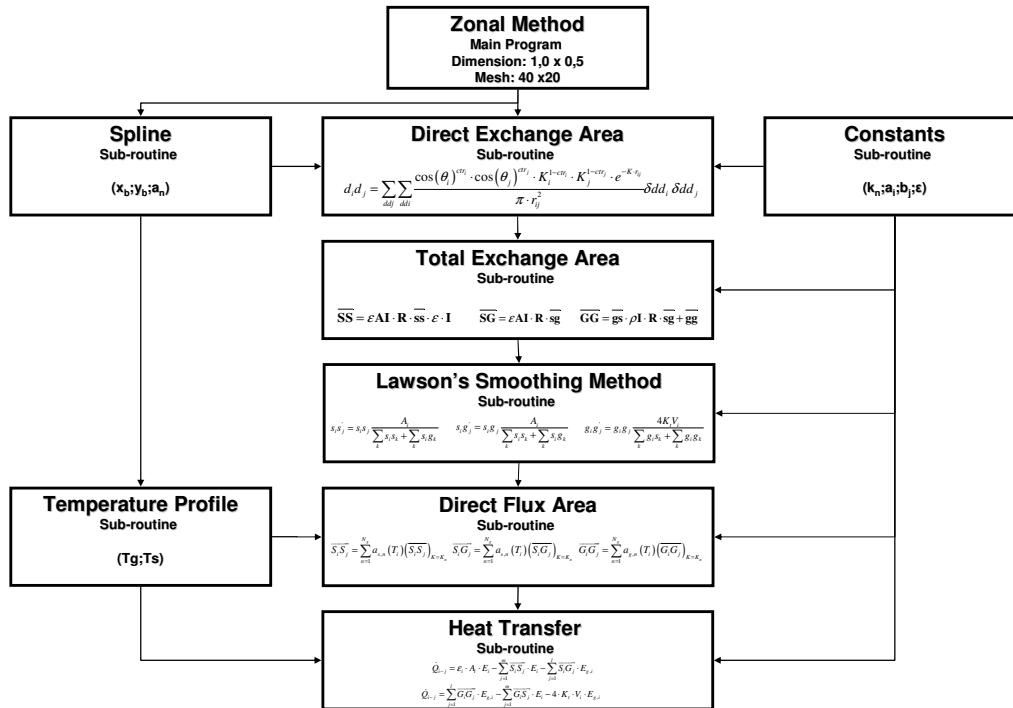
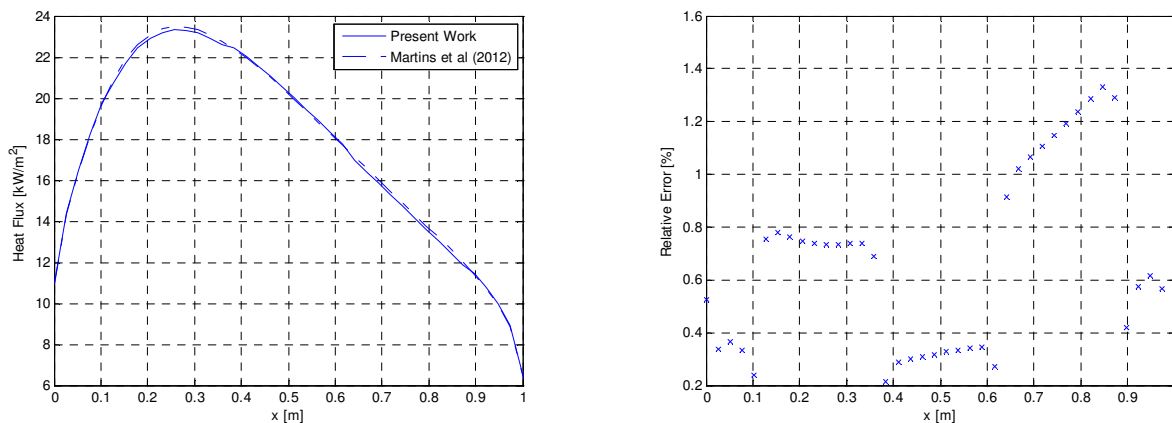


Figure 4 - Implemented Code flowchart

### 3. RESULTS

Firstly to validate the implemented code, was used plane surface modeled by spline considering amplitude of 0,0m and step of 0.1667m with temperature profile established by Goutiere et al. 2000, using the method of weighted sum of gray gases. The results are compared with Martins et al. 2012, that implemented a zonal method specific code associated with weight sum of gray gases for the theoretical furnace established by Goutiere et al. (2000), as showed in fig (5).



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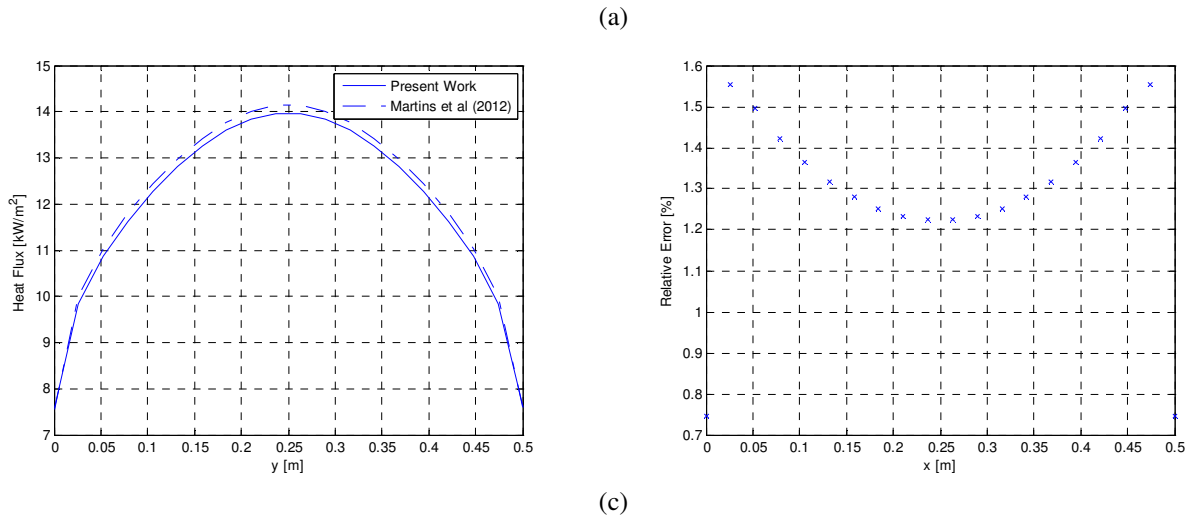


Figure 5. Radiative heat flux and relative error for the bottom (a) and the right (b) walls for plane surface

Figure 5 represents satisfactory description of radiative heat transfer in the surfaces by the implemented code, with small errors at the bottom wall and at the right wall. The relative error showed some tendency errors that may be associated to Lawson's smoothing method. As showed by Martins, et al. 2012 the results presented by the zonal method with weighted sum of gray gas, using integration by discrete sum and Smith, et al. 1982, parameters, shown accurate results close to the statistic narrow band method developed Goutiere et al. 2000.

To assessment the physical analysis for corrugated surface, firstly it was plotted the temperature profile for enclosure, as showed in fig (6).

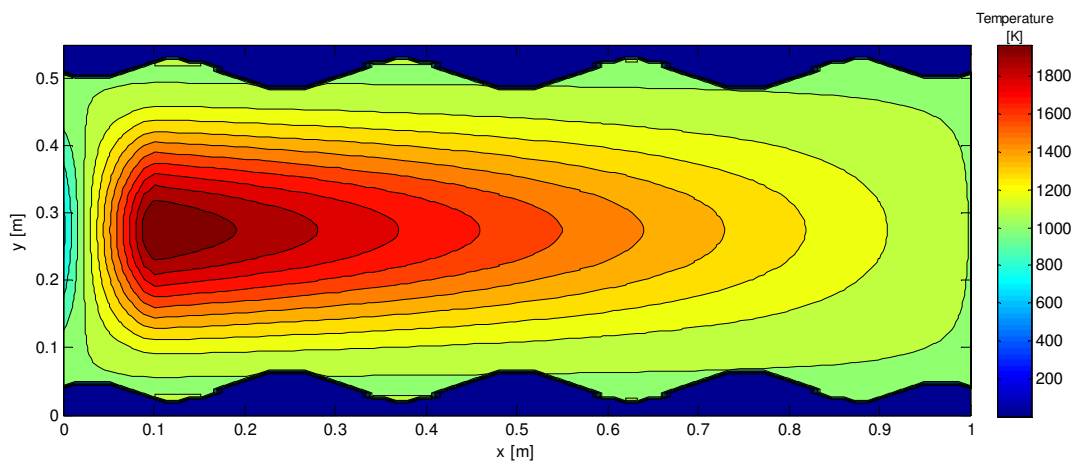


Figure 6. Temperature profile in theoretical furnace

The temperature profile, in figure 6, show a typical temperature distribution of combustion furnace, in which high temperature was found near the left wall. Figure 6 also shows that temperature profile is not affected by the corrugated surface because the mathematic correlation did not accounted the flow dynamics in combustion furnace.

The results for radiative heat transfer for corrugated surface with amplitude of 25mm e step of 125mm using the temperature profile determined give by fig (6), are showed in fig (7).

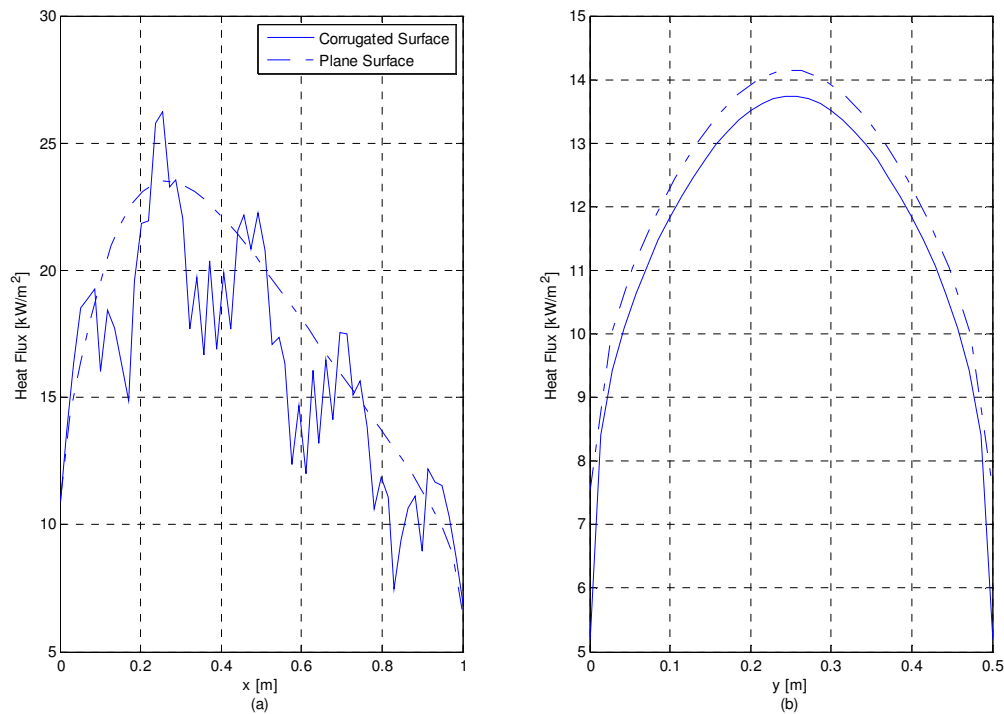


Figure 7. Radiative heat flux for the bottom corrugated surface (a) and the right wall (b)

The figure 7 shows the radiation heat transfer peak in the maximum amplitude surface, as showed at the ordinate of 0.24m, the maximum radiative heat flux in fig(7) represent the peak of amplitude most close to the highest temperature in fig (6). In the same way, the figure 7 show that low radiation heat transfer corresponded to the shaded surface from greatest temperature profile. The amount of heat transfer improved 5% at bottom corrugated surface and show differences at the right wall mainly near edges, showing the effect of shaded areas.

#### 4. CONCLUSION

The zonal method implementation with weighted sum of gray gas showed as a method with low computational cost and with easy development for complex surface applied spline to modeling corrugated surface. A validation with plane surface presented satisfactory errors when compared with results developed by Martins (2012). The results for corrugated surface, although no literature results, represent the physic coherence with improvement of amount of radiative heat transfer.

Further improved in methodology may also be realized by using a combustion modeling that account the effect flow dynamics. In this case, temperature profile is determined by use of iterative heat transfer between the energy equation for flow dynamics and the radiative heat transfer.

#### 5. ACKNOWLEDGEMENTS

Financial support of Faculty of Integrated Sciences of Pontal and Graduate Program in Mechanical Engineering at the Federal University of Uberlândia and also to CAPES and FAPEMIG.

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