



SWING-BY MANEUVERS COMBINED WITH AN IMPULSE

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Abstract. *The Swing-By maneuver is important because it generates an economy of fuel that makes feasible many space missions. This maneuver can be studied with or without the application of propulsive forces. In this paper, the main goal is to study of Swing-By maneuvers combined with the application of an impulse out of the periapsis. For this, we developed an algorithm to calculate the energy variation of this maneuver. The method was then applied to the Sun-Jupiter System. The results obtained from the numerical simulation showed that the most efficient point to apply the impulse to obtain the maximum or the minimum energy variation is out of the periapsis ($\theta \neq 0^\circ$) and inside the region of influence of secondary body. This impulse also has to be applied in a direction that is not tangential to the orbit ($\alpha \neq 0^\circ$).*

Keywords: *Swing-By maneuver, trajectory, spacecraft, astrodynamics, orbital maneuver.*

1. INTRODUCTION

The passage of a spacecraft near a celestial body with the use of the gravity of this body to gain or lose energy is called Swing-By maneuver. When, during this passage, it is applied an impulse to the satellite to add velocity to the natural variation of velocity caused by the gravity it is called a powered Swing-By. The combination of the natural Swing-By with the impulse becomes the more efficient maneuver, considering that the goal is to reach the desired trajectory using the minimum possible fuel.

There are works available in the literature about this subject, like Prado (1996) who studied the Swing-By maneuver with an impulse applied exactly at the periapsis of the orbit and in a point outside the region of influence of the secondary body for the Earth-Moon system. The goal of Prado (1996) was to study the efficiency of a combined maneuver and analyze where the best point of the trajectory to activate the thruster is. Mcconagh et. al. (2003) studied optimization of low-thrust for trajectories with Swing-By maneuvers. The first step was a search through a wide range of possible trajectories. The second step optimizes the most promising trajectories using an efficient method for parameter optimization. Sukhanov et al. (2010) presented a project to send a spacecraft into a near-Earth triple asteroid, called Aster project. This is the main motivation of using Swing-By maneuvers in this project. Other references are Dunham and Davis (1985), Weinstein (1992), Prado e Broucke (1993), Petropoulos and Longuski (2000), Solorzano et. al (2006) and Gomes e Prado (2009).

The maneuver is studied using the model given by the restricted problem of three bodies. The main body is M_1 , M_2 is the secondary body and the spacecraft is M_3 . The secondary body orbits M_1 in a Keplerian orbit and M_3 orbits M_1 and makes a Swing-By with M_2 . The mass of the M_3 is considered negligible.

The Swing-By maneuver is characterized by three parameters: V_{inf} , the approach speed of the spacecraft, r_p , periapsis' radius, which is shortest distance between M_2 and M_3 and ψ , the approach angle, which is the angle between the line connecting the two primaries and the periapsis' radius. The powered Swing-By includes two more parameters: δV , the magnitude of the impulse applied and α , the angle that defines the direction of the application of the impulse, which is the angle between V - and δV .

The main goal of this paper is to study the effect of the impulse before and after the Swing-By, out of the periapsis of the orbit, but in the vicinity of the sphere of influence (Araújo et. al., 2008) of M_2 .

The position r of the point where the impulse δV was applied was specified through its true anomaly, called θ . We adopted, for the displacement of θ , the counterclockwise direction as positive and the clockwise direction as negative, both measured from the vector r_p (radius of the periapsis). Varying the point of application of the impulse, it is possible to analyze the effect of changing this variable. Then, it become possible to map optimal regions to apply the impulse from the options: a) at the time of the passage of the spacecraft by the periapsis b) outside the periapsis, but within the sphere of influence of M_2 c) outside the sphere of influence of M_2 .

The results obtained from the numerical simulation showed that the most efficient point to apply the impulse to obtain the maximum and minimum energy variations is out of the periapsis of the orbits ($\theta \neq 0^\circ$) and inside the region of influence of secondary body, with the impulse applied in a direction that is not tangential to the orbit ($\alpha \neq 0^\circ$).

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The results were calculated by numerical simulation varying θ and α , in other words, analyzing the cases with impulse is out of the periapsis and in the direction not tangential of the orbit.

2. SYSTEM DYNAMICS

The Swing-By maneuver is studied under the model given by the planar restricted problem of three bodies, where M_1 is the most massive body, in this case the Sun, M_2 is the secondary body, Jupiter, and M_3 is the spacecraft, considered with negligible mass. The equations of motion are (Murray and Dermott, 1999):

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} \quad (1)$$

$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} \quad (2)$$

$$\Omega = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \quad (3)$$

The Lemaître Regularization was used to avoid the singularity in the simulations caused due to r_1 and r_2 to be in the denominator of Ω , where r_1 is the distance M_1 - M_3 and r_2 is the distance M_2 - M_3 . When these distances are small, close to zero, the singularity occurs. The method given by the Lemaître regularization uses substitution of variable to solve this problem (Lapa, 2008).

The goal of this work is to find the optimal point to apply the impulse in a maneuver. This analysis is made by varying the application point and the direction of the impulse and then calculating the maximum and minimum variations of energy in each point (ΔE).

The position where the impulse is applied is a function of θ . This angle θ can be obtained from the scalar product of the vectors r and r_p (see Eq. (4)).

$$\theta = \arccos\left(\frac{x_p x + y_p y}{r_p r}\right) \quad (4)$$

The Fig. 1 shows the geometry of this maneuver:

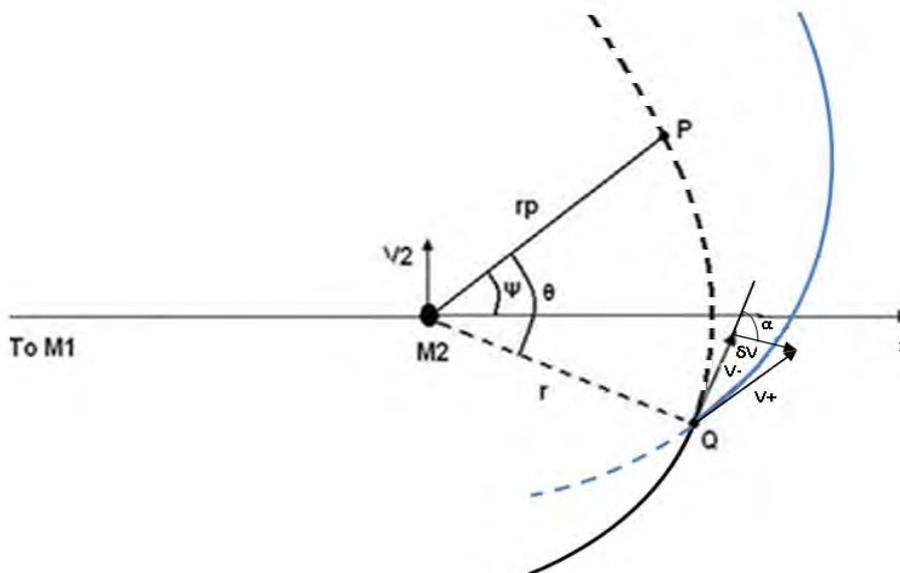


Figure 1. Geometry of the Swing-By with the application of the impulse out of the periapsis.

The variables shown in the figure are: P, the periapsis, r_p , periapsis' radius, V_2 , the linear speed of M_2 with respect to the center of mass of M_1 - M_2 , ψ , the approach angle, θ , the true anomaly, α , the angle that defines the impulse direction, δV , the magnitude of the impulse, r , the radius of the first orbit, Q, the point to apply the impulse, V^- and V^+ , the velocity of the spacecraft at the first and second orbit, respectively.

The maneuver works as follows:

- I. The first orbit is known from the initial conditions r_p , ψ and V_{inf-} ;
 - II. Change the value of r_p by θ . If θ is negative, the moving direction is backward. If positive, the move is forward (Fig. 1 represents θ negative). Then there is the point Q;
 - III. Then, it is made an integration from the point Q in reverse in time (Vieira Neto and Winter, 2001), without the impulse. Then, it is obtained the energy of the first orbit;
 - IV. From the point Q the equations of motion is integrated again, but now forward in time and with the application of the impulse (which is defined by δV and α). The interruption point of the numerical integration is given by a distant defined as half of the distance M_1 - M_2 . It is then obtained the energy of the new orbit;
 - V. Then it is possible to calculate the variation of the energy. This is done for each condition.
- Finally, it is possible to obtain the maximum variation of the energy as a function of θ , δV and α .

3. RESULTS

From Eq. 5 it can be concluded that: if the spacecraft passes in front of the secondary body ($0^\circ < \psi < 180^\circ$), it is braked by the body and loses energy. The maximum loss occurs when $\psi = 90^\circ$. If the spacecraft passes behind the secondary body ($180^\circ < \psi < 360^\circ$), it is accelerated by the body and it gains energy. The maximum gain occurs when $\psi = 270^\circ$. This information is important because it will be used many times during the analysis of this work.

$$E_o = E_i - 2V_2 V_{inf-} \sin \delta \sin \psi \quad (5)$$

In the Eq. (5), E_o is the energy of the spacecraft after the close approach with M_2 , E_i is the energy before the close approach, V_{inf-} is the magnitude of the velocity of approach; δ is half of the curvature angle of the first orbit.

For the simulations shown below, the values $r_p=1.1$ Jupiter's radius was used, and different values for ψ and δV . Figs. 2 to 5 show the energy variation as a function of α , θ and $\delta V = 0.5^\circ$. The x-axis represents the angle α , which defines the direction of the impulse and the y-axis represents θ . The curves show the energy variations.

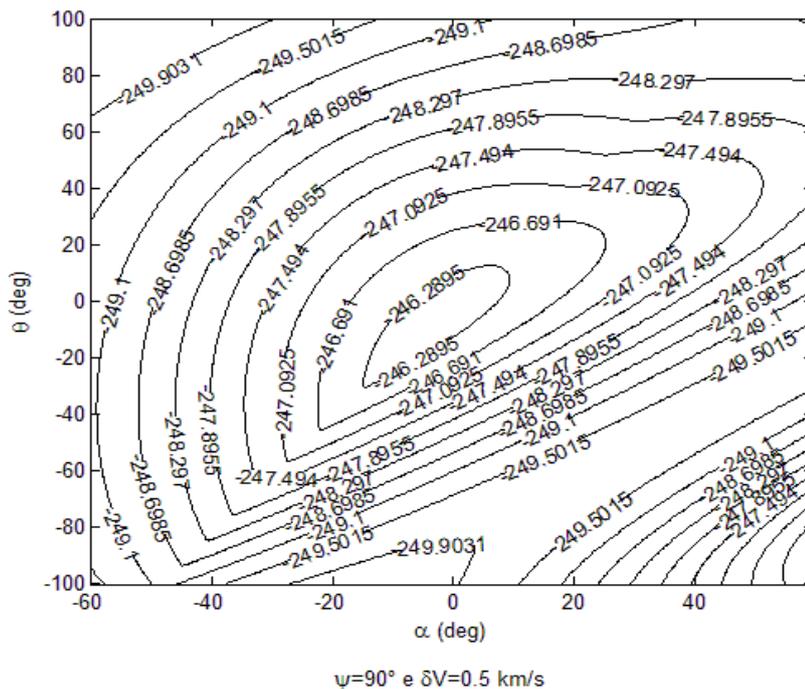


Figure 2. Energy variation for $\psi=90^\circ$ and $\delta V=0.5$ km/s.

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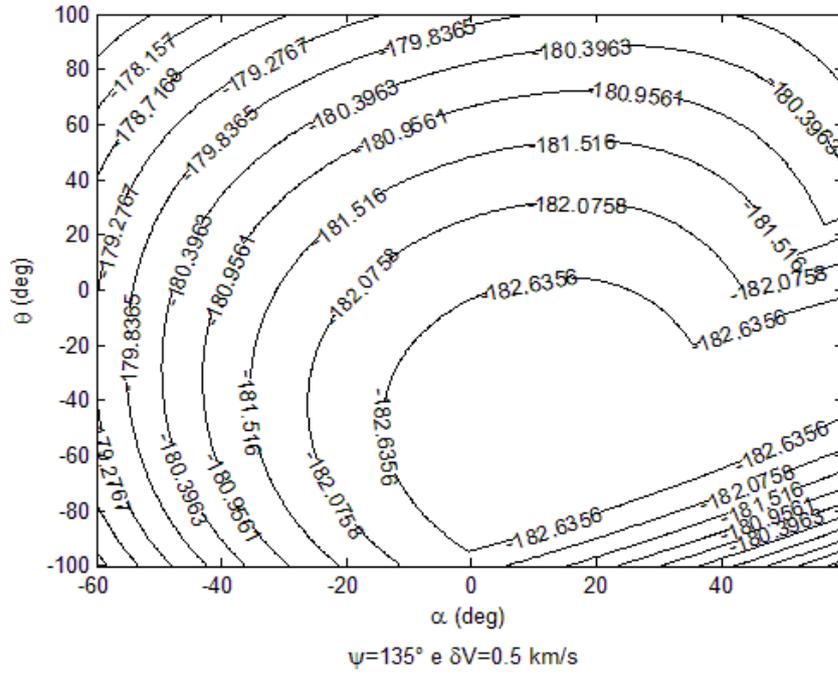


Figure 3. Energy variation for $\psi=135^\circ$ and $\delta V=0.5$ km/s.

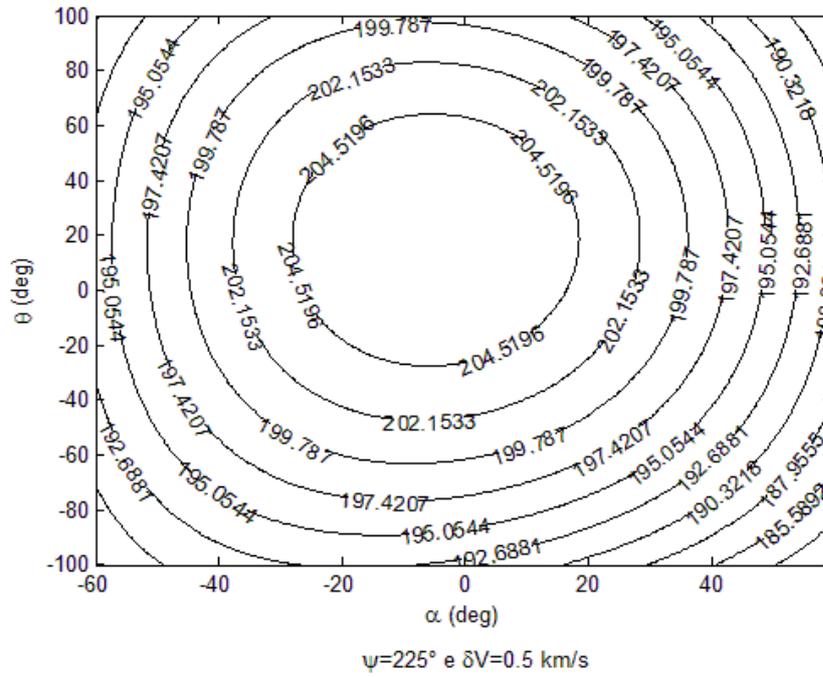


Figure 4. Energy variation for $\psi=225^\circ$ and $\delta V=0.5$ km/s.

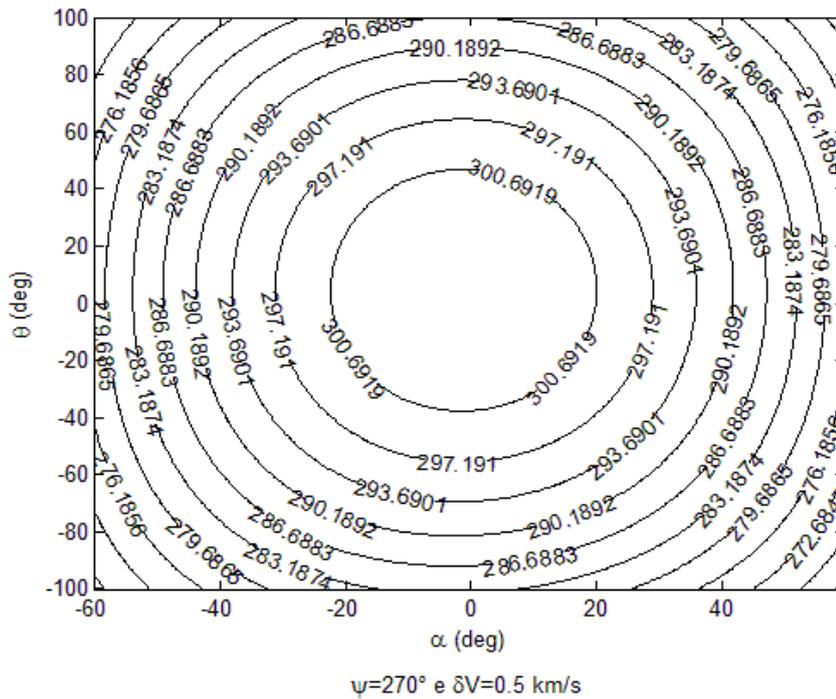


Figure 5. Energy variation for $\psi=270^\circ$ and $\delta V=0.5$ km/s.

In Fig. 4, the impulse was applied in the region where the spacecraft gains energy from the Swing-By, when the goal is the maximum variation of energy, $\theta = 18.50456^\circ$ and $\alpha = -5^\circ$. According to the geometry of the system (see Fig. 6), the impulse was applied in the region where it gains energy from the Swing-By, $180^\circ < \psi < 360^\circ$. The impulse was applied for the minimum variation of energy in the region that there are loss in the energy due to the Swing-By, $0^\circ < \psi < 180^\circ$ (see Fig. 7), $\theta = -100.01817^\circ$ and $\alpha = -59.5^\circ$.

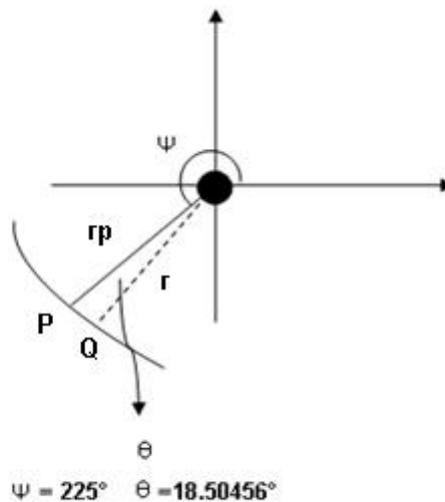


Figure 6. Geometry of the system for the case of maximum variation of energy for $\psi=225^\circ$ and $\delta V=0.5$.

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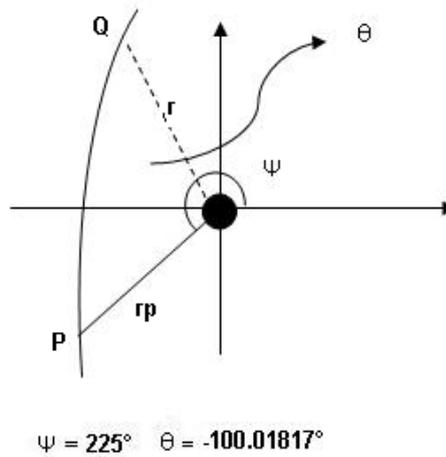


Figure 7. Geometry of the system for the case of minimum variation of energy for $\psi=225^\circ$ and $\delta V=0.5$.

The solid black line represents the periapsis radius (r_p) of the original orbit. The point P is the periapsis. This is the point from where θ is displaced. The dashed black line represents the position vector (r). Q is the point where the impulse is applied. The point Q is calculated as the addition of the angles ψ and θ .

In Figure 6, the impulse is applied when displaced 243.50456° of the horizontal axis. In Figure 7 the impulse was applied when the total displacement is 124.01817° from the horizontal axis. In this case the goal is to minimize the variation of energy. Figs. 8 to 11 show the energy variation as a function of α and θ for the case $\delta V = 1.5^\circ$.

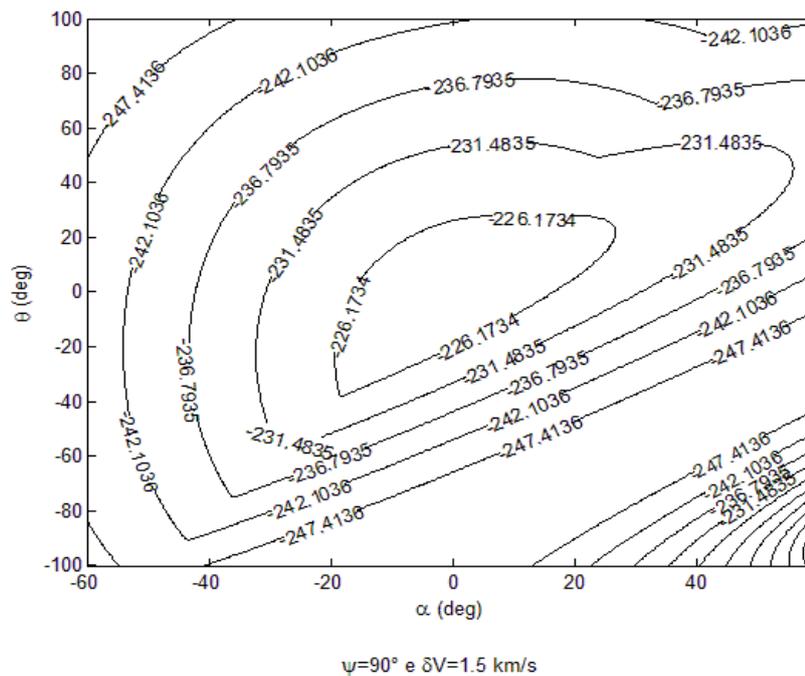


Figure 8. Energy variation for $\psi=90^\circ$ and $\delta V=1.5$ km/s.

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 November 3-7, 2013, Ribeirão Preto, SP, Brazil

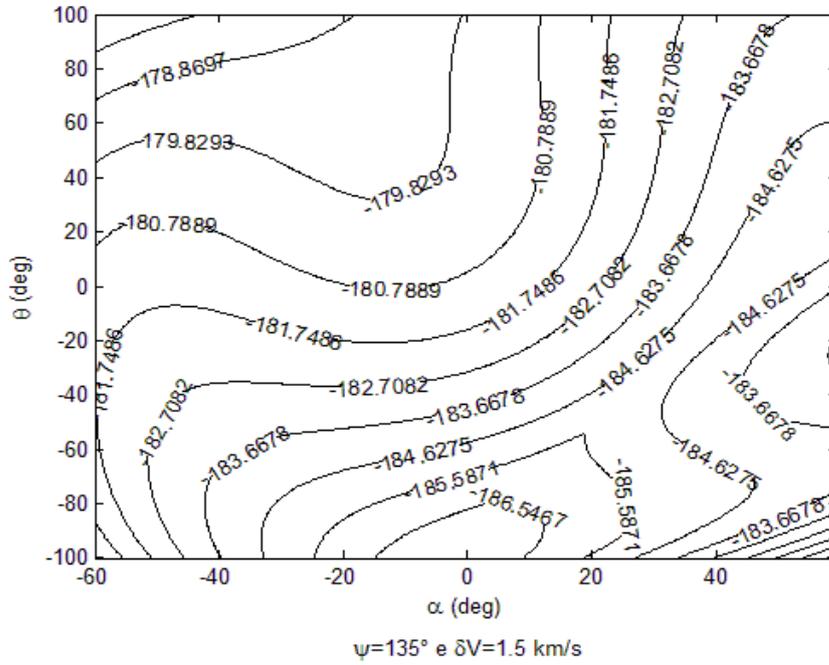


Figure 9. Energy variation for $\psi=135^\circ$ and $\delta V=1.5$ km/s.

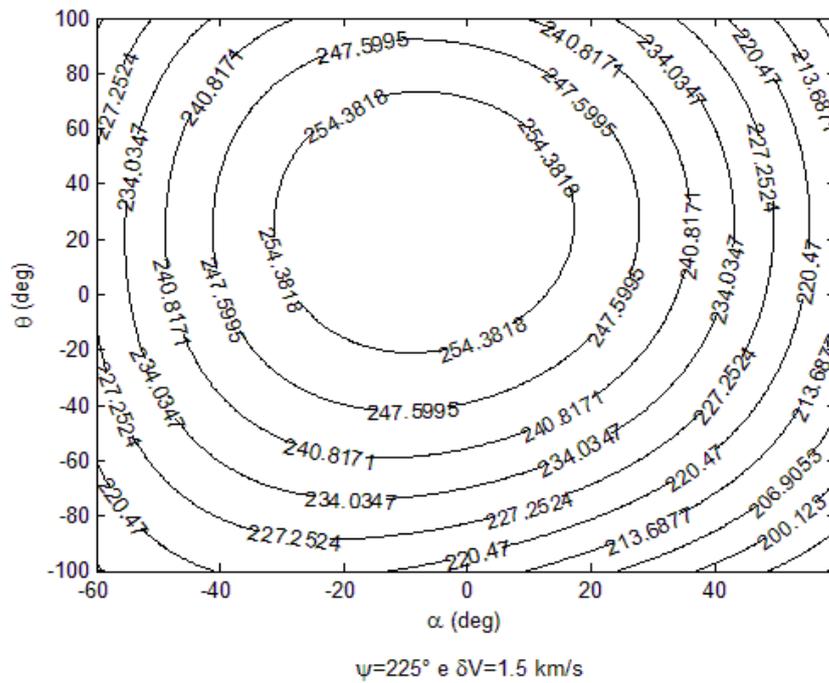


Figure 10. Energy variation for $\psi=225^\circ$ and $\delta V=1.5$ km/s.

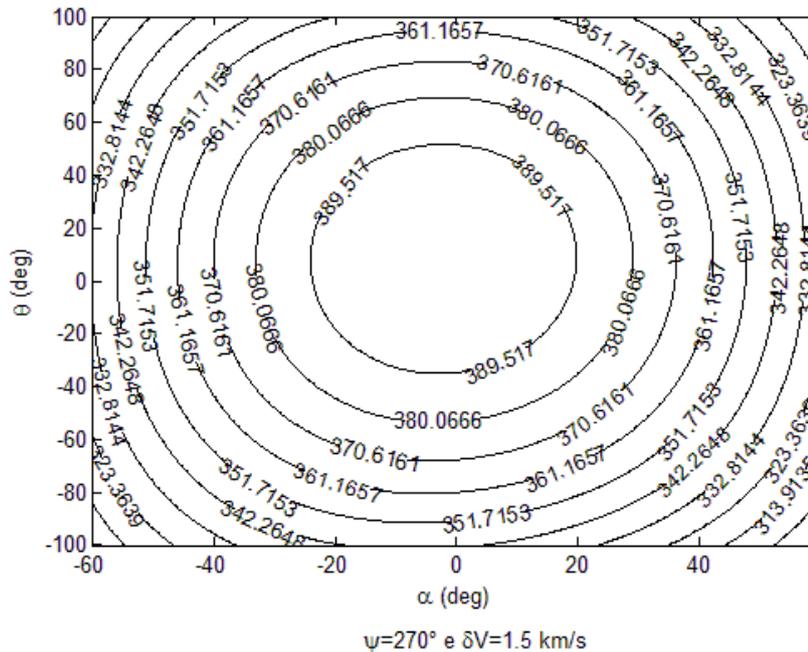


Figure 11. Energy variation for $\psi=270^\circ$ and $\delta V=1.5$ km/s.

In Fig. 9 the maximum variation of energy occurs for $\theta = 100.01817^\circ$ and $\alpha = -60^\circ$ and the minimum variation of energy occurs for $\theta = -100.01463^\circ$ and $\alpha = -3^\circ$.

For all values of δV , the highest value of the maximum variation of energy occurs when $\psi = 270^\circ$. This is expected because the orbit with a new r_p remains close to the region of $\psi = 270^\circ$, and this is the region where there is maximum energy gain from the Swing-By.

It would be expected that the best place to apply the thrust would be at the periapsis ($\theta = 0^\circ$) and in the direction of the motion of the spacecraft ($\alpha = 0^\circ$). But this is not what happens, because the impulse is applied in different conditions, in order to increase or decrease the energy variation due to the Swing-By, thus compensating the losses in energy due to the change in the direction of the impulse. This is because the orbit changes instantaneously, then there is a new value of r_p , which can be smaller and thus causes a Swing-By that is more efficient. The result of this change is positive.

To analyze whether the impulse was applied inside or outside the region of influence of M_2 during the simulations, we calculated the value of R , which is the distance between the spacecraft and M_2 at the time of the application of the impulse. We compare the values obtained for each trajectory that resulted in the minimum and maximum variations of energy with the radius of influence of Jupiter, which is 0.062 canonical units (Murray and Dermott, 1999), and found that in all cases the impulse was applied within the region of influence of M_2 .

4. CONCLUSIONS

The best place and the best direction to apply the thrust, when the goal is to get the optimal energy variation (maximum or minimum) is different from the pair (0, 0), i.e., outside the periapsis of the orbit but within the region of influence M_2 (for the initial conditions studied), and in a direction not tangential to the orbit.

When this happens, the satellite gains less energy due to the impulse, but this is compensated by the new Swing-By, so the net result between losses and gains is positive. This is a flexible and realistic method.

As expected when the goal is to gain energy, the best place to apply the impulse is $180^\circ < \psi < 360^\circ$, because this region gives increase of energy from the Swing-By. But, if the goal is the minimum variation of energy, the best region to apply the impulse is $0^\circ < \psi < 180^\circ$, for the same reason.

With this method studied, we will make available in the literature results that maps all optimal regions to apply the impulse: at the periapsis orbit; out of the periapsis orbit but inside the region of influence of the secondary body; and outside the region of influence of the secondary body.

5. ACKNOWLEDGMENTS

The authors wish to express their appreciation for the support provided by grants # 473387/2012-3 and 304700/2009-6, from the National Council for Scientific and Technological Development (CNPq);

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

grants # 2011/09310-7 and 2011/08171-3, from São Paulo Research Foundation (FAPESP) and the financial support from the National Council for the Improvement of Higher Education (CAPES).

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