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GENERALIZED FINITE ELEMENT METHOD TO APPROACH THE FREE VIBRATION PROBLEM IN MINDILN THICK PLATES MODEL

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Abstract. *One of the limitations of low regularity approximation space constructed using the Finite Element Method (FEM) is the accuracy in the determination of modes and natural frequencies relatively high. In this paper, we present an alternative to approach to the problem of natural frequencies of Mindlin thick plate model using the high regularity Generalized Finite Element Method (GFEM). In this work the approximation space is obtained from explicit enrichment of partition of unity (PU) of high regularity, with polynomials functions. In this work the PU´s in 2D, are obtained from tonsorial product of rational polynomials PU´s of high regularity in 1D. In the examples are analyzed: the problem of locking for the first natural frequency; the problem of convergence with "p" version for a target frequency, for regular and distorted mesh; and influence of the regularity of approaching spaces in obtaining relatively high frequencies (up to ten percent of frequencies approximated numerically). The analysis is performed for simply supported rectangular plate, comparing the results with those obtained with GFEM and the high order FEM.*

Keywords: *Natural frequencies, Mindlin plate, GFEM*

1. INTRODUCTION

The problem of propagation of mechanical waves in solid media has gained significant importance in recent decades in the automotive and aeronautical sectors. The simulation of the propagation phenomena of mechanical waves produced by impulsive forces is critical in the design of components that are impacted. On the other hand in elliptic problems eigenvalues/eigenvectors the possibility of obtaining a high percentage of eigenvalues approximate numerically with satisfactory accuracy is still an open research topic for which should arise numerous proposals for its approach.

In Finite Element Method (FEM) one of the factors with the highest incidence in the low efficiency in approximating eigenvalues/eigenvectors of elliptic problems is directly related to low regularity and high order of approximation spaces. This fact can be verified by a priori estimator for the Euclidean norm of the error in the eigenvalues for version "h" MEF (see: Hughes (1987) and Givoli (2008)). The use of the unconventional numerical methods as: Finite Element Method p-Fourier proposed by Leung and Chan (1998); Recently the Generalized Finite Element Method (GFEM) shown in the work of "Arndt *et ali. (2009)"* in addressing the problem of free vibrations in Benoulli-Euler frames showed excellent results with respect to FEM. Still the use of non conventional numerical methods in approach the problem of the undamped free vibration in thin plates and shells has been the work of: "Ferreira *et al. 2005*", approach the problem of free vibrations in laminated composite plates modeled by FSDT and using Multi-quadric radial basis functions (MQRBF); "Liew K. M. *et al. 2003*", addresses the problem of free vibrations in composite plates with kinematics defined by first shear deformation theory (FSDT) and the moving least squares differential quadrature method; "Chen J. T., *et ali. (2004)*" approach the problem of free vibration and rectangular and circular plates using radial basis functions (RBF); "Liu G. R., *et ali. (2001)"* approach the static and free vibration of thin plates of complicated shape using the moving least square method (MLSM); "Liu L., *et ali. (2002)*", approach the static and free vibration problem of the spatial shell structures using the Element Free-Galerkin Method (EFGM). In the works cited above the authors show initial results of performance of the approximations spaces, built according to different methodologies, in addressing the problem of free vibrations in different structural components. However there was no attention in exploring the potential of these methods for greater accuracy in numerical approximation of problems eigenvalues / eigenvectors.

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In this sense, recently there has been some research with surprisingly results as: "Cottrell ***et al. 2007a-2007b*** "using k-method to builds approximation spaces of regularity and order desired to approach the free vibration problem in road bars and aluminum conical shell. The results showed very similar results to the analytical solution to the problem of the bar. In this instance there arisen the acoustic and optics branch's characteristic of the approaches made by FEM. Other approaches by using high regularity and high order approximation spaces can be seen in Garcia and Rossi (2012), here the authors use spaces built by the Generalized Finite Element Method (GFEM) to obtain the natural frequency associated with axis symmetric modes for thick plates and shells of revolution.

In the present work high order and high regularity approximation spaces are build by the GFEM to approach the free vibration problem to Mindlin thick plate model. The high regularity and high order approximation spaces are build by explicit enrichment by polynomial functions of partition of unity´s (PU´s) with regularity C^0, C^2 and C^4. The 2D high regularity (PU´s) is build by tensor product of 1D high regularity (PU´s) obtained by rational polynomial functions. This work is presented in six sections as follow: introduction, approximation space by GFEM, Free vibration problem; numerical result; conclusions, bibliographic reference.

2. APROXIMATION SPACE (GFEM)

Enrichment of approximation spaces with PU properties has been studied by several authors over the last fifteen years. The methods have been given different names; for instance, one can find the names the Generalized Finite Element Method (GFEM) proposed by "Duarte ***et al. 2000***"; eXtended Finite Element Method (XFEM) proposed by Merle and Dolbow, (2002), Element Free Galerkin Method (EFGM) proposed by "Belytschko ***et al. 1994***" all this methods build the approximation space with extrinsic enrichment of the PU´s functions. The enrichment procedure used in this work consists of the multiplication of a rational polynomial based PU shape function, defined on a nodal position of the element of the integration mesh, by a set of complete monomials of ***p*** order. The nodes to be enriched can be either selectively selected, by means of an error estimator, or simply homogeneously selected.

The enriched approximation space is composed of all possible linear combinations of a finite dimension space generated by the product of functions Φ, which defines the PU, by a set of functions Q_{α}^{p}. Here, is the node number. Some important definitions are presented in order to aid in the presentation of the global approximation space.

2.1 Partition of Unity of regularity $C^k(\Omega)$, $k = 0, 2, 4, ...$

In this work, for construction of the approximation space are used the set functions $\{\Phi\}_{\in \Upsilon}$ with Υ a set index functions, which represent a partition of unity (PU) subordinate to an open cover $\Omega \subseteq \bigcup \Omega$ such that $\exists M \in \mathbb{N}\ \forall \boldsymbol{x} \in \Omega\ \text{card}\{\ \ | x \in \Omega\} \leq M$, thus a partition of unity, of the type *Lipschitz*, has the following properties according Melenk and Babuska (1996):

$$\operatorname{supp}(\Phi) \subset \Omega, \ \forall \ ; (1)$$

$$\sum_{\in \Upsilon} \Phi(x) = 1, \ \forall \boldsymbol{x} \in \Omega ; (2)$$

$$\|\Phi\|_{L^{\infty}(\mathbb{R}^n)} \leq C_{\infty} ; (3)$$

$$\|\nabla \Phi\|_{L^{\infty}(\mathbb{R}^n)} \leq \frac{C_G}{diam(\Omega)} ; (4)$$

where $\|\cdot\|_{L^{\infty}(\mathbb{R}^n)}$ is the infinite norm and C_{∞} e C_G are constants.

In this work the High regularity ($C^k(\Omega)$, $k = 0, 2, 4, ...$**)** PU´s functions**,** are builds by tensor product, defined in Eq. (5) , of PU´s builds in 1D domain shown in Garcia and Rossi (2012) as follows,

$$\mathbf{P} = \mathbf{\Phi} \otimes \mathbf{\Phi} ; (5)$$

In the Eq. (5) has;

$$\mathbf{\Phi} = \left[{}_{1}(\ ) \quad {}_{2}(\ ) \right]; (6)$$

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$$\mathbf{\Phi} = \left[\ _1(\ )\quad _2(\ )\right];\ (7)$$

In the Eq. (6)-(7), $\left\{\ _i(\ )\right\}_{i=1}^{2}$ and $\left\{\ _i(\ )\right\}_{i=1}^{2}$ are the PU´s function on 1D domain defined in Garcia and Rossi (2012). In Eq. (8) $\mathbf{P}$ is a matrix defined as;

$$\mathbf{P} = \begin{bmatrix} _1(\ ,\ ) & _2(\ ,\ ) \\ _4(\ ,\ ) & _3(\ ,\ ) \end{bmatrix};\ (8)$$

In the Eq. (8) for the PU´s $C^2(\Omega_e)$ functions are defined by de Eq. (9)-(12);

$$_1(\ ,\ ) = ((\ -1)^2(\ +3)^2(\ -1)^2(\ +3)^2)/(4(\ ^4-2\ ^2+9)(\ ^4-2\ ^2+9))\ ;\ (9)$$

$$_2(\ ,\ ) = ((\ +1)^2(\ -3)^2(\ -1)^2(\ +3)^2)/(4(\ ^4-2\ ^2+9)(\ ^4-2\ ^2+9))\ ;\ (10)$$

$$_3(\ ,\ ) = ((\ +1)^2(\ -3)^2(\ +1)^2(\ -3)^2)/(4(\ ^4-2\ ^2+9)(\ ^4-2\ ^2+9))\ ;\ (11)$$

$$_4(\ ,\ ) = ((\ -1)^2(\ +3)^2(\ +1)^2(\ -3)^2)/(4(\ ^4-2\ ^2+9)(\ ^4-2\ ^2+9))\ ;\ (12)$$

The function of Eq. (9)-(12) over the natural domain Ω_e of finite element, are shown in the Fig.1c.

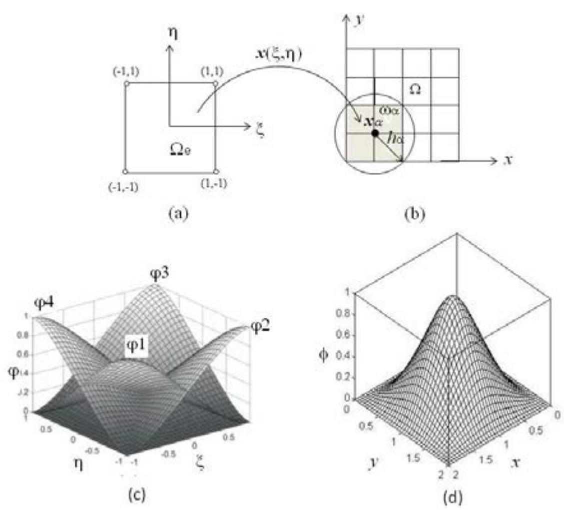


Figure 1. PU global function $\Phi(x, y)$ obtained by geometric mapping $\Omega_e \to \Omega$. Fig. 1(a) and (b): Mapping defined by Eq.(13)-(14). Fig. 2(c) and (d) PU functions defined by Eq. (9)-(12).

The $\Phi(x, y)$ is the global function of PU obtained by geometric mapping to $\Omega_e \to \Omega$ (see Fig.1a-b) defined by Eq.(13)-(14) of the PU´s functions defined by Eq. (9)-(12).

$$x(\ ,\ ) = \sum_{i=1}^{4} x_i N_i(\ ,\ );\quad (13)$$

$$y(\ ,\ ) = \sum_{i=1}^{4} y_i N_i(\ ,\ );\quad (14)$$

In the Eq. (13)-(14) $N_i(\ ,\ )$ are the shape functions of the bilinear element.

2.2 Local Approximation Space Q_{α}^{p}

The local approximation space of order *p* associated with the PU is defined by

$$Q^{p} = span\left[\left\{ P_{k} \right\}_{k=1}^{p}\right]; \ (15)$$

where $P_{k\alpha}$ are the complete monomials (defined in the Pascal triangle) of order *k* with origin set at the th node of the mesh (central coordinates of th clouds). The local approximation space Q^{p} is constructed based on a monomials basis.

2.3 Enriched approximation space $\Im_{N}^{p}$

Let $\{\Phi_i\}_{i=1}^{N}$ be a Partition of Unity subordinated to an open covering $\{\Omega\}_{\alpha=1}^{N}$, then, the global approximation space of order *p* is defined as,

$$\Im_{N}^{p} = span\left[\left\{ \Phi Q^{p} \right\}_{=1}^{N}\right]; \ (16)$$

where for $p = 2$ has

$$Q^{p=2} = \left\{ 1, \bar{x}, \bar{y}, \bar{x}^{2}, \overline{xy}, \bar{y}^{2} \right\}; \ (17)$$

In the Eq. (17) $\bar{x}, \bar{y}, \ldots$ are the normalized coordinates value defined by,

$$\begin{cases} \bar{x} = \dfrac{x - x}{h} \\ \bar{y} = \dfrac{y - y}{h} \end{cases}; \quad (18)$$

The (x, y) is the point overlapped for th cloud $(x\ , y\ )$, with h radius (see: Fig. 1b).
For the uniform local enrichment of PU functions with $Q^{p=2}$ the global space is defined as,

$$\Im^{p=2} = \left\{ \ _1 \quad _2 \quad _3 \quad _4 \quad _5 \quad _6 \right\} = \left\{ \Phi \quad \Phi\bar{x} \quad \Phi\bar{y} \quad \Phi\bar{x}^{2} \quad \Phi\overline{xy} \quad \Phi\bar{y}^{2} \right\}; \ (19)$$

One feature of the spaces $\Im_{N}^{p}$ constructed with the PU's above is that the functions of the local approximation spaces are not linearly dependent. This feature result in the mass and stiffness matrix, although ill-conditioned, for high “p” enrichment, are not singular (see: Garcia and Rossi (2012)).

3. FREE VIBRATION PROBLEM

The formulation of the problem of undamped free vibrations to thick plate, is obtained from the elliptic eigenvalue / eigenvectors problem whose weak formulation is defined as: find $(\ , w) \in$ such that $\neq 0, w \neq 0$ and $\Omega \in \mathbb{R}$, $\Omega > 0$ such that,

$$\int_{\Omega} \boldsymbol{M} \cdot \nabla \hat{\ } d\Omega + \int_{\Omega} \boldsymbol{Q} \cdot \left(\nabla \hat{w} - \hat{\ } \right) d\Omega - \Omega^{2} \left(\int_{\Omega} \frac{Ph^{3}}{12} \cdot \hat{\ } d\Omega + \int_{\Omega} Phw \cdot \hat{w} d\Omega \right) = 0, \ \forall \hat{\ }, \hat{w} \in Var; \ (20)$$

In the Eq. (20), $\boldsymbol{M}$ is the flexural moment tensor, $\boldsymbol{Q}$ is the shear vector, is the rotation vector, w is the transverse displacement, h and P are the thick and density respectively.

The set of the test functions is defined by,

$$=\left\{ \ , w \in H^{1}\left(\bar{\Omega}\right) \middle| \ \boldsymbol{=} \ ^{-}, w=\bar{w}, \forall\left(x, y\right) \in \Gamma_{D}\right\}; (21)$$

For this example is adopted $^{-} \boldsymbol{= 0}$ and $\bar{w}=0$.

In Eq. (20) $\boldsymbol{Var}$ is the set of weight functions defined by,

$$Var=\left\{ \hat{} , \hat{w} \in H^{1}\left(\bar{\Omega}\right) \middle| \hat{} \boldsymbol{= 0}, \hat{w}=0, \forall\left(x, y\right) \in \Gamma_{D}\right\}; (22)$$

The discretized formulation for the problem defined by Eq. (20) using Bubnov-Galerkin method is defined by Eq. (23) as follow,

$$\left[\left(\int_{\Omega} \boldsymbol{B}_{b}^{T} \boldsymbol{D}_{b} \boldsymbol{B}_{b} d\Omega+\int_{\Omega} \boldsymbol{B}_{s}^{T} \boldsymbol{D}_{s} \boldsymbol{B}_{s} d\Omega\right)-\Omega^{2}\left(\int_{\Omega} \frac{Ph^{3}}{12} \boldsymbol{N}^{T} \boldsymbol{N} \ d\Omega+\int_{\Omega} Ph \boldsymbol{N}_{w}^{T} \boldsymbol{N}_{w} d\Omega\right)\right] \boldsymbol{U}=\mathbf{0}; (23)$$

In Eq. (23) one has to,

$$\boldsymbol{N} \ =\begin{bmatrix} {}^{1}_{1} & 0 & 0 & \cdots & {}^{i} & 0 & 0 & \cdots & {}^{p}_{N} & 0 & 0 \\ 0 & {}^{1}_{1} & 0 & \cdots & 0 & {}^{i} & 0 & \cdots & 0 & {}^{p}_{N} & 0 \end{bmatrix}, \ i=1,..,p, \quad =1,..,N; (24)$$

$$\boldsymbol{N_{W}}=\begin{bmatrix} 0 & 0 & {}^{1}_{1} & \cdots & 0 & 0 & {}^{i} & \cdots & 0 & 0 & {}^{p}_{N} \end{bmatrix}; (25)$$

$$\boldsymbol{U}^{T} \ =\left\{ {}^{1}_{1x} \quad {}^{1}_{1y} \quad w_{1}^{1} \quad \cdots \quad {}^{i}_{x} \quad {}^{i}_{y} \quad w^{i} \quad \cdots \quad {}^{p}_{Nx} \quad {}^{p}_{Nx} \quad w_{N}^{p}\right\}; (26)$$

For homogeneous "p" enrichment the dimension of $\boldsymbol{U}$ is $n=3pN$, where, N is the number of nodes of the mesh.

In Eq. (20), the curvature matrix $\boldsymbol{B}_{b}$ and the shear deformation matrix $\boldsymbol{B}_{s}$, are defined in the natural domain of element. Therefore the partial derivatives must be mapped to the physical domain of the problem defining $\boldsymbol{B}_{b}$ and $\boldsymbol{B}_{s}$ as follows:

$$\boldsymbol{B} \ = \boldsymbol{HJ\partial} \ \boldsymbol{N} \ ; (27)$$

$$\boldsymbol{B_{S}}=J^{-1} \boldsymbol{\partial_{S}} \boldsymbol{N}_{w}-\boldsymbol{N} \ ; (28)$$

In the Eq. (27)-(28), $\boldsymbol{H}$, $\boldsymbol{J}$ and $\boldsymbol{\partial}_{s}$, $\boldsymbol{\partial}$, are the Boolean operator, the Jacobean operator and the differential operator, respectively. Theses operators are defined in Eq. (29)-(31) as follow,

$$\boldsymbol{\partial}^{T}=\begin{bmatrix} \frac{\partial(\cdot)}{\partial} & \frac{\partial(\cdot)}{\partial} & 0 & 0 \\ 0 & 0 & \frac{\partial(\cdot)}{\partial} & \frac{\partial(\cdot)}{\partial} \end{bmatrix}; (29)$$

$$\boldsymbol{\partial}_s = \begin{bmatrix} \dfrac{\partial(\cdot)}{\partial} \\ \dfrac{\partial(\cdot)}{\partial} \end{bmatrix}; \ (30)$$

$$\boldsymbol{J} = \begin{bmatrix} J^{-1} & 0 \\ 0 & J^{-1} \end{bmatrix}; \ (31$$

In the Eq. (31) J is the Jacobean matrix defined by,

$$J = \begin{bmatrix} \dfrac{\partial x}{\partial} & \dfrac{\partial y}{\partial} \\ \dfrac{\partial x}{\partial} & \dfrac{\partial y}{\partial} \end{bmatrix}; \ (32)$$

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}; \ (33)$$

Equation (20) can be expressed in matrix form by,

$$\left(\boldsymbol{K} - \Omega^2 \boldsymbol{M}\right)\boldsymbol{U} = \boldsymbol{0}\ ; \ (34)$$

In Eq. (43) that has,

$$\boldsymbol{K} = \int_\Omega \boldsymbol{B}_b^T \boldsymbol{D}_b \boldsymbol{B}_b d\Omega + \int_\Omega \boldsymbol{B}_s^T \boldsymbol{D}_s \boldsymbol{B}_s d\Omega\ ; \ (35)$$

$$\boldsymbol{M} = \int_\Omega \frac{Ph^3}{12} \boldsymbol{N}^T \boldsymbol{N}\ d\Omega + \int_\Omega Ph \boldsymbol{N}_w^T \boldsymbol{N}_w d\Omega\ ; \ (36)$$

In Eq. (3) $\boldsymbol{D}_b$ and $\boldsymbol{D}_s$ are the constitutive matrix of bending and shear respectively shown bellow,

$$\boldsymbol{D}_b = \frac{Eh^3}{12\left(1- \ ^2\right)} \begin{bmatrix} 1 & & 0 \\ & 1 & 0 \\ sym & & \dfrac{\left(1- \ \right)}{2} \end{bmatrix}; \ (37)$$

$$\boldsymbol{D_S} = \frac{k_s hE}{\left(1- \ ^2\right)} \begin{bmatrix} \dfrac{1-}{2} & 0 \\ 0 & \dfrac{1-}{2} \end{bmatrix}; \ (38)$$

In the Eq. (38) $k_s = 4/5$ is the shear correction factor.

If the Eq. (34) defined the symmetric problem of eigenvalue/eigenvector, then, $\boldsymbol{K}$ is, at least, positive semi-definite and $\boldsymbol{M}$ is positive definite. In this case has: $0 < \Omega_1 \le \Omega_2, \ldots \le \Omega_i \le, \ldots \le \Omega_n$, $i = 1, \ldots, n$ and for $\Omega_i \ne \Omega_j$ has mass orthogonal eigenvectors in other words,

$$\boldsymbol{U}_i^T \boldsymbol{M} \boldsymbol{U}_j = {}_{ij} m_{ij},\ i, j = 1,..,n \text{ ; (39)}$$

and,

$$\boldsymbol{U}_i^T \boldsymbol{K} \boldsymbol{U}_j = {}_{ij} k_{ij} \text{ ; (40)}$$

In the next section we analyze the numerical results obtained with high order FEM and high order and high regularity GFEM to approach the undamped free vibrations problems for Mindlin thick plates.

4. NUMERICAL RESULT

In this section are analyzed de performance of high regularity and high order approximation spaces build by GFEM in the approach undamped free vibrations problem for Mindlin thick plate model. For this example the numerical results are obtained for the thick plate with mechanical properties, boundary conditions and discretization of the domain shown in the Fig. 3 as follow,

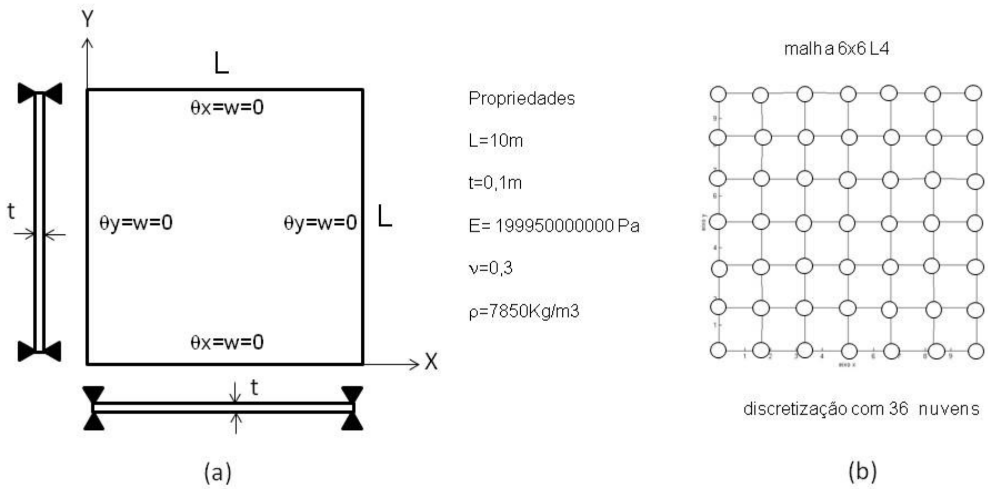


Figure 3: a) campled thick plate; b) mesh with 6x6 bilinear elements.

The analysis of convergence is made for the first Ω_{150} natural frequencies of the plate obtained through the relative error given in Eq. (41). In turn, the relative error is obtained from the analytical solution defined in Eq. (42).

$$E_r = \frac{\left|\Omega_h - \Omega_{mn}\right|}{\left|\Omega_{mn}\right|} \text{ ; (41)}$$

In the Eq. (41), Ω_h are the approximated frequency and Ω_{mn} the analytical frequency.

$$\Omega_{mn}^2 = \frac{12D}{P^2 h^4 \left(1 - {}^2\right)} \left[\left(\frac{m}{L}\right)^2 + \left(\frac{n}{L}\right)^2\right] \left[1 - \left(1 + \frac{E}{\left(1 - {}^2\right) kG}\right) S^2 \left(m \quad\right)^2 \Gamma_{mn}\right] \text{ ; (42)}$$

Further details of Eq. (42) can be seen in Dym and Shames (1973).

The case studies will be made for a discretization with uniform mesh shown in Fig.3b and a discretization made with distorted mesh shown in Fig.6. For both cases of the domain discretization in the construction of approximation space are used "p" homogeneous enrichments as follow,

A. The approximation space is build with 6x6 fourth order Lagrangian elements (25 nodes element). The problem is approximated with 1779 degrees of freedom;
B. Approximation space built second GFEM by explicit enriched with complete polynomial “p=4”, of the C^0 PU functions;

C. Approximation space built second GFEM by explicit enriched with complete polynomial "p=4", of the C^2 PU functions;

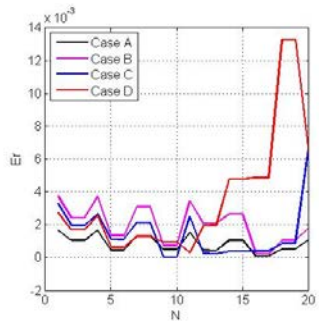
D. Approximation space built second GFEM by explicit enriched with complete polynomial "p=4", of the C^4 PU functions;

For the cases B-C the problem is approximated with 2069 degree of freedom.

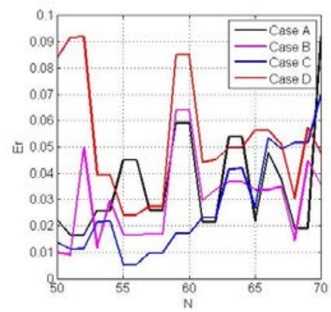
The numerical results for the relative error are shown for the uniform and distorted mesh as follow.

4.1 Result for umiforme mesh

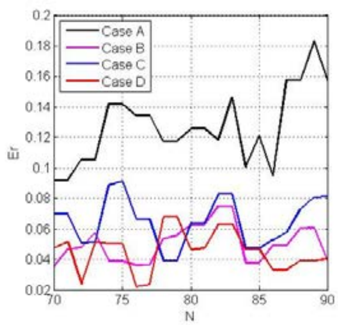
The results for uniform grid (see. Fig.3b) for the cases A-D are shown in the Fig. 5a-f as follow.



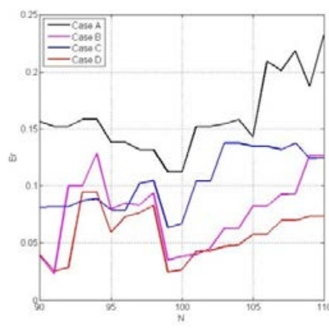
(a)



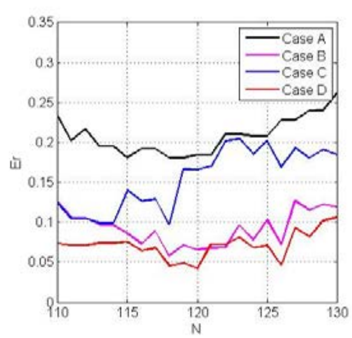
(b)



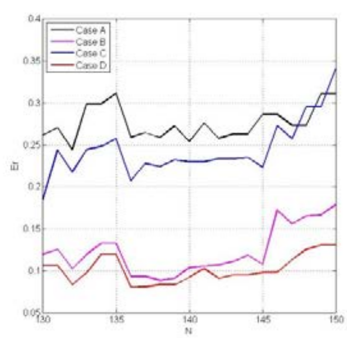
(c)



(d)



(e)



(f)

Figure 5: Relative error in natural frequencies; a) for order $N=[1,20]$; b) for order $N=[50,70]$; c) for order $N=[70,90]$; d) for order $N=[90,110]$; e) for order $N=[110,130]$; for order $N=[130,150]$.

The results shown in the figures above show a good performance of the strategy A (high order FEM) about the strategies obtained with GFEM for the first N = 20 frequencies. However since N = 50, the relative error increases continuously reaching more than thirty percent.

An opposite behavior is noticed in BD strategies. In these cases the loss of convergence is significantly less pronounced especially for strategy B and D. Note that the high regularity approximation spaces build in accordance GFEM present the flexural mode stiffer than those obtained using FEM, this can be concluded by value of the first frequency in Fig. 5a.

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4.2 Result for distorted mesh

The results for the distorted mesh shown it the Fig.6 for the cases A-D are shown in the Fig. 7a-f as follow.

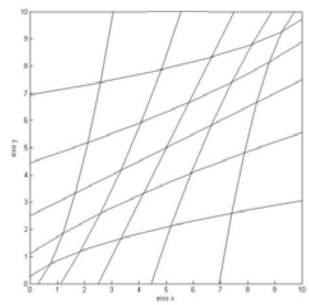
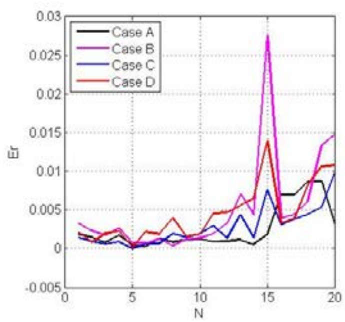
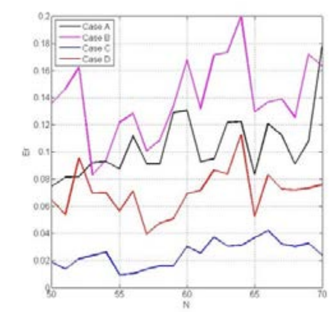


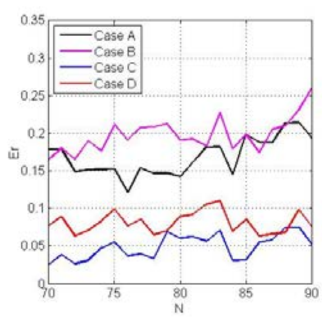
Figure 6: Distorted mesh for 6x6 bilinear elements.



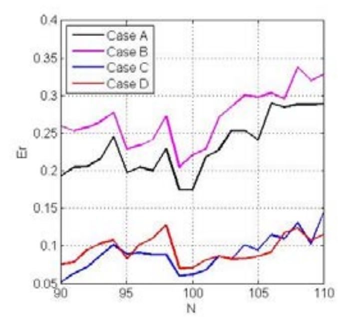
(a)



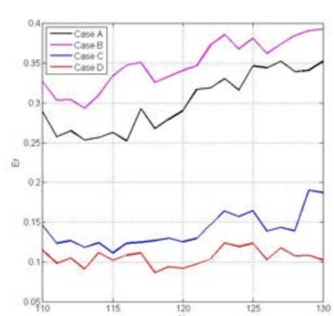
(b)



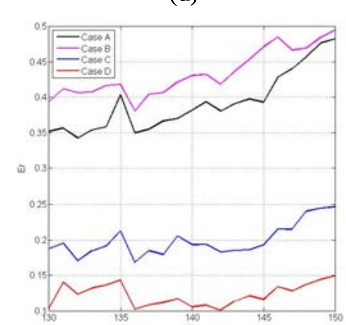
(c)



(d)



(e)



(f)

Figure 5: Relative error in natural frequencies; a) for order $N = [1, 20]$; b) for order $N = [50, 70]$; c) for order $N = [70, 90]$; d) for order $N = [90, 110]$; e) for order $N = [110, 130]$; for order $N = [130, 150]$.

The effects of mesh distortion in the relative error generally produces a loss of accuracy in all strategies used, however, this loss of convergence is significantly pronounced in strategy B. On the other hand the severe distortion of the mesh, observed in Fig.6, resulted in little loss of accuracy in the strategy D.

It can be concluded that the loss of accuracy, produced by distortion mesh, to approximation spaces build according GFEM are sensitive to the same regularity. That is, the lower regularity of the approximation spaces build according GFEM increases the loss of precision produced by distortion of the mesh.

5. CONCLUSION

The results confirm the observations made in single and two-dimensional free vibrations and wave's propagation problems in elastic medium. The better performance of approximation spaces of high regularity space build according GFEM with respect to the approximation spaces obtained with MEF are evident also in the free vibrations problems of Mindlin thick plates model. On the other hand it was observed a decrease in sensitivity to distortion of the mesh with increasing regularity of the approximation space.

The results, although preliminary, show the potential of smoothness spaces constructed second GFEM approach in addressing problems of undamped free vibrations in thick plates. The result observed for free vibrations, point to a possible good performance of the high regularity approximation spaces in problems of forced vibrations in thick plate's model.

6. REFERENCE

Arndt, M., 2009. "Método de Elementos Finitos Generalizados aplicado ao problema de vibrações livres de estruturas reticuladas", Tese de Doutoramento, Universidade Federal do Paraná, Curitiba.

Belytschko T., Lu Y. Y., Gu L., 1994. "Element Free Galerkin Method", *International Journal for Numerical Method in Engineering*, 37: 229-256.

Cottrell, J.A., Hughes, T.J.R., Reali, A., Sangalli, G., 2007a. "Isogeometric discretizations in structural dynamics and wave propagation", *ECOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, Crete, Greece, 13–16.

Cottrell, J.A., Hughes, T.J.R., Reali, A., 2007b. "Studies of refinement and continuity in isogeometric structural analysis", *Computer methods in applied mechanics and engineering*, 196:4160–4183.

Duarte, C.A, Babuska, I. and Oden, J.T., 2000. "Generalized finite element method for three-dimensional structures mechanics problems", *Computer and Structures*, 77: 215-232.

Dym L. C., Shames I. H., 1973. "Solid Mechanics a variational approach". McGraw-Hill book company, New York.

Garcia, O.A., Rossi, R., Linzmayer, P.R., 2010. "Elementos Finitos Generalizado de alta regularidade na análise de vibrações livres e propagação de ondas em meios sólidos elásticos", *Mecánica Computacional*, 29:931–947, Asociacíon Argentina de Mecanica Computacional, Buenos Aires, Argentina.

Garcia, O. A., Rossi, R., Avila C. R., 2012. "Enrichment of rational polynomial family of shape functions with regularity $C_0^k, k = 0, 2, 4...$, Applications in axisymmetric plates and shells", *Engineering Computations*, 29: 636-660.

Givoli, D., 2008. "On the number of reliable finite-element eigenmodes", *Communications in Numerical Methods in Engineering*, 24:1967–1977.

Gu. Y. T, Liu G. R., 2001. "A meshless local Petrov-Gelerkin (MLPG) method for free and forced vibration analysis for solid", *Computational Mechanics* 27: 188-198.

Ferreira, A.J.M., Roque, C.M.C. and Jorge, R.M.N, 2005. "Free vibration analysis of symmetric laminated composite plates by FSDT and radial basis functions", *Composite Structures*, 77:296-305.

Hughes, T.J.R., 1987. "The Finite Element Method – Linear Static and Dynamic Finite Element Analysis", Dover Publications.

Linzmayer P. R., Garcia O. A., Rossi R., 2011. "MEFG de alta regularidade na abordagem de vibrações livres em problemas axissimétricos de cascas compostas" XXXII Iberian Latin-American Congress on Computational Methods in engineering, Ouro Preto, SP, Brazil.

Liu G. R., Chen X. L., 2001, "A mesh-free method for static and free vibration analyses of thin plates of complicated shape", *Journal of Sound and Vibration*, 241(5), 839-855.

Liu, L., Liu G. R., Tan V. B. C., 2002, "Element Free method for static and free vibration analysis of spatial thin Shell structures" *Computer Methods in Applied Mechanics and Engineering*, 191:5923-5942.

Liew K. M, Huang Y. Q., Reddy J.N., 2003. "Vibration analysis of symmetrically laminated plates based on FSDT using the moving least squares differential quadrature method", *Computer Methods in Applied Mechanics and Engineering*, 192:2203-2222.

Leung, A. Y. T., Chan, J. K. W., 1998. "Fourier p-element for the analysis of beam and plates". *Journal of Sound and Vibrations*, vol. 212, n.17, pp 179-185.

ISSN 2176-5480

Melenk J. M., Babuska I., 1996"The partition of unity finite element method: Basic theory and applications", *Computer Methods in Applied Mechanics and Engineering*, 139:289-314.

Merle R., and Dolbow J., 2002. "Solving thermal and phase change problems with the eXtended finite element method", *Computational Mechanics* 28, 339-350.

Zeung, P., 1998(a). "Composite element method for vibration analysis of structures, part I: C^0 element (bar)". *Journal of Sound and Vibrations*, vol. 218, n.4, pp 619-658.

Zeung, P., 1998(a). "Composite element method for vibration analysis of structures, part II: C^1 element (beam)". *Journal of Sound and Vibrations*, vol. 218, n.4, pp 619-658.

7. RESPONSIBILITY NOTICE

The authors Oscar Alfredo Garcia de Suarez, Rodrigo Rossi, Rudimar Mazzochi and Claudio Avila da Silva Junior are the only responsible for the printed material included in this paper.