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AN ALGORITHM TO DETERMINE THE AERODYNAMIC CHARACTERISTICS OF SMALL WIND TURBINES

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Abstract.*The BEM – Blade Element Momentum method, which combines the blade element method and momentum theory, has been the most used for calculation of the aerodynamic characteristics of wind turbines, giving still competitive results if compared with others more sophisticated and expensive. For this reason, it has been the main reference method adopted in the current bibliography about wind turbines design. In the present work, it is proposed an algorithm to determinate the aerodynamic characteristics of the wind rotors by BEM method. The main feature of the algorithm is the use of an equation derived using manipulation of classic equations of the BEM method. The most relevant difference, in comparison with the current procedure, is the need of solving just one nonlinear equation involving only one parameter, while the current algorithm involves the numerical solution of a system of equations. The remaining aerodynamic parameters are obtained by direct formulas without iterative procedures. Therefore, a more efficient algorithm is obtained. It is shown a practical example in the end of the article.*

Keywords:*wind turbines, blade element momentum method, wind energy.*

1. INTRODUCTION

Since the beginning of the researches related with wind energy conversion into electric energy, several models were proposed as design tools. The most used and studied in this article is the BEM – Blade Element Momentum method (Burton, 2011).

Even with the large use of BEM method for the analysis of wind turbines performance, the literature doesn't present an optimized procedure to solve the related equations. The common to do is to reduce then to a set of 2 equations in 2 unknowns, which should be solved simultaneously for obtaining the axial and radial induction factors in each section of the blade (Teixeira, 2012). The main problem with this procedure is that, in some cases, the values of the induction factorscan diverge along the iterative process.

In the present article, it is proposed aneffective procedure to evaluate the power coefficient of wind turbines. The aim of the procedure is the deduction of one nonlinear equation involving only oneunknown parameter, which is the angle between the local aerodynamic velocity in the blade section and the turbine rotation plane.This equation needs to be solved for each blade station individually.All remaining parameters needed to characterize the blade aerodynamics are obtained a posteriori without the necessity of additional iterative processes, which effectively diminish calculations. Indeed, differently of the previous procedures, by using the algorithm of this article, the necessity of solving a set of nonlinear equations involving several unknowns is eliminated.

To exemplify the use of the procedure, in the end of this paper, it is presented the solution of a practical problem of determining the performance of a wind turbine developed in the Centro de EstudosAeronáuticos da Universidade Federal de Minas Gerais (CEA/UFMG).

2. TRADITIONAL BEM METHOD EQUATIONS

As can be find in the literature (Burton, 2011), the blade aerodynamics of a wind turbine is described by the following equations:

$$C_P = \lambda^2 \int_0^1 8a'(1-a)\,\mu^3 d\mu \qquad (1)$$

where:

$$\mu = \frac{r}{R} \qquad (2)$$

$$\alpha = \varphi - \beta \tag{3}$$

$$\frac{a}{1-a} = \frac{\sigma_r}{4\,\mathrm{F}\sin^2\varphi} c_x \tag{4}$$

$$\frac{a'}{1+a'} = \frac{\sigma_r}{4\,\mathrm{F}\sin\varphi\cos\varphi} c_y \tag{5}$$

$$c_x = C_L\cos\varphi + C_D\sin\varphi \tag{6}$$

$$c_y = C_L\sin\varphi - C_D\cos\varphi \tag{7}$$

$$\sin\varphi = \frac{U_\infty}{w}(1-a) \tag{8}$$

$$\cos\varphi = \frac{\Omega r}{w}(1+a') \tag{9}$$

$$w = \sqrt{U_\infty^2(1-a)^2 + \Omega^2 r^2 (1+a')^2} \tag{10}$$

$$\sigma_r = \frac{N}{2\pi r} c \tag{11}$$

$$F = \frac{2}{\pi}\cos^{-1}\left[exp\left\{-\left(\frac{N/2\left(1-\frac{r}{R}\right)}{\frac{r}{R}\sin\varphi}\right)\right\}\right] = \psi(\varphi) \tag{12}$$

Where r is the radial position of the section, R is the blade radius, a denotes the axial induction factor, a' denotes the radial induction factor,β is the torsion of the blade station,C_L is the airfoil lift coefficient, C_D is the airfoil drag coefficient, φ is the summation of the angle of attack with the torsion angle, U_∞ is the flow velocity far upstream the turbine, Ω is the blades rotation, W is the effective velocity in the blade sections, N is the number of blades, c is the chord at a given blade section,σ_r is the blade solidity and F is the correction for lift losses, due to the formation of a vortex in the tip of the blade (Ingram, 2005).

The aerodynamic coefficients are known functions of attack angle (α):

$$C_L = C_L(\alpha) \tag{13}$$

$$C_D = C_D(\alpha) \tag{14}$$

Usually it's necessary to solve Eq.4 and Eq. 5in the unknowns $a$anda', and the other ones are obtained by the equations 6 to 12. The main problem with this procedure is that the values of a anda' are very unstable, because they can lead to values that aren't valid, according to the theory (Burton, 2011).

3. DERIVATION OF THE NEW EQUATION

To obtain a more efficient algorithm seems to be a good idea, if possible, to derive an equation in just one unknown. This simplifies drastically the iterative process, making the algorithm more robust and fast.

Indeed, manipulating the previous equations of the BEM method, more specifically, Eq.4, Eq. 5, Eq. 8, Eq. 9 and Eq. 12, leads to the resulting equation in the unknown φ:

$$\cos\varphi - \lambda\sin\varphi - \frac{\sigma_r}{4F}\left[(\lambda C_D + C_L) + \frac{\cos\varphi}{\sin\varphi}(\lambda C_L - C_D)\right] = 0 \tag{15}$$

where the aerodynamic coefficients C_L, C_D are functions of φ, since, according to Eq. 3

$$C_L = C_L(\alpha) = C_L(\varphi - \beta) \tag{16}$$

$$C_D = C_D(\alpha) = C_D(\varphi - \beta) \tag{17}$$

andβ is previously known for each station.

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Numerical experiments show that this equation has some singularities that turn difficult to find the solutions for some situations. To overcome this obstacle, Eq. 15 can be multiplied by Fsin φ, giving:

$$\mathrm{F}\sin\varphi\cos\varphi - \lambda\mathrm{F}\sin^2\varphi - \frac{\sigma_r}{4}[\sin\varphi\,(\lambda C_D + C_L) + cos\varphi(\lambda C_L - C_D)] = 0 \tag{18}$$

Equation 18 is stable and leads to acceptable values ofφ, and is the base for the algorithm describedbelow.

4. THE ALGORITHM

Step 0: Set values for: velocity ratio (λ), blade radius (R), number of blades (N).

Step 1: For all sections along the blade length:

Step 1.0: Set values for: chord (c), torsion (β), radial position of the section (r).

Step 1.1: Find φ that solve Eq. 18, by using some simple numerical method, like the bisection method (Burden andFaires, 2010).

Step 1.2: With the true value of φ, found in *Step 1*, calculate:

$$\alpha = \varphi - \beta$$

$$F = \psi(\varphi)$$

$$C_L = C_L(\alpha)$$

$$C_D = C_D(\alpha)$$

$$C_x = C_L\cos\varphi + C_D\sin\varphi$$

$$C_y = C_L\sin\varphi - C_D\cos\varphi$$

$$a = \frac{\sigma_r C_x}{\sigma_r C_x + 4F\sin^2\varphi}$$

$$a' = \frac{\sigma_r C_y}{\sigma_r C_y + 4F\sin\varphi\cos\varphi}$$

Step 2: Evaluate the power coefficient numerically (Burden and Faires, 2010)

$$C_P = \lambda^2\int_0^1 8a'(1-a)\,\mu^3 d\mu$$

5. PRACTICAL EXAMPLE

The implemented procedure was checked by calculating the power coefficient of a wind turbine with a known geometry, which means that its radial chord and torsion distributions along are previously given, and plotting it´s λ versus C_P curve.

The wind turbine used as example for the study was CEA WT-01 (Ribeiro, 2006), which uses the NACA 23015 airfoil along all its length. Figure 1 illustrates the airfoil lift curve and drag polar (Abbott and Doenhoff, 1958).

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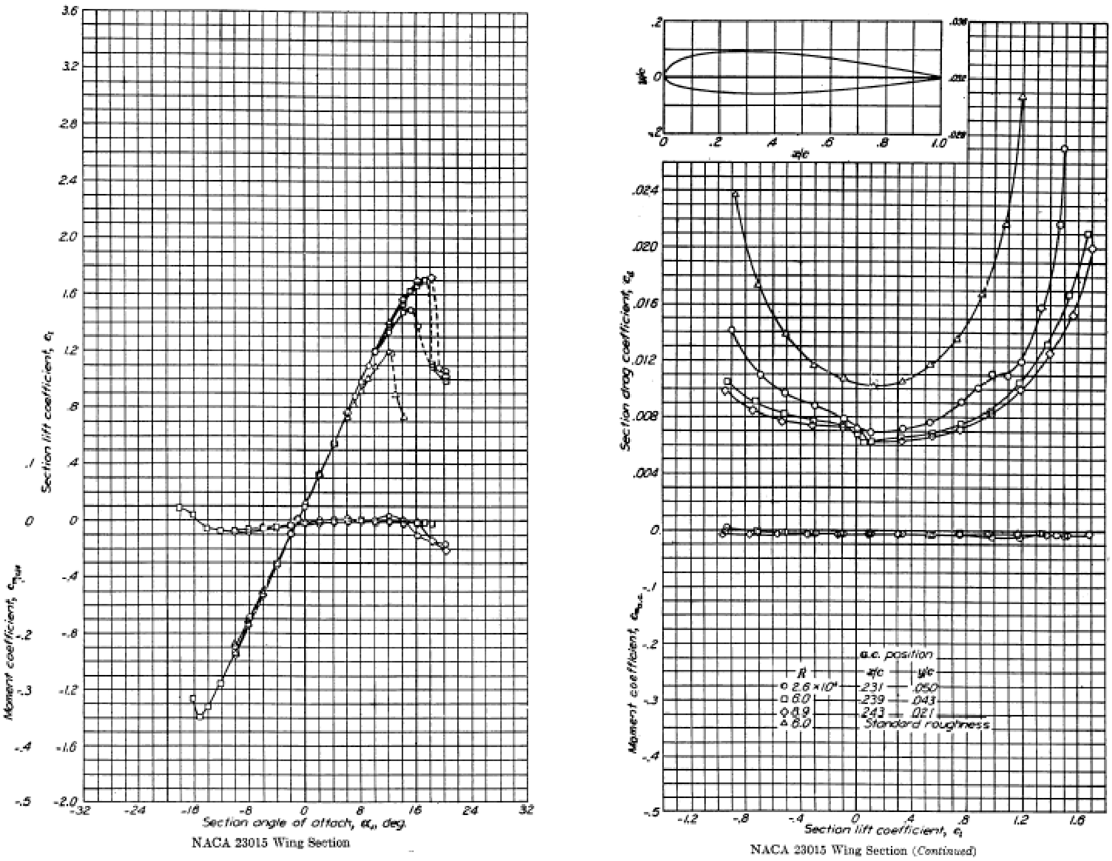

Figure 1 – NACA 23015 lift curve and drag polar (Abbott and Doenhoff, 1958).

The geometric characteristics of the wind turbine are shown in Tab. 1.

Table 1.Geometric characteristics of the studied wind turbine.

Radial position (m)	Chord (m)	Torsion angle (°)
0,525	0,445	24
0,7	0,435	17,64
0,875	0,421	11,27
1,05	0,406	8,36
1,225	0,391	5,45
1,4	0,376	4,4
1,575	0,362	3,35
1,75	0,347	2,49
1,925	0,332	1,63
2,1	0,317	1,16
2,275	0,303	0,68
2,45	0,288	0,44
2,625	0,274	0,19
2,8	0,259	0,1
2,975	0,244	0
3,15	0,229	0
3,325	0,215	0
3,5	0,2	0

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Figure 2 shows the obtained curve for the CEA WT-01 wind turbine for two cases, being one considering the tip loss factor (F) in the calculation, and the other without this consideration, which means that its value is made equal to 1 in the equations that describe the model.In both cases an error less than 1×10^{-4} was adopted as convergence criteria. The curves show the power coefficient as a function of the velocity ratio (λ), that is the ratio between the tangent and the axial flow velocities.

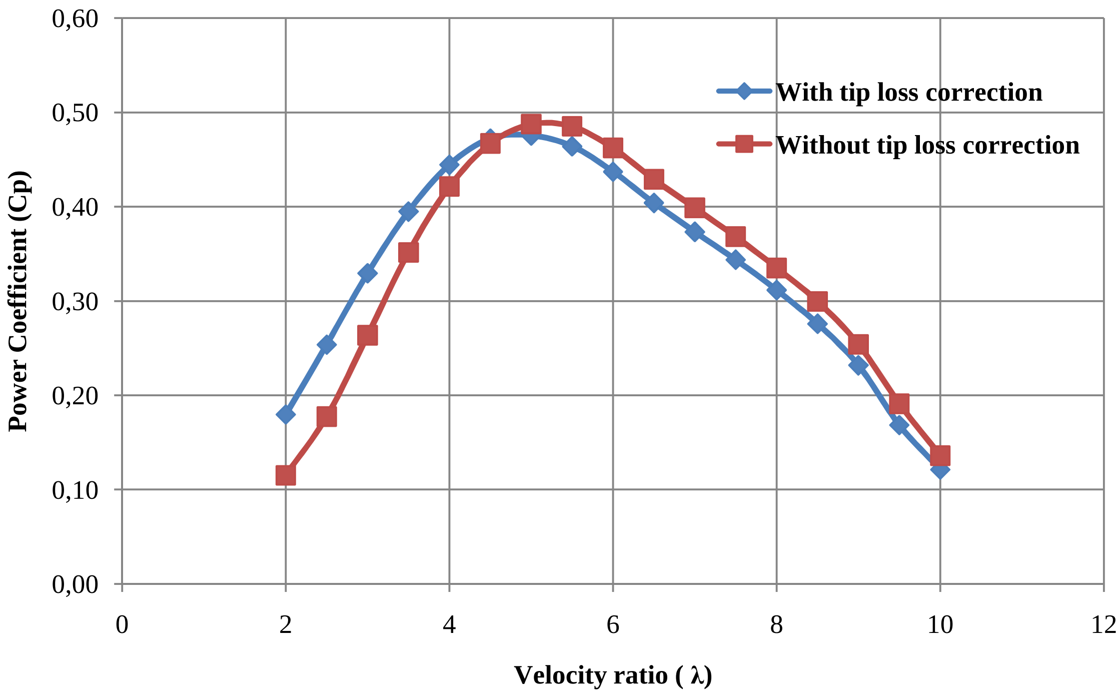

Figure 2 – Performance curves for the CEA WT-01 wind turbine.

To check the residual values produced by the solution of the original system of equations, the values found for φ were substituted in the Eq. 19 that represents the quotient between Eq. 8 and Eq. 9.

$$\tan\varphi = \frac{1}{\lambda}\frac{(1-a)}{(1+a')} \qquad (19)$$

Table 2 shows the residual values for Eq. 19, considering the tip loss correction in the calculations and $\lambda = 5$.

Table 2. Residual obtained for the variable φ with the proposed algorithm (Values multiplied by 10^4).

Blade section	Residue
1	0,0006
2	0,0031
3	0,0011
4	0,0007
5	0,0020
6	0,0000
7	0,0008
8	0,0003
9	0,0010
10	0,0010
11	0,0009
12	0,0004
13	0,0000
14	0,0007
15	0,0002
16	0,0006
17	0,0007

It can be noticed that all the residual values are very small, what indicates that the method solves the basic equations correctly.

6. COMMENTS

The graphs plotted in Fig. 2 show the tendency of the rotor to present low values of power coefficient for low velocity ratios, and the C_P value get larger when λ is approximating of the design velocity ratio, that is the one considered in the calculation of the blade geometry. After this λ value, there's a tendency of the C_P get smaller again. This result is consistent with the literature (Burton, 2011; Ribeiro, 2006).

7. CONCLUSIONS

In the literature the BEM method for determination of the power coefficient of wind turbines is presented as a set of equations in several unknowns that, usually are in practice reduced to a set of 2 equations in the unknowns a and a'. The rest of the unknowns can be obtained a posteriori.

In this paper was shown that the equations can be manipulated to derive an equation in just one unknown, which is the φ angle. That enables to simplify the algorithm for evaluate the performance of the rotor, where we need to solve iteratively just one equation in one unknown. The other parameters can be found after, directly by the other equation without the need of iterative procedures.

The authors hope that this procedure will be adopted to analyze the performance of wind turbines by the BEM method, once that the proposed algorithm is more concise, simple and effective.

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9. REFERENCES

Abbott, I.H., Doenhoff, A.E., 1958. "Theory of Wind Sections". Dover Publications, Inc.
Burden, R. L., Faires, J. D., 2010."Numerical Analysis".Brooks/Cole, Cengage Learning.
Burton, T., 2011."Wind Energy Handbook".John Wiley & Sons, Ltd.
Ingram, G., 2005. "Wind Turbine Blade Analysis using the Blade Element MomentumMethod. Version 1.0".http://www.dur.ac.uk/g.l.ingram/. Accessed in 10/02/2011.
Ribeiro, R. P., 2006. "Metodologia de projeto de pás de turbinas eólicas de eixo horizontalpara baixa potência". Tese de doutorado, Universidade Federal de Minas Gerais.
Teixeira, L. B., Utsch, R. L., Ribeiro, R. P., Bortolus, M. V., 2012 - "Análise aerodinâmica de uma turbina de pequeno porte usando o método BEM – BladeElement Momentum.", Proceedings of SIMMEC – Simpósio de Mecânica Computacional.

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