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NUMERICAL STUDY OF FORCED CONVECTION OF A POWER-LAW FLUID ACROSS A SQUARE CYLINDER

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Abstract. *Non-Newtonian fluids are present in nature and industry. Examples are food products (yoghurt, soft cheeses, jams, chocolate), paints, rubber, polymer melts, polymer solutions, adhesives, gum, sludge, blood and others. These fluids exhibit nonlinear relation between the shear stress and the shear rate, that is, its viscosity is not constant. In cases where viscosity decreases with increasing shear rate, the fluids are classified as shear-thinning, whereas the fluids that exhibit the inverse behavior are classified as shear-thickening. The power-law model is used in engineering to model both behaviors. Computational Fluid Dynamics (CFD) is a tool used in the numerical simulation of Newtonian and non-Newtonian fluid flow. Numerous free and commercial codes are used today, including the free and open source MFIX, which was developed aiming the numerical simulation of reactive multiphase flows. The goal of this work is to implement the power-law model in MFIX, validate the implementation and conduct a case study using the model implemented. With the implementation of a non-Newtonian model to the code, a new possibility for the simulation of multiphase flows of solid-non-Newtonian liquids is opened, as well as there is an increase in the capability of the code regarding the study of single-phase fluid flows of non-Newtonian fluids subject to heat transfer. The model was implemented and validated by comparison with literature results for the flow in a lid driven cavity. Subsequently, simulations were carried out concerning isothermal and non-isothermal flows around a square cylinder immersed in a channel. Parameters of analysis consisted of Prandtl number, power-law index and blockage ratio, for a fixed Reynolds number. It was found that the Nusselt number is strongly influenced by the blockage ratio and decreases with the increase of the power-law index. The Prandtl number also directly influences the process. With its increase, the dependence of the Nusselt number with the power-law index and the blockage ratio is more pronounced.*

Keywords: *Generalized Newtonian Liquid, non-Newtonian fluids, MFIX.*

1. INTRODUCTION

Non-Newtonian fluids are present in nature and industry. Examples are food products (yoghurt, soft cheeses, jams, chocolate), paints, rubber, polymer melts, polymer solutions, adhesives, gum, sludge, blood and others. These fluids exhibit nonlinear relation between the shear stress and the shear rate. In the case of purely viscous non-Newtonian fluids, viscosity is a function of the shear rate. In cases where viscosity decreases with increasing shear rate, the fluids are classified as shear-thinning, whereas the fluids that exhibit the inverse behavior are classified as shear-thickening. These fluids find many industrial applications. The Ostwald-de Waele model, or power-law fluid, is much used in engineering to model both behaviors, in view of the simplicity of its equation and its ability to adjust the flow curve of many fluids of interest (Bird, *et al.*, 1987).

Computational Fluid Dynamics (CFD) is a tool used in the numerical simulation of Newtonian and non-Newtonian fluid flow. Numerous free and commercial CFD codes are available today. One of them is the software MFIX (Multiphase Flow with Interphase eXchanges), developed by the National Energy Technology Laboratory of the U.S. Department of Energy (Syamlal, *et al.*, 1993). This code was developed aiming the numerical simulation of reactive multiphase flows. Since it is free and open source, it has been used by researches around the globe as a test-stand for testing and developing multiphase flow constitutive equations. As MFIX does not have the capability of predict non-Newtonian behavior, the goal of this work was to implement a non-Newtonian model - namely the power-law model - in the code. This improvement of code capability opens the horizon to future applications of numerical simulation of non-Newtonian multiphase flows.

After the power-law model was implemented in the code MFIX, the implementation was validated by comparison with literature results and a case study was conducted. This case study consisted of the convection heat transfer from a square cylinder immersed in a channel to power-law fluids. The effects of blockage ratio, Prandtl number and fluid parameters on the Nusselt number were studied, for a fixed Reynolds number.

According to Aboueian-Jahromi, *et al.* (2011), the study of power-law fluids on square cylinders began with the steady flow regime in unconfined and confined geometries, and, afterwards, flow and heat transfer phenomena in the laminar vortex shedding regime became the focus of attention, with the works of Dhiman, *et al.* (2008) and Dhiman

(2009). The latter was a study of the heat transfer of shear-thickening fluids for a single value of the blockage ratio of 1/8. It was demonstrated that the average Nusselt number is decreased by increasing the value of power-law index at fixed values of the Reynolds and Prandtl numbers for this blockage ratio. Sahu, *et al.* (2009) studied the isothermal flow of power-law liquids (power-law index from 0.5 to 1.8) over a confined square cylinder for a Reynolds number range from 60 to 160 and blockage ratios of 1/6, 1/4 and 1/2. They concluded that confinement imparts stability to the flow and delays the onset of periodic vortex shedding and that increasing the blockage ratio increases the critical power-law index (power-law index corresponding to the maximum critical Reynolds number in each blockage ratio). Aboueian-Jahromi, *et al.* (2011) conducted a study of the effect of the inclination angle on the heat transferred from a square cylinder immersed in a channel to power-law liquids. They studied the effect of Reynolds number (1 to 40), power-law index (0.4 to 1.8) and cylinder inclination (0^o to 45^o) for a fixed Prandlt number of 50 and blockage ratio of 1/4. The onset of vortex shedding has been predicted for a Reynolds number less than 40 only for the case with power-law index equal to 0.4 and inclination angle of 45^o. The total drag coefficient increased with the increase of the power-law index, irrespective of the value of the Reynolds number, fluid behavior or inclination angle. The Nusselt number increased with a decrease in the power-law index and/or with an increase in the inclination angle. They concluded that using larger inclination angles and/or lower power-law indices economically improved the thermal efficiency of the process, i.e., increased the Nusselt number. This work complements the work of Aboueian-Jahromi, *et al.* (2011) in view that it extends the study to other values of blockage ratio (3/4, 1/2 and 1/4) and to Prandtl numbers of 10 and 50.

2. MATHEMATICAL MODELING

The problems addressed in this work are governed by the mass conservation, momentum balance and energy conservation principles assuming steady state and the absence of body forces and heat sources. Assuming that Cauchy tensor is decomposed in spherical and deviatoric portions, $T_{ij} = -p1_{ij} + \tau_{ij}$, for i, j = 1, ..., N, and p denoting a mean pressure, these governing equations may be expressed in Cartesian tensor notation as

$$\begin{aligned} &\partial_{x_i} u_i = 0 \\ &\rho u_j \partial_{x_j} u_i = -\partial_{x_i} p + \partial_{x_j} \tau_{ij} \\ &\rho c u_i \partial_{x_i} T = -\partial_{x_j}(k \partial_{x_i} T), \quad \text{for}\, i, j = 1, \ldots, N \end{aligned} \tag{1}$$

where ρ is the fluid density, u_i the i-component of velocity field, τ_{ij} the ij-component of the deviatoric tensor, c the fluid specific heat capacity, T the temperature, k the fluid thermal conductivity and N the number of space dimensions.

The stress-strain relation that is assumed herein is the generalized Newtonian liquid (GNL) (Bird, *et al.*, 1987), which is given by

$$\tau_{ij} = 2\eta(\dot{\gamma}) D(u)_{ij}, \quad \text{for}\, i, j = 1, \ldots, N \tag{2}$$

where $\eta(\dot{\gamma})$ represents the GNL-viscosity function (Bird, *et al.*, 1987) and D_{ij} the ij-component of the strain rate tensor, whose magnitude is given by $\dot{\gamma} = (2D(u)_{ij} D(u)_{ij})^{1/2}$.

In order to predict alternatively shear-thinning and shear-thickening behavior, the Ostwald-de Waele model, or power-law fluid, is used to model the relation between shear stress and shear rate. This model may be written as a viscosity function in the form

$$\eta(\dot{\gamma}) = K \dot{\gamma}^{n-1} \tag{3}$$

where K is the consistency index and n is the power-law index. When the power-law index is less than one, viscosity decreases with strain rate increase (shear-thinning), while when the power-law index is greater than one the opposite behavior occurs (shear-thickening). When the power-law index is equal to one, the model is identical to the Newtonian fluid model with the viscosity given by the constant K.

3. CODE VALIDATION

At first, numerical simulations were performed, which served to validate the MFIX as a code for simulating single-phase flow of Newtonian fluids (Siqueira, 2013).

Subsequently, the power-law model was implemented in MFIX by modifying the routine *calc_mu_g,* on the source file calc_mu_g.f.

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Then the implementation of the power-law model in the software MFIX was validated using the classical lid-driven cavity problem and the results of Neofytou (2005) for power-law indexes of 0.5 and 1.5 and Reynolds number of 100. For this problem, the Reynolds number is defined as

$$\mathrm{Re}_{PL} = \frac{\rho L^n}{K U_\infty^{n-2}} \tag{4}$$

where U_∞ is the lid velocity and L the square cavity side, as depicted in Fig. 1.

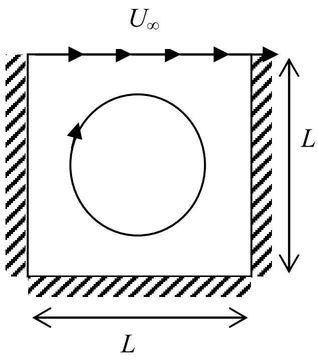

Figure 1: Problem statement for the lid-driven cavity problem.

Figure 2 depicts the horizontal velocity profile verus vertical position (Fig. 2(a)) and vertical velocity versus horizontal position (Fig. 2 (b)), where the lines represent the results of the present study, with the model implemented in the code MFIX, and the dots represent the results of Neofytou (2005). These results show that the implementation of the power-law model in MFIX was able to predict the same results as the reference.

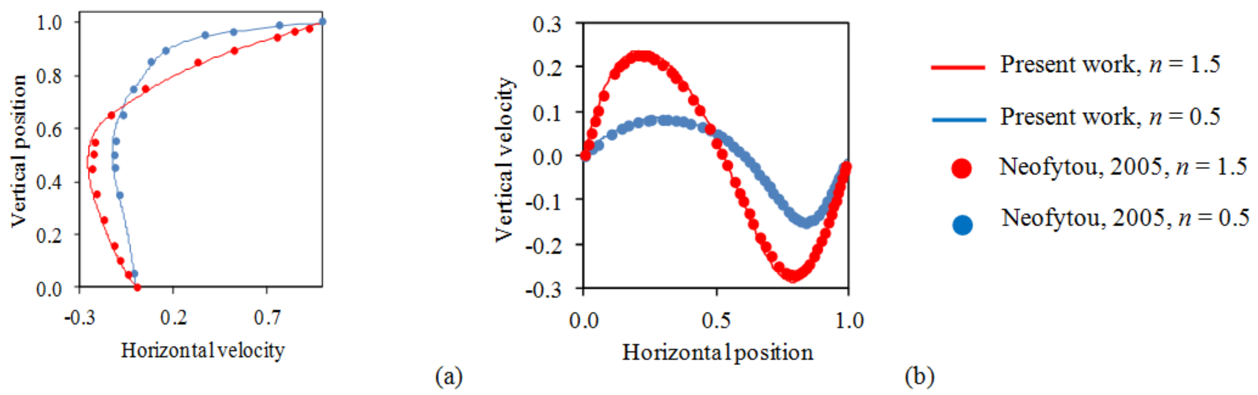

Figure 2: Results of present work and Neofytou (2005) for the lid-driven cavity. (a) Horizontal velocity versus vertical position and (b) vertical velocity versus horizontal position.

4. PROBLEM STATEMENT

After the power-law fluid model was implemented in MFIX, a case study was conducted, addressing the problem of forced convection from a confined square cylinder to power-law fluids. The 2-D geometry and boundary conditions are summarized in Fig. 3. The fluid enters the domain with a parabolic fully developed velocity profile and temperature equal to T_∞. The upper and lower boundaries are adiabatic walls and the cylinder wall is kept at temperature T_w.

A careful study was performed in order to define distances L_u and L_d in order to guarantee fully development of the flow. The distances L_u = 3.75 H and L_d = 7.5 H were chosen. A mesh refinement study was also performed in order to achieve a mesh which would not affect the accuracy of results. A mesh of 622 x 142 cells, refined near the cylinder walls, was chosen for the simulations. In all simulations, the convergence criterion was of 10^{-10}.

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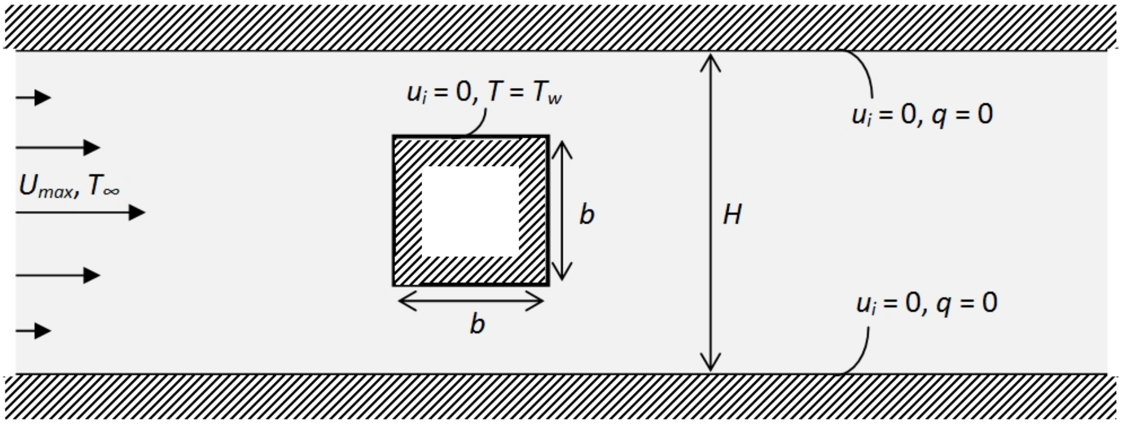

Figure 3: Problem statement.

The dimensionless numbers which govern the problem are the Reynolds number, Prandtl number, blockage ratio and power-law index, given below:

$$\mathrm{Re} = \frac{\rho U_{\max}^{2-n} b^n}{K}, \quad \mathrm{Pr} = \frac{Kc}{k}\left(\frac{U_{m\acute{a}x}}{b}\right)^{(n-1)}, \quad \beta = \frac{b}{H}, \quad n \tag{5}$$

In the present work, the parameters tested were Re = 1; Pr = 10 and Pr = 50; $\beta = 0.25$, $\beta = 0.5$ and $\beta = 0.75$; and $n = 0.4$, $n = 0.6$, $n = 1$ and $n = 1.4$.

5. RESULTS AND DISCUSSION

The dimensionless energy loss, W^*, was analyzed considering β variation. Figure 4 shows that an increase in β causes a greater increase in W^* for the shear-thickening fluid ($n > 1$), while for the shear-thinning ($n < 1$) this increase is low. This is because an increase in β causes higher shear rates, which for the shear-thickening fluids increases viscosity and resistance to flow, while for the shear-thinning fluids it decreases viscosity and the effect of energy loss caused by an increase in β is compensated by the decrease in flow resistance.

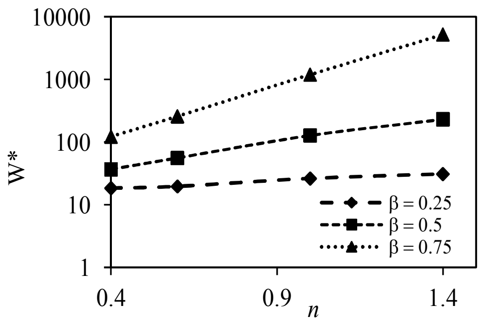

Figure 4: Dimensionless energy loss as a function of β.

The effect of β and n on the Nusselt number is analyzed in Fig. 5, for Prandtl numbers of 10 (Fig. 5a) and 50 (Fig. 5b).

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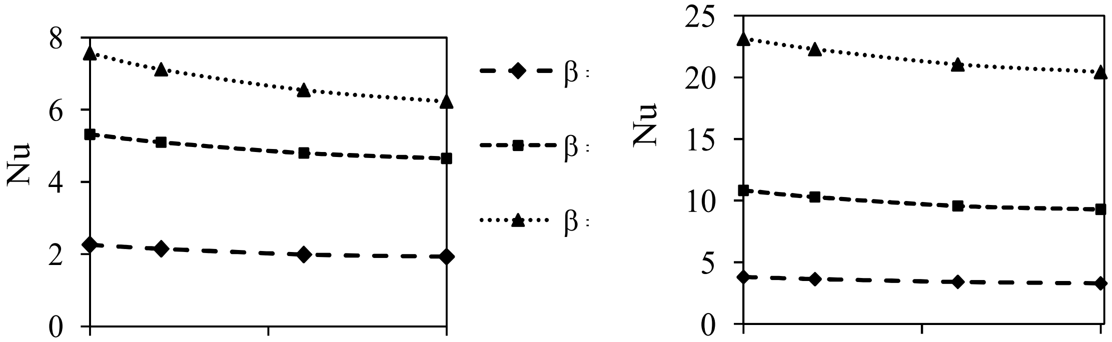

(a) (b)

Figure 5: Nusselt number as a function of n and β. (a) Pr = 10 and (b) Pr =50.

The highest Nu occurs for the highest Pr, as expected. As β increases, Nu also increases, due to the higher heat exchange area available. Also, for larger values of β, the flow is accelerated and the convective effects are more pronounced, resulting in higher Nu.

Figures 6 and 7 are pictures of the velocity and temperature fields that are used to elucidate the behaviors observed in Figs. 4 and 5.

Figure 6 shows the velocity field around the square cylinder for all the blockage ratios and power-law indexes tested. It may be observed that the higher values of n cause an elongated velocity profile around the square cylinder, which smoothes the velocity gradients and reduces the convective effects of heat transfer.

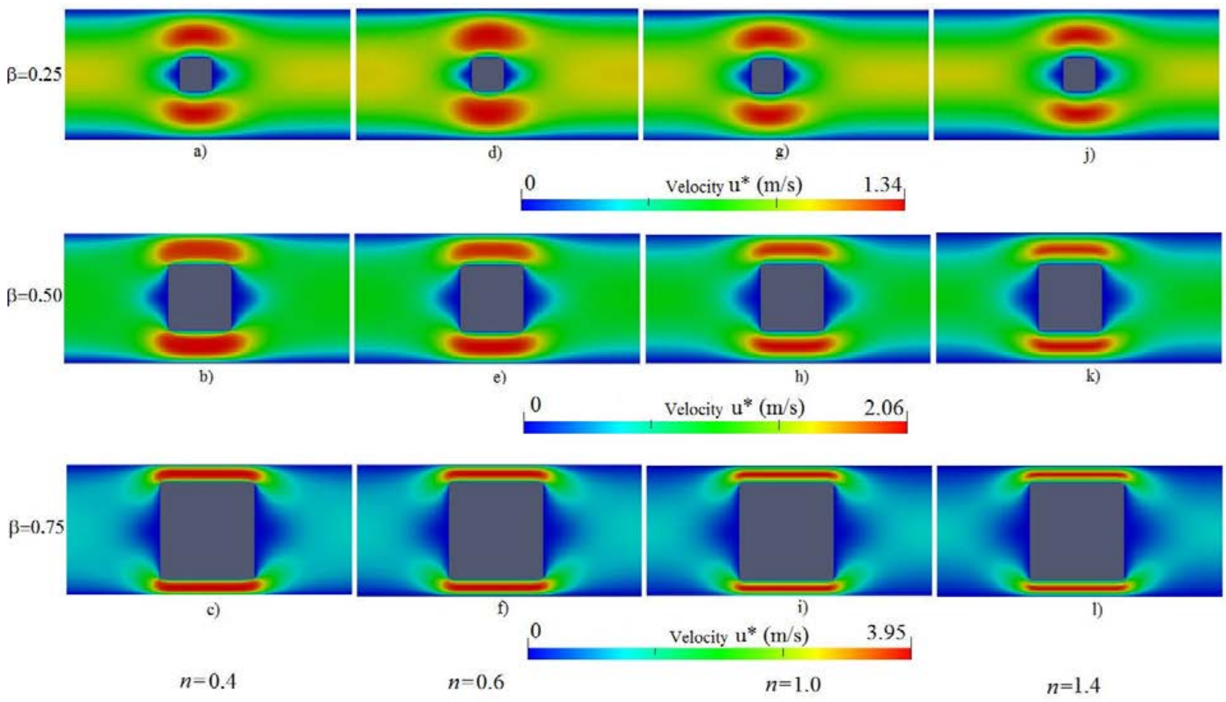

Figure 6: Velocity magnitude field.

Figure 7 shows the temperature field in the problem domain for the case of n = 4 and Pr = 10. It may be observed from this picture how the blockage ratio influences heat transfer. Besides an increase in the blockage ratio accelerates the flow around the square cylinder, increasing the effects of convection, the area subjected to heat transfer is much higher for the higher blockage ratio, increasing heat transfer and consequently increasing the resulting Nusselt numbers.

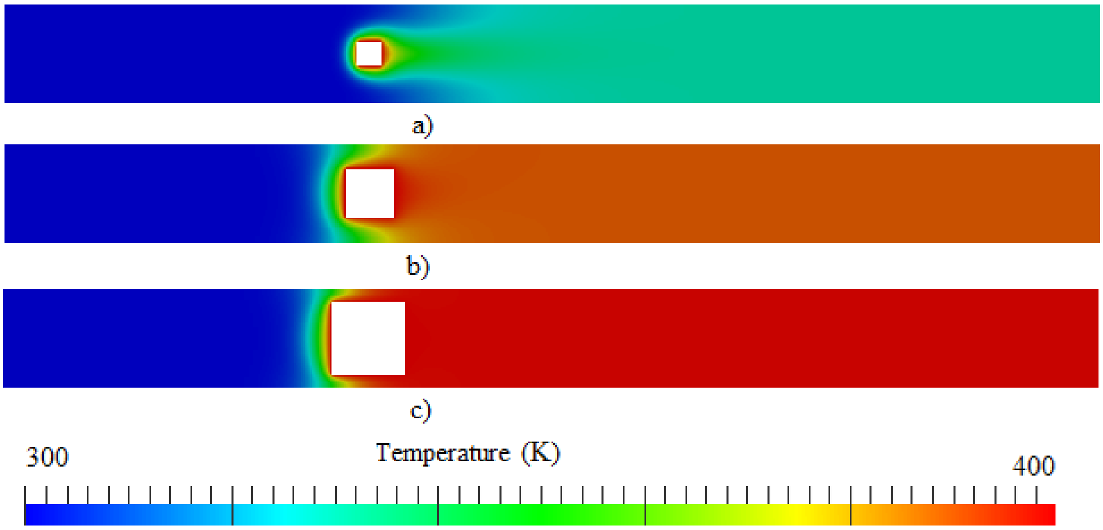

Figure 7. Temperature field for n = 0.4, Pr = 10.

Figure 8 shows how much the Nusselt number increases, when the Prandtl number ranges from 10 to 50, for each n value. The columns represent the variation of the Nusselt numbers for Prandtl numbers 10 and 50, in other words,

$$\Delta \mathrm{Nu}\% = \frac{\mathrm{Nu(Pr=50)} - \mathrm{Nu(Pr=10)}}{\mathrm{Nu(Pr=50)}} \mathrm{x}100 \tag{6}$$

It is observed that the Nusselt number always increases with the increase in the Prandtl number, with more intensity for bigger values of β. However, this increase is not influenced by the power-law Index.

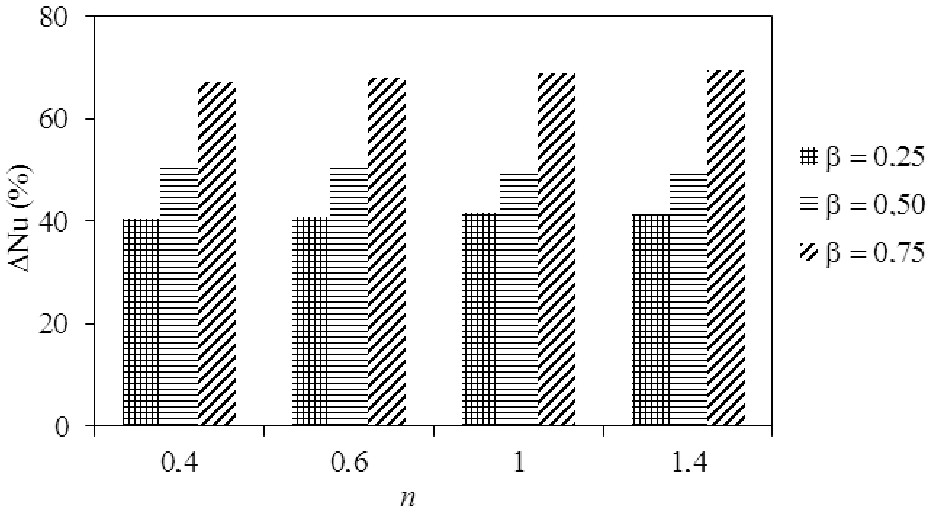

Figure 8. Variation of the Nusselt number between the numbers Prandtl 10 and 50, according to n and β.

There were analyzed temperature profiles at the midpoint ($x = L_u$) beneath each square prism for the three different values of β and the four values of n studied. For Pr = 10, graphs were constructed that are seen in Figure 9, where (Fig. 9a) referes to β = 0,25, (Fig. 9b) β = 0,5 e (Fig. 9c) β = 0,75. The vertical axis represents the ratio of the distance from the bottom wall of the channel to the cylinder wall and the horizontal axis the fluid temperature. Figure 10 illustrates the same cases, but for Pr = 50.

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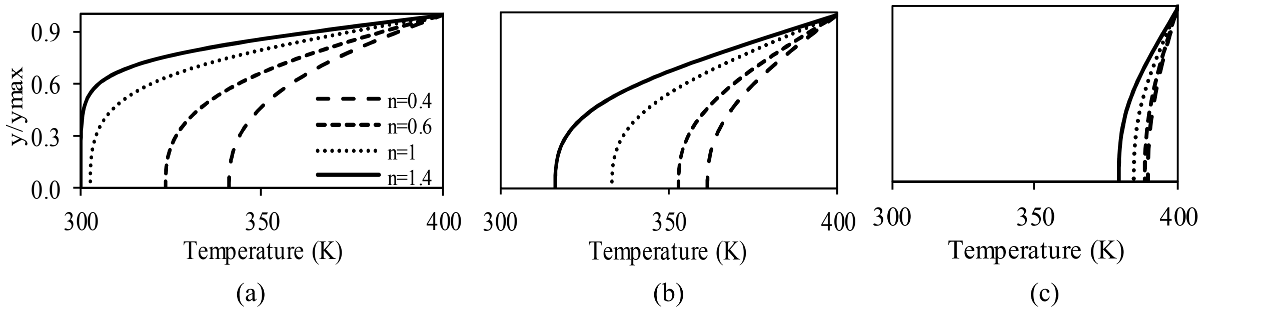

Figure 9. Temperature profile between the channel and cylinder wall, Pr = 10. (a) $\beta = 0.25$, (b) $\beta = 0.5$ and (c) $\beta = 0.75$.

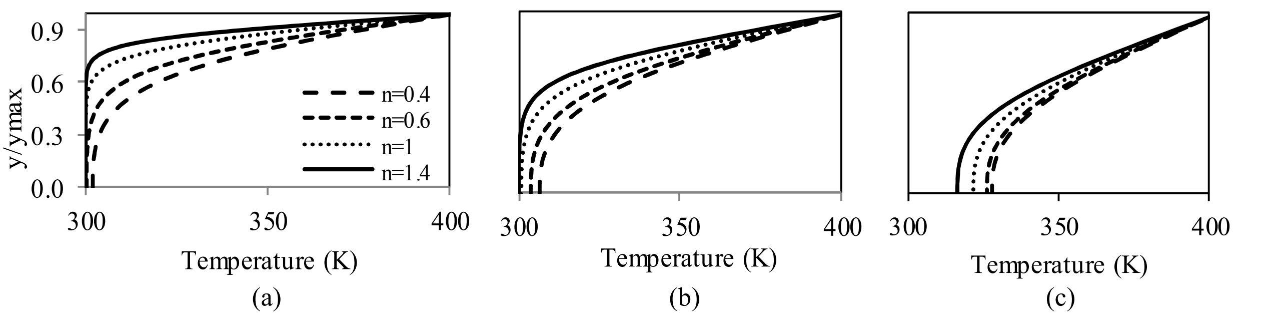

Figure 10. Temperature profile between the channel and cylinder wall, Pr = 50. (a) $\beta = 0.25$, (b) $\beta = 0.5$ and (c) $\beta = 0.75$.

Figure 9 and 10 show that the temperature profiles near the channel wall are perpendicular to it, this is due to the boundary condition of adiabatic wall. We can also see that the smaller the blockage ratio, the more the temperature profile change with the power-law index. It is also observed that, the lower the *n,* the higher the average temperature reached by the fluid, what makes the resulting Nusselt number higher to fluids with lower *n* according to (Fig. 9a) and (Fig. 9b). Comparing Figures 9 and 10 is evident that for higher Prandtl number, the influence of the blockage ratio on the temperature profile decreases, because the higher Prandtl numbers cause the reduction of thermal boundary layer, causing the convective process to occur in a region closer to the cylinder, even when the block ratio is small.

6. FINAL REMARKS

This work arose from a proposition to open up space for simulating non-Newtonian fluids in a free and open source software, MFIX. The implemented power-law model is a relatively simple model, but with wide applicability in engineering.

Numerical simulations were performed, which served to validate the MFIX as a code for simulating single-phase flow of Newtonian fluids (Siqueira, 2013). Subsequently, the power-law model was implemented in MFIX by modifying the routine *calc_mu_g,* on the source file calc_mu_g.f. A case study considering the flow of a power-law fluid around a square cylinder immersed in a channel was conducted. The model was validated by comparison with literature (Aboueian-Jahromi, *et al*., 2011).

In the flow analysis, it was observed that, the greater the power-law index, the greater the pressure drop caused by the presence of the prism, and consequently greater the dimensionless energy loss, due to higher viscosities predicted by the model in regions of high shear rate. It was also observed that increasing the blockage ratio does not cause a substantial increase in energy loss for fluids with lower power-law index, because reduced viscosity in the region around the prism is much more pronounced for the larger blockage ratios.

In the thermal analysis, an increase in the Nusselt number with the Prandtl number was observed. As for the power-law index, the opposite behavior was observed, that is, shear-thinning fluids ($n < 1$) increase the heat transfer relative to Newtonian fluids, while shear-thickening fluids ($n > 1$) reduce it. It was also observed that the lower the blockage ratio, the more affected is the temperature profile between the channel wall and cylinder by the type of fluid.

With the implementation of a non-Newtonian model to the code, a new possibility for the simulation of multiphase flows of solid and non-Newtonian liquids is opened, as well as there is an increase in the capability of the code MFIX regarding the study of single-phase fluid flows of Non-Newtonian fluids subject to heat transfer.

7. ACKNOWLEDGEMENTS

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