

DYNAMIC BEHAVIOR COMPARISON OF AN ACTIVE MAGNETIC BEARING USING SLIDING MODE WITH VOLTAGE AND CURRENT CONTROL

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Abstract. Magnetic bearing is a product based on magnetic levitation and has been researched for more than forty years. At first, their function is the same of mechanical bearings, but it acts with no contact, using field forces provided by electromagnets to keep rotor's position. Its advantages over other types of bearings are several, like the possibility of reaching incredibly high rotational speed, vibration control, high efficiency, low costs of maintenance, no contamination of fluids in turbo-machinery, and others. This high performance is only possible thanks to the improvement of control instrumentation and theory, which is the base behind the active magnetic bearing (AMB). Since then, several control laws are being implemented to improve the stability of the rotor dynamics. The objective of this paper is to use the Sliding Mode Control (SMC) technique within an experimental model of a rotor supported by two AMBs. The SMC is based on making the space-state function "slide" over a straight line, changing the behavior of the system dynamics. This is a robust technique that allows good results even if there is some uncertainty or variation of physical parameters. This uncertainty can be in rotor characteristics or in the model itself, due to the hypotheses made. Considering this system being highly non-linear and with low fault tolerance, a robust control law brings many advantages and makes easier to achieve good results when dealing with a real prototype. However, the model relate the magnetic force linearly with the input current, which should be the variable of control, but current control involves a more complex electronic implementation that is harder to project in a system with inductive load. A voltage control simplify the control signal amplification, but raise the order of the system, making it more unstable, since the inductance of the AMB coils tends to oppose the current change. Thus, a voltage control requires an accurate control and a comparison with current control using SMC in both cases is tested to determine the advantages and disadvantages of each. Finally, some results will be shown for a experimental prototype with simplistic electronic project, using PWM signal for current control.

Keywords: magnetic bearing, sliding mode, rotor, voltage control.

1. INTRODUCTION

Bearings are machine component present in every rotational machine. Its function is to keep the desired direction of the rotor. This function must be made whatever is the disturbance; it can be vibration, random load or acceleration of the machine itself. At a first look, this seems to be simple, but due to physic aspects this control becomes quite complex, since the mechanical components have finite stiffness. Thus, unbalance of the rotor and external excitation start a vibration problem which can be easily seen in rotational machines. So, bearings must be designed to support all kind of loads and ensure that the desired direction will be achieved under the project tolerances. However, due to all effects related, mechanical bearings need constant maintenance and have to be replaced after a period of life, leading to high costs.

The work principle of a magnetic bearing is based on levitation made by a magnetic field. There are basically two groups of forms to make the rotor hover in the air: by reluctance force or Lorentz force (Hannes, 1992). Reluctance force acts when we have materials with different magnetic permeability and an attraction or repulse force appears between them under a magnetic field. This concept is the one used in this paper for ferromagnetic material, and can be extended in applications with diamagnetic or superconductor materials. The Lorentz force is used in electrodynamic devices, when a force appears when we have a conductor material moving under a magnetic field. This is the concept behind the levitation of MAGLEV, the magnetic levitated vehicle.

However, the most commonly method used in industrial applications is the electromagnetic levitation with high permeability material, the ferromagnetic ones. The idea is to stabilize a full levitation of the rotor, making it spin in a contact free condition. As can be seen in practice and mathematically, the model of the reluctance force is unstable and according to Earnshaw theorem (Earnshaw, 1842) it would be impossible to levitate a body and keep it stabilized with this kind of force. However, last decades research have shown several ways to make it possible, like rotating object under constant magnetic field (Simon *et al.*, 1997) or using superconductor materials (Moon, 2008). The model discussed in this paper is based on controlling the magnetic field using a displacement sensor and an electromagnet, which together constitute the so-called active magnetic bearing (AMB). R. S. Siqueira, F. A. N. C. Pinto Dynamic Behavior Comparison of an Active Magnetic Bearing using Sliding Mode with Voltage and Current Control

There are many advantages of AMB over mechanical ones. First, it is the extremely low friction, caused basically by air resistance, or even none in vacuum applications. This allows reaching incredible high rotational speed, up to 300 kHz (Beams, 1937). There is no need for a precise alignment in this kind of bearing, since it can be adjusted in the control. There is also the non-need of lubrication, which is great for applications with contamination issues like blood pumps and some kinds of gas compressors (Schweitzer, 2009) and brings great cost reduction for maintenance. The active control leads to several advantages, like the possibility of controlling vibration or fix problems during the operation of the machine by regulation of control parameters. A further research topic would be the concept of smart machine, which makes the bearing capable to make self-diagnoses of its operational conditions in real time and change the parameters itself to optimize the performance. With all of these advantages, it is clear the advantages of AMB over other kinds, and since the maintenance costs are much lower, it pays the higher costs of project and implementation.

2. THEORY

The process of designing an AMB model can be divided in three parts: the rotor dynamics, reluctance forces equation and the control law. The rotor dynamics will provide how the rotor behaves, such as its position and velocity for a given force. The reluctance force equation makes a relation of this force with the position of the rotor and the current on the electromagnetic coils. Than, a control law may provide the necessary current to make the reluctance force keep the rotor in a stable position.

2.1 Rotor Dynamics

Due to the wide bibliography related to rotor dynamics, this part of the construction of the model will be omitted. The modeled rotor was implemented using Maple Software (Maplesoft, 2011) with Sophia toolbox (Lesser, 1995) and is considered rigid with general unbalance, which means the presence of static and dynamic unbalance. The axial displacement was not considered.

Also, a linearized version of this model was made on MatLab (MathWorks, 2011), since this step will be needed for current control.

2.2 Magnetic Force

To make possible the levitation of the rotor by the bearings, we need a force field that act by distance and can be controlled. The levitation principle of AMBs is the use of an electromagnet, which has a magnet field induced by a controlled current. This magnetic field will form a closed loop through the electromagnet, the rotor and the air gap, creating an attraction force between the two ferromagnetic parts. So, it is needed the relation between the input current and the output force. Figure 1 illustrate the path made by the magnetic field. This path depends on the air gap *s* between the rotor and the electromagnet, which makes the force also dependent of this gap, beside the input current i.



Figure 1. Force generated by the magnetic field induced by current *i* and dependable of the air gap *s*.

The current passing through the coil will induce what is commonly called H field, a distinct kind of magnetic field. With the raise of H, the ferromagnetic material will start to align its magnetic dipoles and create the loop of magnetic field itself, called B. So, value of B depends on the material it is going through, which could be iron and air in this case, and its relation with H is considered linear, given by the magnetic constant μ_0 and the magnetic permeability relative to the material μ_r . In fact, it exists also an hysteresis between B and H, which is neglected, and there is a limit for the rise of B, called magnetic saturation, when all magnetic dipoles of the material are aligned.

Using Ampere's Law, it is possible to calculate the magnetic field induced by the current i passing through the coil with n, considering the area A of the magnetic field's path a constant. The attraction force appears because of the presence of two materials with different permeability, the iron and the air. This force is formulated from the energy of the magnetic field in the volume of the air gap. Using the principle of virtual work, the force can be written as the derivative of the

energy to s, as following:

$$f = \frac{\partial E}{\partial s} = \frac{B^2 A}{\mu_0} = \mu_0 A \left(\frac{ni}{2s}\right)^2 \tag{1}$$

Equation (1) the simplest approximation for the magnetic force, that considerate a linear magnetization of the ferromagnetic material, the path of the magnetic field with constant sectional area and no escape of this field through the boundaries. This model may lead to some errors, but it gives a good estimation of how the force varies with current and gap.

It is reasonable to consider this force actuating around an operating point, since it is desirable that the rotor stays in constant position. So, an linearized form of Eq. (1) is commonly used for simplification, as shown in Eq. (2), using now coordinate x to represent position instead of the gap s and deviation variables.

 $f = -k_s x + k_i i \tag{2}$

The terms k_s and k_i are dependent constants of physical parameters, since they come from Eq. (1), which have many errors included caused by the formulation. Estimation of these parameters can present uncertainty on the order of 20%. This can lead to some problems, if the whole system doesn't have high tolerance and it is very sensitive.

2.3 Sliding Mode Control

The negative signal before k_s in the previous model of the magnetic force proves the instability of this system, working with an opposite behavior of a mechanical spring. So, a feedback of the rotor position, given by a displacement sensor, is necessary to control this current and stabilize the system.

The most commonly used controller is PID (Proportional Integral Derivative), and its theory can be found in any control literature. The stabilization of the system is guaranteed and can be proved mathematically (Ogata, 2010). However, a real experiment may not work as the simulated model with PID control if there is uncertainties in the parameters of the system.

A robust control is a control capable to ignore the uncertainties, and this is the expected behavior and proposal of SMC (Sliding Mode Control). Equation (3) shows its work principle (?).

$$i = J \cdot sign(\sigma) \tag{3}$$

$$\sigma = \lambda x + x \tag{4}$$

Terms J and λ are the parameters of control. It has only two signal values as output, sending an on/off type of signal. It leads the system to reach the point where $\sigma = 0$, which is a straight line in the state space plane where the result will "slide" on, called sliding line. Thus, system dynamics is forced to follow that line, making system behave in much simpler way, and here it's where the robustness of this control is found. Taking $\sigma = 0$ in Eq. (4), the result will be:

$$x = e^{-\lambda t} x(0) \tag{5}$$

Which means that the system is stabilized for any $\lambda > 0$. As higher λ is, higher is the sensibility of control to correct error position, but it will require also higher capacity of actuator. If this capacity is not high enough, the change of signal in the control law can't lead the system to reach $\sigma = 0$ and it will not "slide" as expected. The value of λ is working only as a damping factor. When it is correctly settled, the system will first reach the sliding line and after it will "slide" to null position error.

Parameter J is the gain related to the capacity of the actuator. In the case of AMBs, is the variation of the current itself, that can vary in the electromagnet. Obviously, there is an upper value for how much current can be passed through the coil, limiting the load capacity and consequently the max value for λ , as explained before.

This standard SMC has a control signal that will never be zero and make the response keep crossing the sliding line in state space. This phenomenon is called chattering. It may consist a problem, since a constant signal for a stable situation is never reached. However, there is some ways to avoid it by making some improvements in the basic control law of SMC. A possible way for that is the use of a softer function instead of signal one, making the control signal not be on/off anymore, but have a continuous performance between that, as shown in Eq. (6). The idea of this is make the actuation of control weaker when state space is near sliding line.

$$i = J \cdot \tanh(\gamma \sigma) \tag{6}$$

Constant γ is related to how much soft the transition between on and off will be. For high γ , this law becomes closer to the standard one. Chattering reduces drastically with this alternative, since actuation is smaller when error is near zero.

3. SIMULATION

Since the objective of this paper is compare the use of SMC with current and voltage control, it is needed previously a study about that. The Equation (1) shows a direct dependence between force and current. So, for a current control, all that has to be done is replace the magnetic force as the bearing forces in the rotor's dynamics model and control this current to stabilize the system. In this case, the current has to be straightly controlled, but the inductance of the coils make this work harder.

Control signals are commonly sent to the actuator (an amplifier in our case) by voltage. However, the current doesn't follow this voltage linearly, since the inductance of the coils will delay this change. So, for a current control, a robust amplifier must be built to guarantee that the current in the coil follows the control signal correctly. It is not the objective of this paper to discuss about amplifiers, but this amplifier needs a current measure feedback and a control loop itself to give a good result. If an efficient current amplifier is provided, these following simulations give good approximations, since the electric dynamic will be much faster, being possible to neglect it.

In a voltage controller, the electric dynamics must be implemented. Equation (7) shows the relation between the current and the voltage (Schweitzer *et al.*, 2009).

$$u = Ri + L\frac{di}{dy} + k_u\frac{dx}{dt} \tag{7}$$

As seen, there is also a dependency with the first derivative of the rotor's position, cause it's movement also generates current. The inductance L is considered constant and the constant k_u , according to (?) has theoretically the same value as k_i . The inception of this new equation to the system increase the order of it. In this case, the current becomes also a state variable, joining the rotor's position and velocity.

The voltage control requires a more sophisticated technique, since there is no straight relation between the control signal and the bearing force. However the vantage of this type of control is that it doesn't required the same robust amplifier as current control, making the electronic project more simple.

Table 1 shows the most relevant parameters of the simulated system. The bearing is considered to have four coils each, being possible to make force in all four direction of each one's plane, the axial movement is neglected. It is applied a torque of 0.001N.m on the rotor.

Parameter	Value
Diameter of the rotor	14.28mm
Length of the rotor	400mm
Mass of the rotor	1.0kg
Air gap between rotor and bearing	1mm
Number of turns of each coil (AWG 21)	260turns
Distance between each bearing	315mm
Coil resistance	$1, 2\Omega$
Coil indutance	20mH
Magnetic force/position constant (k_s)	$2.10^4 N/m$
Magnetic force/current cosntant (k_i)	20N/A

Table 1. Most important parameters of the system rotor/bearings.

3.1 Current Control Results

For the current control results, the non-linear system modeled on Maple was used. It was chosen a maximum of 2A for the current in the coils. This value keeps the operating point very far from magnetic saturation, being possible to use also higher values to achieve higher forces, but this lower value doesn't bring the temperature issues, since this effect is not studied here. So, based on that, the value for J on control needs to be half of that, 1A, so that the control will turn the coil on and off.

We have two control signals for each bearing, being each one related to one direction. So, for a same direction, while one of the coils will be turned on by the SMC control, the other one will be turned off. It was found $\lambda = 200$ as the configuration with best results and the control frequency was fixed at 10kHz, being this an important parameter to stabilize the dynamics. Considering the rotor starting in rest, with its x coordinate at -0.4mm, the results were well stabilized, with the presence of a high chattering.

Basically, what this first simulated system does is the application of an instantaneous big force to one of the sides of the rotor according to the control, making easy to achieve good results, but it uses excess of energy. This excess takes to

an excitation in high frequency, due to chattering. So, better results were found using modified SMC, with $\gamma = 320$, as shown in Fig. 2.



The results achieved for this case were similar to the previous one, but more smooth and with a reduction of the oscillation amplitude. ItŠs seen a behavior change at 18s, when the rotational speed is about 7000rpm. This is the moment that the dynamics leave the sliding mode, although it still keeps a stabilization for some seconds more. It is also seen that the chattering problem is solved and that vibration on high frequencies is over, since this control has a continuous signal.

3.2 Voltage Control Results

As discussed before, current amplifier capable to give this instantaneous on/off on the coils requires a more complex project. For modified SMC, that needs a continuous and well definite signal, this amplifier may get even more complicated. Looking for an easier and functional solution that brings facilities to mechanical designers, we test the voltage control. In this case the force in SMC will not be instant anymore and will require a while to raise, making the stabilization a greater challenge.

Since this model requires the use of Eq. (7) in the model, requiring a harder method for solution and more computational model, the linearized model built in MatLab was used. The control parameters were set so that the same capacity of current control could be done, making the comparison between both more logical. So, for this case, we have J = 1.2and $\lambda = 20$. It's clearly seen how this new dynamics requires a lower λ , which means it needs more relevancy for the velocity to achieve stability due to the delay cause by inductance.

Keeping all other parameters the same as before, we got the results for standard SMC. Since for this case it is voltage that keeps chattering, making the current assume intermediate values, the chattering problem is less visible, but still present. So, the modified SMC presented better results again, as seen in Fig. 3.

This time the system reached only 800*rpm* before destabilize. It is a big reduction of performance comparing to current control. However, it is important to remember that in this case we have three state variables: position, velocity and current. The measurement of this current were not considered in the control, which would provide a 3D state space and

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probably better results, but for a straight comparison, this was not implemented. Although both results for voltage control were quite inferior to the ones seen for current control, it was capable to show that the stabilization can be achievable with a very simplistic amplifier project.

4. EXPERIMENTAL RESULTS

It's been developed at the Laboratory of Acoustic and Vibrations (LAVI), from COPPE - UFRJ, a prototype of an heteropolar mechanical bearing. After the simulations have been made, the experiment project was developed in a iterative process. The first part to be developed was the mechanical part, and after the electronic circuit and the data process system.

The bearing was developed for a axis with length 400mm and diameter of 9/16''. It was designed as a heteropolar structure, made of a set of plates, to avoid eddy current losses, and using silicon steel, same as transformers. Than, according to (Schweitzer *et al.*, 2009), the design was made to optimize the bearing capability. The plates were cut by water jet process and numeric control, to obtain the desired design and low fault tolerance between the plates.

To build in the coils, a reel was made to fit each of the parts of the set of plates. The number of turns, among other factors, were mainly limited by the space available. The construction of the coil was made by hand, so it was not used such a small diameter of cooper string to make this process easier. It was used a "AWG 21" with "class-D" temperature protection. Each of the eight coils of the bearing could have than a maximum of 130turns, so it could fit and be mounted on the silicon-steel structure. After mounting them, each pair was connected in series, forming four systems to control all



the four directions. Figure ?? shows one of the individual plates and the final assembly.

Figure 4. An individual plate of silicon steel and the bearing mounted.

With the bearing done, the axis was made of stainless steel. The internal diameter of the bearing is much bigger than the axis steel on purpose, to raise its force capabilities and make the construction easier. So, in each end of the axis, it's mounted the called rotors, with a diameter that matches the resulting internal diameter of the bearing, respecting the air gap, stipulated in 1mm. So, a set of plates is assembled at the axis, similar to the bearing, plus an aluminium plate to isolate the magnetic field from the sensor and carbon steel plates to make the set more compact. Using a nut, this set was fixed at the axis. This structure can be more easily understood on Fig. 5.



Figure 5. Extremity of one of the sides of the axis showing how the rotor is made.

Finally, a structure was built to fix the two bearing, with the rotor at the right place. To limit the rotor movement so that it doesn't hit the bearings during the work and cause damages to both of them it was made a part of polymeric material with a role in the right diameter, letting the rotor to move freely, but protecting the bearing. The air gap between this protection and the axis needed to be smaller than 1mm and it was set in 0.4mm.

During the mechanical parts fabrication, the electronic project was being made. Some strategies were studied to make the work of amplify the control signal. The control signal has a very low current, working basically by voltage and this circuit must amplify this current. Since the force acts according to the current, it would be more suitable to use a circuit that could convert this voltage signal in a linear analogue current. However, since the actuator has inductive load, it requires a more complex project, with current measurement and feedback control. This was not possible to be made with the given resources, so a simpler technique was developed. Using a Mosfet as a switch, the current of the coil could be controlled by a voltage signal. This would work exactly as the voltage control simulated before using standard SMC, since it gives an on/off response.

So, the circuit implemented was very simplistic, but since it was seen in simulations that it could work, a very low cost and it didn't need a long period to make it, it was the chosen one to make the first tests. It allows two types of control: a control using standard SMC directly or using modified SMC with PWM. The second case can be seen as a kind of current control, cause the resulting current will have a straight relationship with the pulse width, but the delay caused by

its change will still be present. Furthermore, the use of PWM requires a fixed frequency of work to be set by the user, witch can be an advantage on implementation.

It is needed also a power supply to give the required current to the system. This supply is directly related to the J parameter of control, since it sets the maximum current in the system. It was used 5V, witch would allow a maximum current of 4A for each coil. It is possible to raise this value to improve the bearing strength, but it could bring temperature problems, witch is not the case of study. Moreover, it was estimated that a current of 10A would saturate magnetically the material, being this value the maximum allowed.

The position sensor used was of the capacitive type. Among others, this one had the best cost benefit and could be used in a magnetic bearing without further problems. So, finally, a data processor needs to be used to implement the control law. Another simplistic concept was used for the first tests, witch is the Arduino UNO board ?. This board is also not the best choice, since it has a limited velocity and it works with only 10-bits. However, its characteristics were enough for first results and it has a very easy to implement interface and a low cost.

Figure 6 shows the actual state of construction of the whole system.



Figure 6. Prototype of AMB being developed.

At this first fase of tests, only one of the bearings were tested. A stable levitation was achieved in few attempts, but only for a static rotor. First step made was to control only the vertical position of the rotor using only the superior electromagnet. The first issue founded was related to the filtering of the position sensors. It was tested some digital filters to be implemented in the microcontroller. However, the process frequency was achieving a maximum of 5kHz, which is close to the minimum frequency that makes a stabilization be possible. So, the delay caused by this filter was affecting the good behavior of the controller and the best results were achieved without the use of it. In this case of vertical control, due to the geometry of the bearing, the rotor was always touching one of the sides of the protection, affecting the results. These first results using standard SMC can be seen in Fig. 7.

The main limiting factor in this system is the velocity calculation. Although the quantization in 1024 parts made by the 10*bit* ADC is enough for a reasonable reading of the rotor's position, it is not enough for a good estimation of the velocity. Since the velocity is an important parameter for SMC, these error doesn't allow the controller to make the change on system dynamics as was discussed before. As can be seen, the result presents very low chattering frequency, much smaller than the processor frequency, caused by this error on velocity calculation. However, the stabilization was achieved and next step is the implementation of modified SMC.

With a fixed high frequency set and a smooth controller, the modified SMC brought better results, with lower vibration amplitude. Since this controller was quite better than the previous one, we started the tests in both directions with the use only of it. The presence of a horizontal control will for sure make influence on the vertical control, and vice-versa, that's why we started these tests using modified SMC at first.

At this point, some adjustments had to be made. First of all, both superior and inferior electromagnets has to work this time. Than, to compensate the weight, the control of the inferior electromagnet was limited. Another problem founded was the use of Arduino for both controls. The slower part of the process is the ADC conversion and with two sensors to read (vertical and horizontal) the process frequency became too slow, in such a way that the stabilization would not be possible. So, it was used two Arduino UNO boards to make it possible, since these controls are independent. Figure 8 shows the best result achieved and Fig. 9 shows an image of the rotor with full hovering.

Due to the process frequency achieved and the problems discussed before, the capability of the bearing (witch means



Figure 7. Rotor's position with vertical control using standard SMC for 0.5s.



Figure 8. Rotor's position with vertical control using modified SMC for 0.5s.



Figure 9. Image showing the prototype's rotor in full levitaiton.

the maximum current allowed) could not be raised. The raise of this current would provide for straight to the bearing, but with those errors, this raise also increase the amplitude of vibration. With a lower capability, this vibration decreases, but it becomes more easy to destabilize the rotor. Thats the paradox to set this value, but since the spinning of the rotor will later cause many forces that can make the system unstable, the intermediate value of 5V were kept for the power supply.

This results are still initial and presents excessive vibration for a real application. For that reason, experiments with a spinning rotor were not made yet. However, the technique presented in this paper had the expected robust behavior and shown that is possible to reach the simulated results when the electronic improvement be made. Basically the limitations are related to processor velocity, digitalization of the position sensor signal and amplification method. Than, with the right

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tools, it will be possible to make the same comparisons made on this paper in an experimental analysis.

5. CONCLUSIONS

After the simulations made, some conclusions can be made about the use of current and voltage control on AMBs. Since this is a very unstable system, the velocity response is one of the most important parameter to achieve good stabilization. The frequency used on the tests could guarantee that, but when lower rates were used, the levitation control became much more harder to keep.

The Sliding Mode Control techniques used brought satisfactory results in a very simple way. This control allows a very intuitive method of parametrization, bringing good results very quickly. The robust behavior is great for this kind of system where there is a lot of uncertainties, and this good characteristic could be proved at the first experiments at the prototype.

The ideal current control brings the best results, since we have a force that acts instantaneously on the bearing. The control by voltage showed to be possible for first results. Since on of the advantages of the magnetic bearing is the possibility of high rotational speed, this type of control brings some limitations, but it's still a powerful tool for mechanical engineers achieve the first results for testing.

The first results with the developed prototype were very satisfactory. The stabilization were achieved with few tries, based on the simulations made previously. Although the amplifier used could provide only an effective voltage control, the system using PWM, witch is closer to a current control, showed much better results.

However many improvements must still be done. As can be seen, the control method used can truly give good results, but the main limiting factor for the experimental project was the data processing. With the same simplistic concept of amplifier, but a better, processor, the results would get closer to the ones seen on simulations.

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