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ERROR ANALYSIS IN THE ESTIMATION OF THERMOPHYSICAL PROPERTIES OF METAL SAMPLES

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Abstract. *Thermophysical properties such as thermal conductivity, λ, and volumetric heat capacity, ρc_p are estimated for metal samples of titanium ASTM B265 and stainless steels AISI 304 and 316. Different intensities of heat flux were used in the same experiment with the purpose of increasing significantly the sensitivity for the thermophysical property estimation. To determine the properties, a square error objective function is minimized defined by the square difference between the experimental and numerical temperatures by applying the optimization technique BFGS. This paper also presents a study on the thermal contact resistance and an analysis of the uncertainties that occur in the simultaneous estimation of λ and ρc_p. The thermal contact resistance is calculated considering the distance between the resistive heater and the sample and a kapton layer of the resistive heater. Satisfactory results are obtained for this analysis because these influences result in a temperature difference of around 0.1 ° C, which is equal to the uncertainty of the thermocouple. The uncertainty analysis is based on the propagation of uncertainty by taking the experimental thermal contact resistance and numerical errors into account. The uncertainty analysis is considered satisfactory because the obtained result was lower than 5%.*

Keywords*: thermophysical properties, conductivity, heat transfer, volumetric heat capacity, experimental technique.*

1. INTRODUCTION

An indispensable, correct and efficient knowledge of the temperature of a certain material has become essential with the development of more accurate new methodologies. Thus, once the temperature is known, studies of the intense use of the material analyzed are carried out reducing the probability of a problem to the material when it is submitted to its regular use. In addition, any abnormal behavior of the material may be dealt with more efficiently and quickly.

In engineering, there is a constant search for the development of materials that display better thermophysical properties and are relatively cheaper than those in use. Hence, it is of paramount importance that there are studies as presented in this paper to determine concisely the thermophysical properties and closely collaborate to the development of more efficient methods in the heat transfer area. In the automobile industry, the brake mechanism of a car may be cited as an example of where it is essential to know how high the temperature of the brake disc will reach. Thus, it is important to carry out studies to avoid accidents during the braking of a vehicle and hence project materials that bear high temperatures when halting the vehicle taking a safety coefficient into account. Studies may be carried out with the aim to find a more appropriate material (lower cost, equivalent mechanical features, etc) to produce the brake system. Therefore, the manufacturer may benefit from the drop in cost without losing the quality. In this way, every engineer has to be attentive to the characteristics and behavior of the materials used in their projects.

Thermal conductivity, λ, which is the capacity of the material to conduct heat, may be cited as an important property to be studied within this context. Thermal conductivity provides an indication of the rate at which energy is transferred by the diffusion process. The volumetric heat capacity ρc_p, represents the capacity of a material to store thermal energy. These two properties are of extreme importance in heat conduction problems.

A method to estimate simultaneously λ and ρc_p of three different materials are presented in the work: Titanium ASTM B265, stainless steel AISI 316, and stainless steel AISI 304. A thermal model based on unsteady one-dimensional heat diffusion equation in Cartesian coordinates is used in this work. Two different, constant and uniform heat fluxes are applied to the top surface of a homogeneous sample, while the opposite surface is insulated. To guarantee the one-dimensionality, the side surfaces are much larger than the thickness and the total time of the experiment is short. A symmetric assembly is used bearing the sample between the resistive heater and the isolator. The properties are estimated from the minimization of a minimum square error function, defined by the square difference between the experimental and numerical temperatures. The experimental temperature is measured by the placement of a thermocouple on the surface opposite the heater whereas the numerical temperature is obtained through the resolution of the thermal problem by using the finite difference method with implicit formulation. An analysis of the sensitivity coefficients, which are defined by the partial derivative of the temperature in relation to the parameter, was also carried out. This analysis is fundamental for the estimation of the properties, for it allows the determination of the best region, the best position of the temperature sensor, the total time of the experiment, the time interval, among other important parameters.

Nowadays, there are several techniques to determine the thermophysical properties of diverse materials (Jannot *et al*., 2006, Jannot *et al*., 2009, Xamán *et* al., 2009, Jannot *et al*., 2010, Thomas *et al*., 2010). These techniques may determine these properties separately or isolatedly, moreover, most of these estimations occur rapidly, safely and precisely. Hence, an experimental assembly is necessary to perform this estimation. Therefore, when experiments are performed, several uncertainties occur related to the thermal contact resistance as well as in the equipment used to measure the temperature, the heat flux, among others. These uncertainties are intrinsic to the process and cannot be avoided. Therefore, the correct procedure is to perform a controlled experiment to quantify these uncertainties. However, authors like Ghrib *et al*. (2007), Borges *et al*. (2008), Le Goff *et al*. (2009), Kravvaritis *et al*. (2011), Sanjaya *et al*. (2011), and others have performed experiments without mentioning these uncertainties. Although these uncertainties are mentioned and quantified in Jannot *et al*., 2006, Jannot *et al*., 2009, Xaman *et al*., 2009, Jannot *et al*., 2010 e Thomas *et al*., 2010, the method and the process to measure these uncertainties are not described.

An uncertainty analysis is also presented in this work as well a study of the contact resistance through the experiment aforementioned to estimate the thermal properties simultaneously. The study on the contact resistance displays the influence of the Kapton layer on the resistive heater and the distance between the heater and the sample. The uncertainty analysis is done by taking into account the influence of the thermal contact resistance and the numerical and experimental temperature errors. In addition, the main purpose of the present work is to present the improvements performed in relation to Carollo *et al.* (2012). The heater, now, is completely symmetrical and the metal samples were rectified.

2. THEORETICAL ASPECTS

2.1. Thermal model and sensitivity coefficient

Figure 1 shows the proposed one-dimensional thermal model, which consists of a sample located between a resistive heater and an insulator. The sample has much smaller thickness than its others dimensions and all the surfaces, except the heated (x = 0), are isolated to ensure the one-direction heat flux. Figure 2 shows the perspective view of the sample. The heat diffusion equation for the problem presented in Figure 1 can be written as:

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\rho c_p}{\lambda} \frac{\partial T(x,t)}{\partial t} \tag{1}$$

subject to the boundary conditions:

$$-\lambda \frac{\partial T(x,t)}{\partial x} = \phi(t) \ \text{ at } \ x = 0 \tag{2}$$

$$\frac{\partial T(x,t)}{\partial x} = 0 \ \text{ at } \ x = L \tag{3}$$

and the initial condition:

$$T(x,t) = T_0 \ \text{ at } \ t = 0 \tag{4}$$

where x is the Cartesian coordinate, t the time, ϕ the prescribed heat flux, T_0 the initial temperature of the sample and L the thickness.

The numerical temperature is obtained through the solution of the one-dimensional diffusion equation by using the finite difference method (FDM) with an implicit formulation.

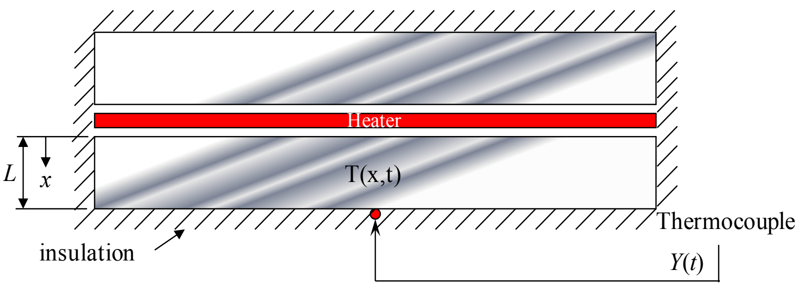

Figure 1 – One-dimensional thermal model.

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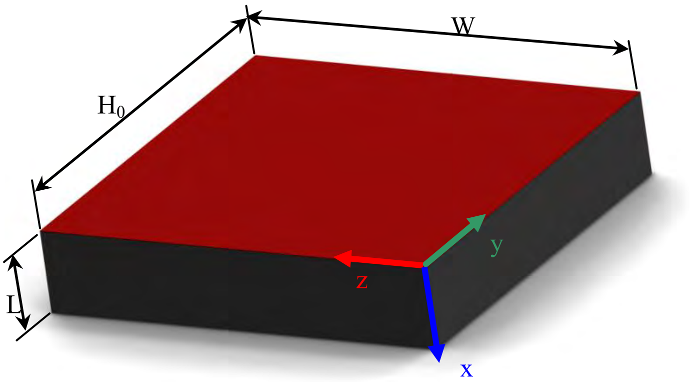

Figure 2 - Perspective view of a sample.

Studies on the sensitivity coefficient for each sample are performed in this work in order to determine the ideal region to estimate the properties and the best configuration of the experimental setup. This study provides information such as: the correct positioning of the thermocouples, the experimental time, and the time interval of the applied heat flux incidence. The higher the coefficient values, the more reliable the results of the properties estimated.

The normalized sensitivity coefficient is defined by the first partial derivative of the temperature in relation to the parameter to be analyzed (λ or ρc_p), being written as follows:

$$X_{ij} = P_i \frac{\partial T_j}{\partial P_i} \tag{5}$$

where T is the numerical temperature, P the parameter to be analyzed (λ or ρc_p), i the index of parameter, and j the index of points. As in this work, only two properties will be analyzed, i = 1 for λ and i = 2 for ρc_p.

2.2. Thermal conductivity and volumetric heat capacity simultaneous estimation and heat flux analysis

To estimate the two properties it is necessary to use an objective function. Usually, the objective function is simply the square difference between the temperatures (Adjali and Laurent, 2007; Borges *et al.*, 2008). However, since the thermal contact resistance is a systematic error, this influence needs to be considered in the analysis, because this value is constant and permanent. So, this influence was included in the objective function with the purpose of considering an initial error, therefore, the objective function will never be equal to zero. Thus, the objective function used in this work is based on the square difference between the experimental and numerical temperatures plus the influence of the thermal contact resistance. This equation can be written as:

$$F = \left[\left(R''_{c,1} + R''_{c,2}\right)\phi_a\right]^2 + \sum_{j=1}^{m}\left(Y_j - T_j\right)^2 \tag{6}$$

To obtain the values for λ and ρc_p in each experiment, the BFGS (Broydon Fletcher Goldfarb Shanno) sequential optimization technique, presented in Vanderplaats (2005), was used. This technique is a particularity of the variable metric methods. The advantages of this technique are its fast convergence and readiness for working with many design variables. Because it is a first order method, it is necessary to know the gradient of the objective function.

3. EXPERIMENTAL PROCEDURE

The experimental apparatus used to determine the properties of AISI 316 and 304 Stainless Steels and ASTM B265 Grade 2 Titanium is shown in Fig. 3. The AISI 304 Stainless Steel plate is 49.89 x 49.98 x 10.76 mm in dimension, the AISI 316 Stainless Steel plate is 49.89 x 49.96 x 9.90 mm and the ASTM B265 Grade 2 Titanium plate is 50.00 x 49.98 x 8.84 mm. The 44.5 x 44.5 x 0.25 mm resistive kapton heater has a resistance of 15 Ω and was used due to its small thinness, allowing faster overall warming. This heater was connected to a digital power supply Instrutemp ST – 305D-II to provide the necessary heat flux. In this work, different intensities of heat flux were used in the same experiment as an attempt to achieve the best condition to estimate the properties simultaneously in accordance to the analyses of the sensitivity coefficients. To achieve this heat flux condition, the digital power supply has a configuration that allows working with parallel or series connection. Then, the series condition was used to provide the highest heat flux for the

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first period of the experiment, and the parallel condition to supply the lowest heat flux for the second part. A symmetrical assembly was used to minimize the errors in the measurement of the heat flux to be generated on the sample surface. In addition, the applied current value was measured by the calibrated multimeter Minipa ET-2042C and weights were used on top of the isolated sample-heater set to improve the contact between the components. To ensure a one-direction flux and minimize the effect of convection caused by the air circulating in the environment, the sample-heater set was isolated with polystyrene plates. Temperatures were measured using type K thermocouples (30AWG) welded by capacitive discharge and calibrated by using a bath temperature calibrator Marconi MA 184 with a resolution of ± 0.01 °C. This thermocouple was connected to Agilent 34980A data acquisition set controlled by a microcomputer. In order to obtain better results, all experiments were performed in controlled room temperature.

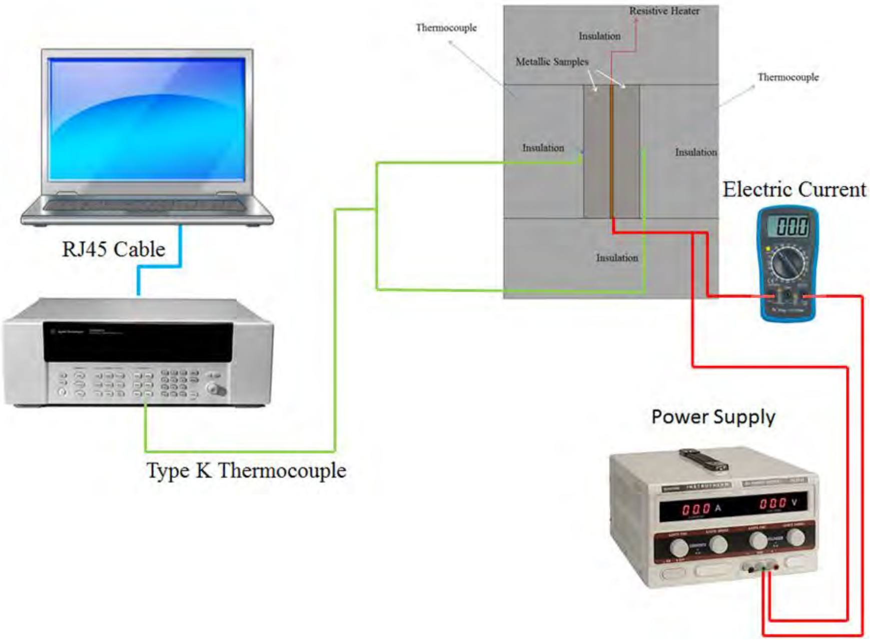

Figure 3 – Sketch of experimental apparatus used to determine the properties.

4. RESULTS ANALYSIS

4.1. ASTM B265 grade 2 titanium

Fifteen experiments were performed to simultaneously estimate the thermal conductivity and the volumetric heat capacity of a titanium sample. This number of experiments was done in order to obtain reliable estimates of standard deviation and average of the data. Each experiment lasted 150 s, and the heat flux was imposed from 0 to 130 s. From 0 to 30 s, the imposed heat flux was approximately 2682 Wm^{-2}; from 30 to 130 s, the imposed heat flux was around 664 Wm^{-2}. The time interval used to monitor the temperature was 0.2 s. To guarantee the hypotheses of constant thermal properties, this configuration for the heat fluxes was chosen to keep the temperature difference lower than 5 K. This temperature difference is based on the difference between the final and initial temperatures which are measured having the thermocouple on the same position of the sample. The sensitivity analyses were performed to determine the best region to estimate the properties. These analyses were performed by using the values of λ and ρc_p obtained from GMTTitanium (2010) and the parameters described above. In addition, several analyses of the objective function (Eq. 6) with the sensitivity coefficient analyses were performed to determine the properties in the selected region (Carollo *et al.*, 2012). This selected region corresponds to a set of points which provides accurate thermal properties estimation. Since this estimation presents an accurate result, it can be said that this region of points presents enough influence to determine these properties. Figure 4 shows the sensitivity coefficients at $x = L$ for λ and ρc_p.

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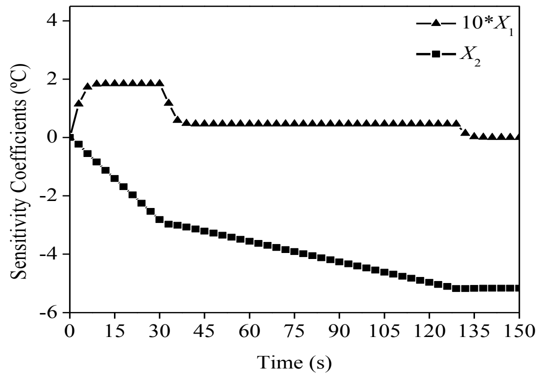

Figure 4. Sensitivity coefficients as a function of time for titanium sample.

X_1 represents the sensitivity coefficient for λ and X_2 represents the sensitivity coefficient for ρc_p, both on the isolated surface. The former is multiplied by a factor to improve the visualization of the curve. In Figure 4, one can see that X_1 increases in the first 30 s and remains constant thereafter until the change of heat flux; and X_2 increases proportionally with the temperature. Because of this behavior, the highest heat flux was imposed in the first period of time, resulting in high sensitivity for λ. The lowest heat flux was imposed in the second part to increase the sensitivity for ρc_p and maintain the sensitivity for λ. This procedure was done, because it is necessary to control the magnitude relation between X_1 and X_2, to guarantee the estimation for both properties, that is, if one coefficient is much larger than the other, the estimation, by using minimization, will occur only for that property presenting the higher coefficient.

Figure 5 presents the distribution of experimental and numerical temperatures for the sample at $x = L$ and the imposed heat flux at $x = 0$. The numerical temperature is achieved by applying the values of the estimated properties, λ and ρc_p, from one of the accomplished experiments. These temperatures present good agreement, which can be proved by analyzing the temperature residuals shown in Fig. 6. These residuals are calculated by the difference between the experimental and numerical temperatures. The maximum value found was lower than 0.08 ºC, which is much lower than the thermocouple uncertainty. This difference will be considered as the imperfection of the thermal insulation in the uncertainty analysis.

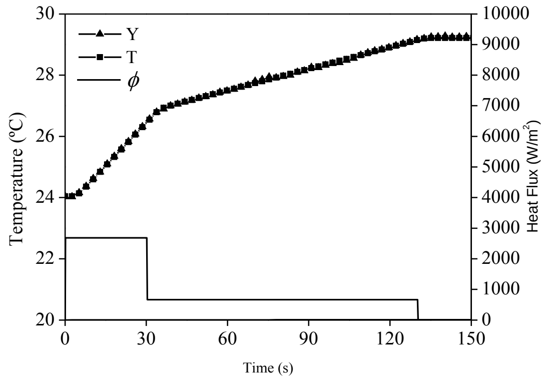

Figure 5. Experimental (Y) and Numerical (T) Temperatures with Heat Flux (ϕ) as a function of time for titanium sample.

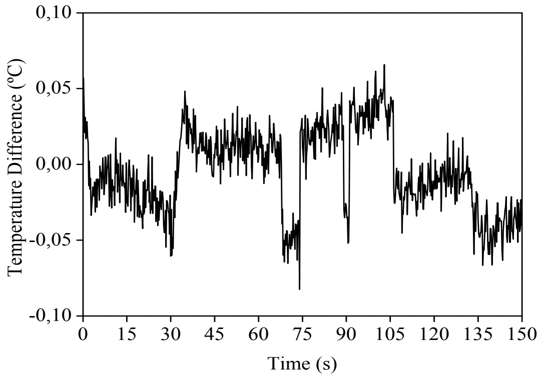

Figure 6. Temperature difference $Y - T$ for titanium sample.

A significant improvement obtained in this work concerns the difference between the measured temperatures in $x = L$ on both samples used for the assembly to guarantee the symmetrical heat flux (Fig. 3). In Carollo *et al.* (2012) this difference presented an average error of approximately 0.2 ºC, with a maximum difference of 0.4 ºC, as may be seen in Figs. 7 and 8. A comparison between the two measured temperatures in $x = L$ on a titanium sample is shown in Fig. 7. Figure 8 presents the difference between the temperatures (Carollo *et al.*, 2012). Thus, three significant changes were made to minimize this difference. The first concerns the asymmetry presented as a manufacturing error of the resistive heater. All Omega heaters (Omega, 2000) previously used displayed this small asymmetrical heat flux when the experiment is performed under unsteady; this problem did not happen under steady condition. For the solution of this problem, resistive heaters purchased from Laboratório de Meios Porosos e Propriedades Termofísicas da Universidade Federal de Santa Catarina were used. Several tests were performed to verify whether the problem of asymmetry had been solved. One of these results will be described in Fig. 9 and 10.

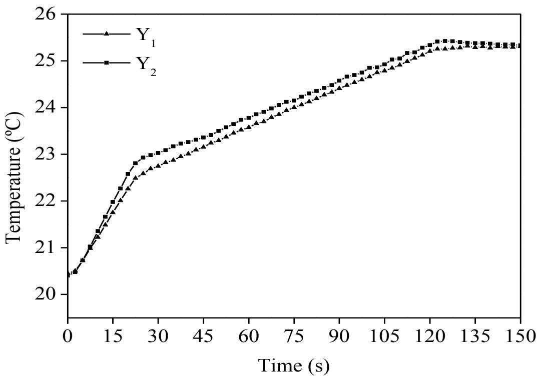

Figure 7. Experimental temperatures comparison in $x = L$ for titanium sample

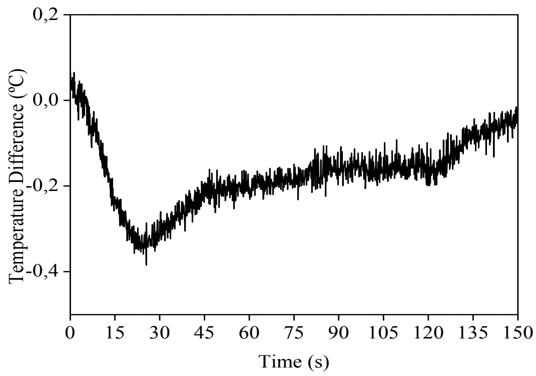

Figure 8.Temperature difference $Y_1 - Y_2$ for titanium sample.

All the samples used for the measurements were rectified aiming to minimize the thermal contact resistance between the heater and the samples. Therefore, Silver Artic thermal paste used in Carollo *et al*. (2012) was unnecessary. This procedure reduced the great uncertainty added to the value of the estimated thermal properties, mainly for the low values displayed by this thermal paste (Narumanchi *et al*., 2008). Finally, the assembly of the samples was made in the vertical to avoid the influence one sample weight on the other. Figure 9 presents a comparison of the experimental temperature at $x = L$ for the titanium as shown in Fig. 7. As already mentioned above, the purpose of these procedures was to solve the asymmetry of the temperatures measured at $x = L$. The effective proof of this significant improvement may be verified in Fig. 10. It may be observed from this figure that the average of the temperature differences is close to 0 ºC and as expected, the difference between these temperatures vary randomly; whereas in Fig. 8, the difference shows a systematic error, especially in the region of highest heat.

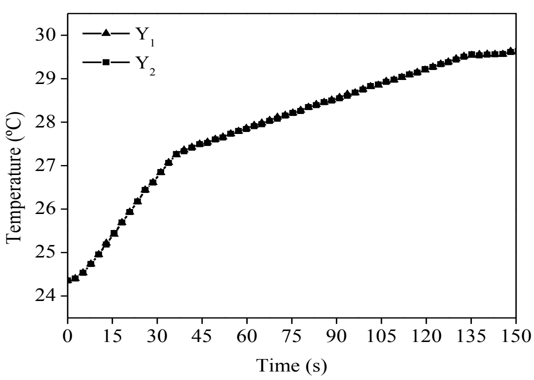

Figure 9. Experimental temperatures comparison in $x = L$ for titanium sample.

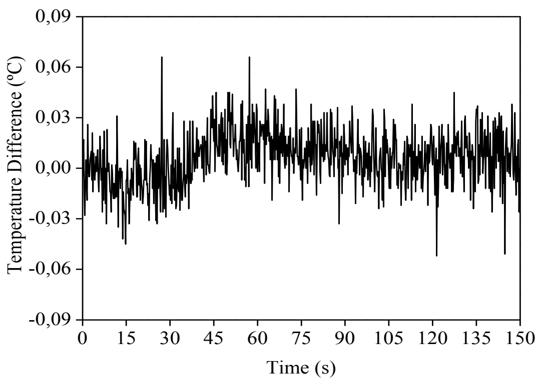

Figure 10.Temperature difference $Y_1 - Y_2$ for titanium sample.

Table 1 displays the estimated property results (λ and ρc_p) for all the 15 experiments carried out on titanium ASTM B265 Grade 2. To prove the symmetric assembly, these properties were estimated for the two temperatures signals measured on the surface of the two samples used at $x = L$. In this table, Th_1 stands for Thermocouple 1 and Th_2 for thermocouple 2. It may be seen that in several cases the results of both properties stand very close to each other, mainly for ρc_p. Table 2 presents the average value, the standard deviation and the percent difference between the estimated average value and the literature values for λ and ρc_p on titanium ASTM B265 Grade 2. It may be observed that the average values for λ and ρc_p for both thermocouples were practically the same. One may see that the results are in accordance with the work found in literature due to the low standard deviation and small percent difference (Tab. 2).

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Table 1 – Obtained values for ASTM B265 Grade 2 Titanium samples.

Experiment	**λ (W/mK)**		**ρc_px10^{-6} (Ws/m³K)**	
	Th$_1$	**Th$_2$**	**Th$_2$**	**Th$_2$**
1	17.7109	17.7106	2.65	2.64
2	17.7112	17.7110	2.64	2.64
3	16.9956	16.3866	2.60	2.60
4	17.7109	17.7104	2.64	2.65
5	17.7106	17.7098	2.65	2.64
6	16.9782	16.5793	2.67	2.66
7	17.0965	17.7121	2.60	2.61
8	17.7113	17.7113	2.63	2.63
9	17.7083	17.7083	2.66	2.65
10	17.7098	17.7099	2.64	2.64
11	17.1110	18.9340	2.67	2.68
12	17.7108	16.8108	2.66	2.67
13	19.4723	18.7302	2.69	2.68
14	17.7097	17.1821	2.66	2.67
15	16.4352	17.7109	2.60	2.61

Table 2. Statistic values obtained for the ASTM B265 Grade 2 Titanium.

Thermocouple	**Property**	**Present work**	**GMTTitanium (2010)**	**S. D.**	**Difference (%)**
1	**ρc_px10^{-6} (Ws/m³K)**	2.64	2.66	± 0.027	0.75
	λ (W/mK)	17.57	18.06	± 0.66	2.71
2	**ρc_px10^{-6} (Ws/m³K)**	2.64	2.66	± 0.025	0.75
	λ (W/mK)	17.60	18.06	± 0.68	2.55

4.2. AISI 304 and 316 stainless steels

10 experiments were carried out for AISI 304 and 316 stainless steels and the same procedures as in section 4.1 (titanium sample) concerning the number of points measured, heating time, time interval, two heating intensities, etc. To avoid repetition of results, only the estimated average values of λ and ρc_p are presented once the difference between the thermophysical property values is small. Moreover, only the values of λ and ρc_p from one thermocouple are presented as the main objective is to estimate just one value of each property of the materials studied. Therefore, Tab. 3 presents the average value, the standard deviation and the percent difference of λ and ρc_p for AISI 304 stainless steel. The same statistical data presented in the previous table are also presented on Tab. 4 for AISI 316 stainless steel.

Table 3. Statistic values obtained for the AISI 304 stainless steel.

Property	**Present work**	**Incropera *et al.* (2006)**	**S. D.**	**Difference (%)**
ρc_px10^{-6} (Ws/m³K)	4.06	3.77	0.76	7.69
λ (W/mK)	15.31	14.9	1.85	2.75

Table 4. Statistic values obtained for the AISI 316 stainless steel.

Property	**Present work**	**Incropera *et al.* (2007)**	**S. D.**	**Difference (%)**
ρc_px10^{-6} (Ws/m³K)	4.00	3.85	0.54	3.90
λ (W/mK)	14.09	13.4	1.84	5.15

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6. THERMAL CONTACT RESISTANCE

The thermal contact resistance was analyzed with the purpose to find out if there is a significant influence during the temperature measurements. This study was divided into two parts: the first part considered the thermal contact resistance caused by the applied thermal compound; the second part took into account the influence of the kapton layer present in the resistance heater.

In Carollo *et al*. (2012) the effect of the thermal contact resistance due to the thermal paste ($R''_{c,1}$) was added as a systematic error to the objective function as presented in Eq. (6). However, since the samples used in this work were rectified and the thermal paste was not used, this effect was disregarded in Eq. (6); only the effect of the kapton layer of the resistive heater was considered.

The influence of the kapton layer was analyzed. An Optical Microscope Jenavert Zeiss (2000x) with the Image Analyzer Olympus Model TVO.5XC-3 was used to measure the thickness of the kapton on the heater. The kapton layer presents a thickness of 10.64 x10^{-6} m since the thermal conductivity of the kapton is 0.12 Wm-1ºC-1 (MatWeb, 2012); a thermal contact resistance equal to 88.67x10^{-6} $m^{2o}CW^{-1}$ was obtained. By considering the applied heat flux average, 1117 Wm^{-2}, this thermal contact resistance corresponds to a temperature difference of 0.10 ºC.

7. UNCERTAINTY ANALYSIS

Uncertainty can be described as a portion of the measurement that cannot be considered as a true value. A uncertainty value depends upon a mechanical, electrical or visual point of reference to be assigned each time a measurement is taken. These values, no matter how carefully they are obtained, contain some uncertainty (Taylor, 1997). The uncertainties are used to evaluate the precision of the result. That is why it is important to keep low values for them. An uncertainty analysis was done to verify whether the obtained results were reliable. This analysis was based on the uncertainty propagation procedure. In this procedure, it is necessary to decide which errors will be analyzed. In this work the procedure to determine the uncertainty in the estimation of λ and ρc_p is based on linear propagation of uncertainties of the variables: temperature measurement, the imposed heat flux, measurement instruments, thermal contact resistance, imperfection of thermal insulation, and the numerical errors (BFGS and finite difference method. The hypothesis of linear propagation is used because the objective function is based on the difference between the experimental and numerical temperatures. This analysis is in accordance with the theory of error propagation extracted from Taylor (1997).

The thermocouple error, the thermal contact resistance, the acquisition data error and the imperfection of thermal insulation were considered when analyzing the experimental temperature of the ASTM B265 grade 2 titanium. Thus:

$$U_Y^2 = U_{aquisition\,data}^2 + U_{thermocouple}^2 + U_{contact\,resistance}^2 + U_{thermal\,insulation}^2 \quad (7)$$

When considering the numerical temperature, the multimeter errors, which were used to measure the current and resistance values, and the numerical error of the finite difference method were used. Therefore:

$$U_T^2 = U_{current\,multimeter}^2 + U_{resistance\,multimeter}^2 + U_{FDM}^2 \quad (8)$$

Finally, the uncertainty of the estimation can be calculated based on the objective function, which is composed of the experimental and numerical temperature, and the BFGS method. Hence:

$$\begin{aligned} U_{final}^2 &= U_Y^2 + U_T^2 + U_{BFGS}^2 \\ U_{final}^2 &= U_{aquisition\,\,data}^2 + U_{thermocouple}^2 + U_{contact\,resis\tan ce}^2 + U_{thermal\,insulation}^2 \\ &+ U_{current\,multimeter}^2 + U_{voltage\,multimeter}^2 + U_{FDM}^2 + U_{BFGS}^2 \end{aligned} \quad (9)$$

After the final uncertainty has been defined, it is necessary to quantify the partial uncertainty. To define these values, the authors decided to consider each uncertainty divided by the mean value of the analyzed parameter. The data aquisiton uncertainty is calculated from the resolution of the equipment which is 0.01 ºC and the maximum difference of the temperature which is approximately 5.00 ºC. This temperature difference is based on the difference between the initial and the highest temperatures which are measured by having the thermocouple on the same position on the sample. Thus, this uncertaintiy is:

$$U_{aquisition\,\,data} = \frac{0.01}{5.00} = 0.2\,\% \quad (10)$$

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Now the uncertainty of the thermocouple is calculated by considering the a oscilation of 0.1 ºC and the same difference of average temperature of 5.0 ºC. Thus:

$$U_{thermocouple} = \frac{0.10}{5.00} = 2.0\,\% \tag{11}$$

The uncertainty due to the thermal contact resistance of 0.1 ºC was estimated in Section 6. Therefore:

$$U_{contact\ resistance} = \frac{0.10}{5.00} = 2.0\,\% \tag{12}$$

As aforementioned, the maximum difference between the numerical and experimental temperatures of 0.08 °C presented in Fig. 6 was considered as uncertainty due to the imperfection of the thermal isolation; hence:

$$U_{thermal\ isnsulation} = \frac{0.08}{5.00} = 1.60\,\% \tag{13}$$

For the calculation of the multimeter uncertainty, the resolution of the digital device divided by the average current and the average resistance is used; the values are 0.67 A and 15.0 Ω. It is worth highlighting that this multimeter was used in the measurement of the current value, with a resolution of 0.01 A and the resistance with a resolution of 0.1 Ω. In this manner, thje following equations are achieved:

$$U_{current\ multimeter} = \frac{0.01}{0.67} = 1.49\,\% \tag{14}$$

$$U_{resistance\ multimeter} = \frac{0.1}{15.0} = 0.67\,\% \tag{15}$$

Finally, the uncertainty of the mathematical methods used must be quantified. For the BFGS method the error of 0.01 ºC was adopted and for the FDM 0.05 ºC as base values for the definition of the uncertainty; thus the following is obtained:

$$U_{BFGS} = \frac{0.01}{5.00} = 0.2\,\% \tag{16}$$

$$U_{FDM} = \frac{0.05}{5.00} = 1.0\,\% \tag{17}$$

Once all the partial uncertainties have been calculated, it is possible to determine the uncertainty of the thermal properties by substituting Eqs. (10) to (17) in Eq. (9).

$$\begin{aligned} U_{final}^2 &= 0.2^2 + 2.00^2 + 2.00^2 + 1.60^2 + 1.49^2 + 0.67^2 + 0.20^2 + 1.0^2 \\ U_{final} &= 3.78\,\% \end{aligned} \tag{18}$$

As it can be seen, the uncertainty value is in accordance with that found in the literature, because the presented value is lower than 5 %.

8. CONCLUSIONS

This paper presented a significant improvement in the technique that uses different intensities of heat flux in the same experiment to estimate the thermal conductivity and the volumetric heat capacity of metal samples simultaneously. These thermophysical properties were estimated for titanium ASTM B265 and AISI 304 and 316 stainless steels samples. Good results for both materials were found. This affirmation can be proved due to the small difference between the literature and estimated values, and the low standard deviation. In addition, an error analysis based on thermal contact resistance and uncertainty analysis in the estimation of λ and ρc_p of ASTM B265 grade 2 titanium was presented. A future proposal is to perform these analyses by considering thermal properties estimation by varying the initial temperature.

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