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UNCERTAINTY CHARACTERIZATION IN MEASURED MODAL PROPERTIES FOR MODEL CALIBRATION ANALYSES

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Abstract. *This work is aimed at characterizing uncertainties in modal parameters extracted from ambient vibration data on a structure through the Time Domain Decomposition (TDD) technique. These modal parameters are intended to be used afterwards in a model calibration procedure in structural dynamic problems. A beam-like prototype was specifically designed for this purpose. It consists of an elastic pinned-pinned beam which is instrumented with 6 accelerometers. Several tests with impact hammers were performed in which long time-recording acceleration histories were considered. It is used a Time Domain Decomposition (TDD) technique to obtain estimates for modal properties and some preliminary analyses concerning their variability are performed.*

Keywords: *Uncertainty Characterization, Model Calibration, Beam, Time Domain Decomposition*

1. INTRODUCTION

To ensure the structural health condition procedures such as inspection, periodical monitoring and maintenance are essential. Currently, several non-destructive techniques have been developed for structural assessment based on vibration tests, extracting the modal parameters from ambient vibration data (Kim *et al.*, 2005), which makes this type of approaches applicable to structural health monitoring. Those non-destructive tests can be associated with other techniques to identify and diagnose structural failures. These techniques may be classified into non-model-based methods and model-based inverse methods (Huang *et al.*, 2012). Accordingly, the main difference between these two types of techniques is that the model-based inverse method requires the use of computational models.

Regardless the estimator used to infer about the model properties, it is always worth to characterize and to quantify their uncertainties. In order to ensure consistence along model validation processes it is essential to characterize and distinguish all sources of uncertainties. Furthermore, different sources of uncertainties should be treated with proper action (Adhikari *et al.*, 2009). For example, Gardoni *et al.* (2002) present an uncertainty quantification approach for fragility estimates for reinforced concrete column problems. They take into account the variations in the basic structural properties of the structure are induced (i.e. material properties and geometry), and consider three sources of uncertainties, (*i*) Modal inexactness related to assumptions in the modeling process of the estimator, (*ii*) measurement error in the vibration response and (*iii*) Statistical uncertainty in the model parameters.

As for uncertainties in computational models, the most traditional way to perform model uncertainty quantification is the Monte Carlo method which has great acceptance due to their stability (Plessis *et al.*, 2000),(Mace *et al.*, 2005) and due to the fact that they are non-intrusive. This approach assumes some PDF for the parameter set and run the computational model until convergence is achieved. The computational costs for these analyses may be extremely high due to possible nonlinear relations between parameters and predictions and also due to the dimension of the parameter space.

Modal parameters of a structure can not be measured directly and acceleration responses from vibration tests are used to estimate them and can be represented as an inverse problem. The goal of this paper is to present some preliminary analysis on the variability of estimated modal properties analyzing the uncertainties on the parameters during the estimation process. Here, the modal properties are estimated with output-only measured data through the Time Domain Decomposition (TDD) technique (Kim *et al.*, 2005). Also the characterization and types of uncertainties was introduced and discussed. An experimental set-up composed of an instrumented pinned-pinned beam was specifically designed for this purpose. A series of several tests on the simply supported beam was performed and time domain data were processed

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by TDD method. The main contribution of this paper is to make uncertainty analysis on modal data based on a large body of measured data.

2. BASICS

Inverse problems are used in situations where retrieving information of unknown quantities by indirect observations of a quantity which can be measured. The problem of characterizing the behavior of a system from observed data and estimate the parameters that govern a given dynamical system is known as identification problem. Once a basic form of a forecasting model is given (parametric or non-parametric model), the model calibration is in general phrased as an Inverse Problem. The information about the unknown parameters are obtained suing iterative processes based on measured data. For this reason, a model based in inverse problems can be established as a problem of identification/estimation. Then, considering the identification process of the modal parameters of a structure as an inverse problem, the modal parameters can not be measured directly and acceleration responses from vibration tests are used to determine them.
Let's consider a physical system instrumented with measurement devices. Let's also suppose that we can measure these output responses and possibly excitation forces. Let's assume that system outputs y may be modeled according to the following observation model

$$y(t) = A(\theta, t) + \nu(t) \tag{1}$$

where $y(t)$ represents the signal response, $A(\theta, t)$ is the operator which provides model predictions which depends on the unknown parameters θ. Here, equation (1) assumes an additive error model which is represented by the variable ν; moreover, it can be interpreted as measurement noise. Based on equation (1) one may note that any strategy to infer about θ will depend on measurement y, model structure through the operations associated to operator $A(\theta)$ and also on the noise. One feasible way would be to seek for the point estimate which provides the greatest probability (or maximum likelihood) given a set of measured data. Therefore, to develop strategies that enable Engineers to detect different sources of uncertainties is extremely important for model calibration processes.

To obtain information about the unknown parameters may be obtained via Statistical Inversion Theory. Assuming that all parameters are random variables, the Baye's rule casts as

$$\pi(\theta|y) \sim \pi(y|\theta)\, \pi(\theta) \tag{2}$$

where $\pi(\theta|y)$ is the posterior conditional density and express the probability of the unknown parameter θ given the observed parameter y take on the values given as the data of the problem and our prior believe. In Bayesian statistical framework, the posterior density is the solution of the inverse problem (Calvetti and Somersalo, 2007). It should be highlighted that it can be shown that the Likelihood mathematical structure $\pi(\theta|y)$ depends on the mathematical structure of the additive noise ν. This fact corroborates the importance of characterizing uncertainties in measured data and the estimation method to be used for Inverse Problems.

3. UNCERTAINTY CHARACTERIZATION

In practical applications, the identification process has associated uncertainties related to modeling of the system, signal processing and noise and those uncertainties may be propagated.

Figure 1 shows the flowchart of the procedure for identification of structural damage models, showing that the uncertainties are accumulated step by step through the processes of physical model parameter estimation and inverse model.

3.1 Uncertainty description

The structural damage model estimation for the modal calibration analyses using modal parameters, involves the following topics:

- vibration tests;
- signal/data acquisition;
- signal processing and filtering;
- modal parameters identification (through an specific algorithm i.e. ERA, ERA-DC, FRF, TDD)

The above topics bring uncertainties related to the nature of the structure (i.e. material parameters, geometry, boundary conditions) and other associated to the process of identification of modal parameters, such as quality of the collected signals, filtering and truncation error and another characteristic of the inverse problem as non-modeling dynamics. Figure 1 shows flowchart for the structural damage model, and presents the measurement and identification stages together with the uncertainties involved in the process.

The main uncertainties in the process of calibration analyses of a structural damage model are described next.

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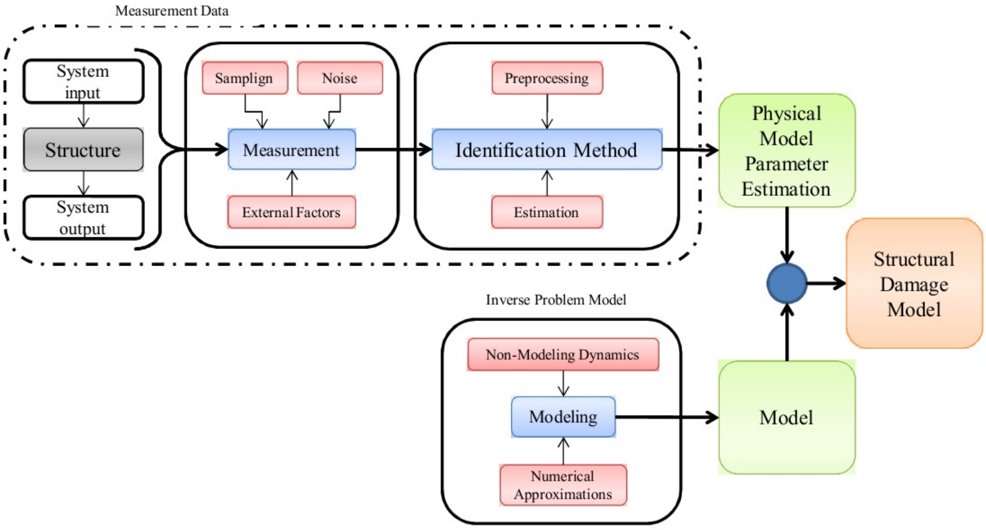

Figure 1: Flowchart of the Structural Damage Model

3.2 Measurement error in the vibration test: Sampling/ Noise

In structural vibration testing generally propagated errors are due to noise, sampling errors in the data acquisition system and data processing (Huang *et al.*, 2012). The signals are measured/quantized with a specific precision and sampled in the discrete time domain (Mao and Todd, 2013). These errors are epistemic and are known as *measurement errors*, they can be reduced by the use of more accurate sensors and improving the data processing. In simulation studies, this kind of error is introduced by adding random Gaussian noise with zero mean and a known variance in the time-history response signals.

3.3 Filtering

Another uncertainty related to the extraction of the modal parameters is the filtering. A filter cannot completely eliminate the effects of other modes in the system and also the filter has its own dynamics that may affect the corresponding signal.

3.4 Extraction modal parameters: Preprocessing

Other sources of error are the numerical approximations per implementation of mathematical algorithms or the use of an arithmetic process with finite precision. Those algorithms have intrinsic errors to the transformation of a continuous into a discrete problem. For example, some errors are related to use of Fast Fourier Transform (FFT) or the Singular Value Decomposition (SVD). This errors are known as *discretization errors*, and they are epistemic errors and can be reduced by improving the numerical processes to compute the mathematical algorithms.

3.5 External factors

These kind of uncertainties are epistemic. Basically include those related to the vibration test as environmental and operational variability, mass and positions of the sensors, uncertainties associated with the position of the excitation force and the actual distance between the supports and another non linear distortions.

3.6 Inverse problem modeling

In the inverse problem there are modeling errors that result from the lack of knowledge about the physical phenomenon. Generally, these uncertainties are related to the considerations in the constitutive equations of the physical or unknown boundary conditions and inaccurate geometry in the model.

4. TIME DOMAIN DECOMPOSITION

In structural analysis, vibration tests are used to estimate modal parameters. Often, acceleration responses are variables that can be easily measured from vibration tests. Kim *et al.* (2005), proposed an extraction method of the modal parameters from acceleration responses, called Time domain Decomposition **TDD** technique. This method is an efficient technique to provide high resolution mode shapes if one has a large array of accelerometers along the structure (Kim *et al.*, 2005).

This approach to identify the modal parameters is based on a set of given mode-isolated signals, and the undamped mode shapes are extracted from the singular value decomposition of the output energy correlation matrix. In this section, we will present the basic characteristics of TDD method, where the main feature is that it allows to use **n** acceleration signal responses simultaneously. A time-history response of the acceleration is given by

$$\ddot{\mathbf{y}}(t) = \sum_{i=1}^{\infty} \ddot{c}_i(t)\mathbf{\Phi_i} = \sum_{i=1}^{n} \ddot{c}_i(t)\mathbf{\Phi_i} + \sum_{i=n+1}^{\infty} \ddot{c}_i(t)\mathbf{\Phi_i} = \sum_{i=1}^{n} \ddot{c}_i(t)\mathbf{\Phi_i} + \varepsilon_f(t) \tag{3}$$

Where, $\ddot{\mathbf{y}}(t) = [\ddot{y}_1(t), \ldots, \ddot{y}_n(t)]$ represents the output acceleration time history, $\ddot{c}_i(t)$ represents the ith modal contribution factor of acceleration at time t, $\mathbf{\Phi_i}$ denote matrix denoting the ith mode shape, and $\varepsilon_f(t) = \sum_{i=n+1}^{\infty} \ddot{c}_i(t)\phi_\mathbf{i}$ denote the truncation error on acceleration.

The first step consists in determining an estimation for the bandwidth frequency associated to a specific mode. The second step consists in apply a digital band-pass filter to isolate the mode *i* in the *n* acceleration responses. Then, Eq. (3) for the ith mode-isolated acceleration response, can be written as

$$\ddot{\mathbf{Y}}_i(t) = \mathbf{\Phi_i}\ddot{c}_i^T + \sum_{k=1}^{n-1} \mathbf{\Psi_k}\ddot{d}_k^T(t) = \mathbf{\Phi_i}\ddot{c}_i^T(t) + \varepsilon_f(t) \tag{4}$$

Where, $\ddot{\mathbf{Y}}_i$ denotes the mode-isolated output acceleration time history that contains only the ith mode. $\varepsilon_f(t) = \sum_{k=1}^{n-1} \ddot{d}_k\mathbf{\Psi_k}$ denotes the error due to both filtering and truncation, $\mathbf{\Psi_k}$ corresponds to the orthogonal noise base, and $\ddot{d}(t)_k$ represents the kth model total error contribution factor.

Defining the cross-correlation of the ith mode-isolated acceleration as $\mathbf{E}_i = \mathbf{Y_i}\mathbf{Y_i^T}$, and assuming an orthogonal bases in the modal space $\ddot{\mathbf{c}}_\mathbf{i} = [\ddot{c}_i(t_1), \ldots, \ddot{c}_i(t_N)]^T$ and an error space represented by $\ddot{\mathbf{d}}_\mathbf{k} = [\ddot{c}_k(t_1), \ldots, \ddot{c}_k(t_N)]^T$, where N is the total time samples, the cross-correlation $\mathbf{E}_j$ can be written as

$$\mathbf{E}_i = \mathbf{\Phi}_i q_i \mathbf{\Phi}_i^T + \sum_{k=1}^{n-1} \mathbf{\Psi}_k \sigma_k \mathbf{\Psi}_k^T \tag{5}$$

Where the scalar values $q_i = \ddot{\mathbf{c}}_\mathbf{i}^T\ddot{\mathbf{c}}_\mathbf{i}$ and $\sigma_k = \ddot{\mathbf{d}}_\mathbf{k}^T\ddot{\mathbf{d}}_\mathbf{k}$, represents the level of energy at the modes i and the noises k respectively. Since the energy associated with noise is lower in comparison with the energy of the corresponding mode is appropriate to assume $q_i > \sigma_1 > \sigma_{n-1}$. Therefore, the ith undamped mode shape $\mathbf{\Phi_i}$, can be extracted from the first singular vector in the singular vector matrix after the Singular Value Decomposition (SVD) of Eq. (5) (Golub and Van Loan, 1996).

To extract the natural frequencies of each ith mode, it is necessary to calculate the time history of the ith modal contribution acceleration factor $\ddot{\mathbf{c}}_i^T$ as follow

$$\ddot{\mathbf{c}}_\mathbf{i}^\mathbf{T} = \frac{\mathbf{\Phi_i^T Y_i}}{\mathbf{\Phi_i^T \Phi_i}} \tag{6}$$

The $\ddot{\mathbf{c}}_i^T$ represents the response of a single-output system for the ith modal behavior, then the frequency at the single peak is the desired damped natural frequency of the ith mode. Therefore, it is possible to use a windowed Fast Fourier Transform (FFT) on $\ddot{\mathbf{c}}_\mathbf{i}$ to extract the modal frequency, where the main source of uncertainty is related to the quantization of the data and the numerical approximation of the FFT (Betta, Liguori and Pietrosanto, 2000).

5. EXPERIMENTAL SET-UP

Let's consider a simple supported steel beam shown in Fig. 2, the length l of the beam is 1.485m, instrumented with N accelerometers ($N = 6$) uniformly spaced with $a = 0.25$m. The second moment of cross sectional area I, cross sectional area A, Young's modulus E and density of the beam are $3.1756 \times 10^{-09} m^4$, $6.0484 \times 10^{-04} m^2$, $207 \times 10^9 Pa$ and $7.85 \times 10^3 Kg/m^3$, respectively.

The excitation for the vibration test was performed by successive impacts in the position of the second accelerometer. The sampling rate was 4000Hz and each of the accelerometers recorded 2.4×10^6 data sets. The set of sensors simultaneously measures the acceleration response, each one of the signals is then divided into N time-windows of equal width to obtain the set of acceleration responses. This set of acceleration responses will then be processed with the TDD algorithm. Table 1 presents the first 5 modal frequencies obtained from the theoretical expression for a simply supported beam (Inman, 2013) and the same system modeled by finite element method (FEM).

In the FEM model were considered mass accelerometers ($m_{ac} = 45$ gr) and the data of Table 1 shows that when this mass is considered, modal frequencies vary slightly. Then, any uncertainty related to the modeling of the system affects the identification of modal parameters. Table 1 compares the theoretical natural frequencies and numerical model, showing that neglecting the mass of the accelerometers in the numerical model can induce uncertainties related to and unmodeled dynamics, bring a ill interpretation of the physical model.

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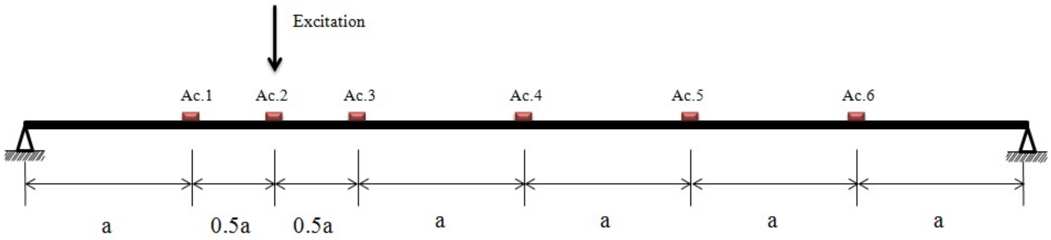

Figure 2: Locations of the sensors on the beam

Table 1: Theoretical modal frequencies (Inman, 2013), FEM Model (150 Beam elements)

Mode	Theoretical Frequency [Hz]	FEM Frequency [Hz]	Variation [%]
1	8.671	8.479	2.214
2	34.683	33.620	3.065
3	78.037	74.560	4.456
4	138.732	136.079	1.912
5	216.769	206.815	4.592

6. RESULTS

The first five mode shapes of N data sets extracted from TDD technique and from finite element model are presented in Fig 3.

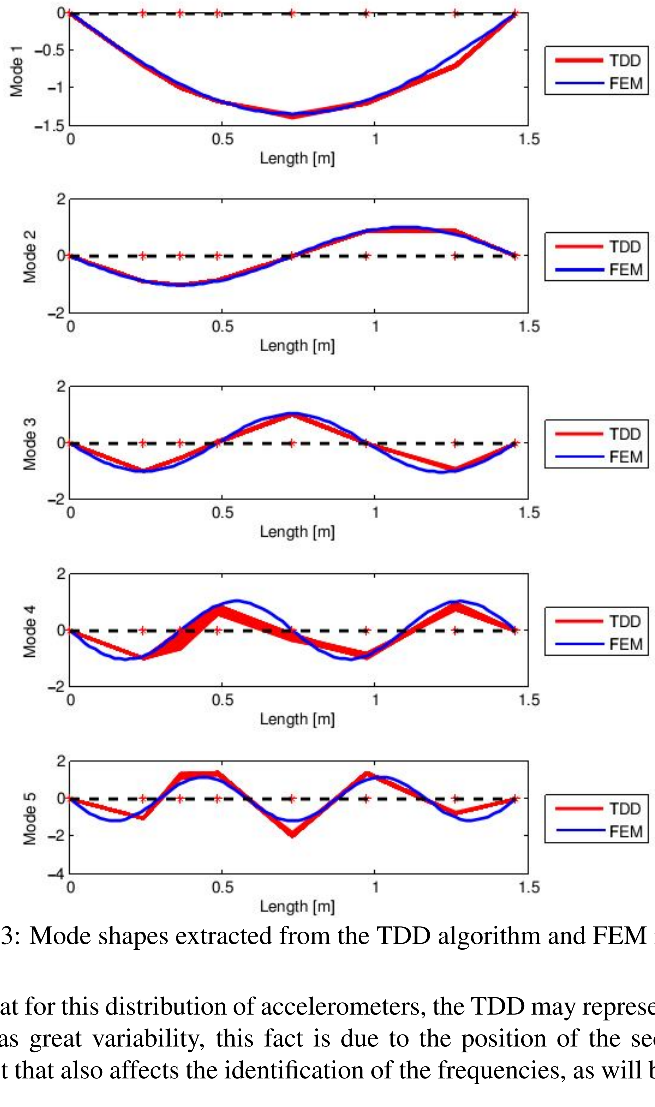

Figure 3: Mode shapes extracted from the TDD algorithm and FEM method

From Fig. 3 is observed that for this distribution of accelerometers, the TDD may represent the first three mode shapes. However, the fourth mode has great variability, this fact is due to the position of the second accelerometer, which is located next to the node, a fact that also affects the identification of the frequencies, as will be seen later. Furthermore, the representation of the fifth mode is acceptable since for higher mode shapes will require a greater number of acceleration sensors, that is, a higher resolution in the mesh.

The acceleration responses were divided into $N = 350$ data sets and they were processed using TDD, thus it is obtained an equal number of modal frequencies for the first five modes. The probability density functions (PDF) obtained from these data are presented in Fig. 4 together with the modal frequency obtained from FEM model.

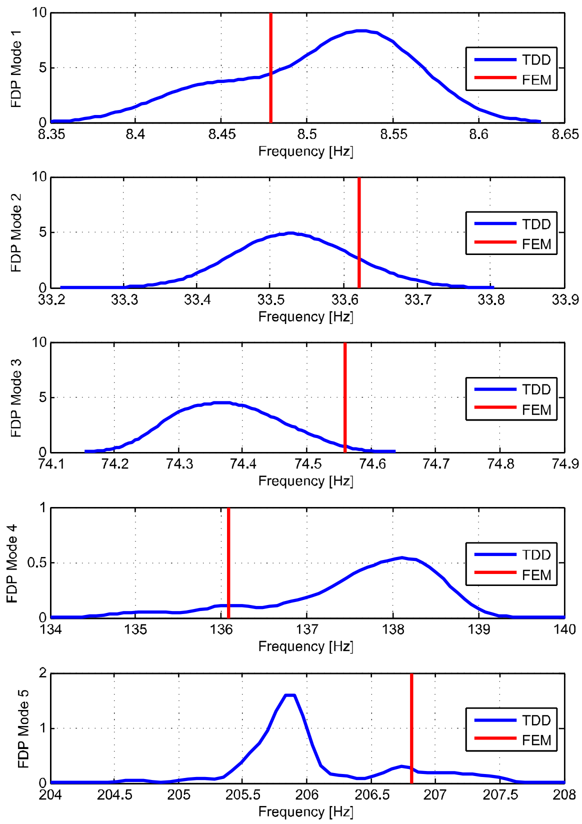

Figure 4: Probability density functions for the modal frequenucies

Figure 4 shows that the PDF of the fourth mode is not well represented, this is due to the fact that in this mode the second accelerometer is located near a point where the mode shape is zero. Figure 4 shows that the modal frequency obtained from FEM model varies according to the probability density function derived from experimental data. This fact may be related to variability in test conditions, for example, position of the accelerometers, support stiffness, variation in the excitation force or even unmodeled dynamics, therefore, it is necessary to perform a model calibration that can be adjusted to the experimental data.

Model calibration can be performed using Bayesian inference, which requires a characterization of the error between the model and the experimental data. Then, for model calibration analyses using the TDD technique it is possible to approximate the uncertainties of the estimator to a Gaussian model. However, the parameters of the approximate Gaussian function should be specific to each structure and especially for each accelerometers mesh, because with a larger number of sensors in the mesh the output energy correlation matrix will have more information available, allowing even estimate modal parameters of the higher modes.

7. CONCLUSIONS

This paper presents some preliminary uncertainty analysis of modal parameters estimated from TDD technique. TDD algorithm was used based on acceleration experimental data out of a simply supported beam. It was presented the first part of analysis in which these estimates were used to obtain PDFs for natural frequencies. Theoretical and FEM natural frequencies were compared, showing that neglecting the mass of the accelerometers in the numerical model can induce uncertainties related to unmodeled dynamics, brings a wrong interpretation of the physical model. In the implementation of the technique TDD shows that the number of acceleration sensors and their location in the structure can affect the estimation of modal parameters. For practical purposes of analysis and calibration of structural model from modal parameter estimation using TDD technique is possible to approximate the uncertainties of the estimator to a Gaussian model. The ongoing research of this project consists in: (i) performing covariance analysis of these estimates in order to infer about model structures for the likelihood $\pi(y|\theta)$; (ii) performing these analysis using modal estimators based on

Short Time Fourier Transforms and (iii) performing uncertainty quantification analysis in the computational model using Monte Carlo based methods.

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