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DESIGN OF STIRLING MOTORS USING MULT-OBJECTIVE OPTIMIZATION

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Abstract. *The Stirling Motor is a device that has great potential for being used in applications where energy (heat) is available in the system. As an example, a Sterling motor can use the energy available in the gases from the combustion process of an automotive engine by using exhaust manifold as hot source. The Sterling motor consists of a piston that can move along a cylinder that is fulfilled by a working fluid and a displacer installed between the hot and cold chambers. Due to the large temperature difference between the chambers, it becomes feasible to use the corresponding energy to drive the Stirling motor. For design purposes, a multi-objective problem is proposed so that the maximization of thermodynamic efficiency, the minimization of energetic loss associated with the movement of the displacer set, and the minimization of energetic cost related to the fluid displacement between the two chambers is obtained for the optimal configuration of the system. To solve this optimal design problem the Non-dominated Sorting Genetic Algorithm is used. The preliminary results demonstrated that the methodology proposed represents a promising approach for the design of Stirling Motors. The theoretical results were used to construct a prototype of a Stirling Motor for evaluating the whole design process.*

keywords: *Stirling Engine, Multi-objective Optimization, NSGA II.*

1. INTRODUCTION

The Government and civil organizations are aware of the growing energy consumption in the world, the reduction of waste and energy regeneration has been the focus of several studies to meet this need (www.ons.org.br). Store energy is no easy task, but to reuse in any way the heat that is being thrown out of a device or simply improve the efficiency of a mechanism that already exists can be possible, contributing significantly to the environment regarding the sustainability of the planet by minimizing energy consumption. In the last decade, there has been a growing interest on the part of researchers to study the products already manufactured in the industry, aiming to improve efficiency at the lowest possible cost, this thought is due to the renewal of designer who obtained access to new methodologies or the robustness of new computers that dramatically reduced the processing time of the data.

The purpose of this procedure was to obtain a project configuration of geometric parameters to build a Stirling engine in the real future beta configuration was adapted in the exhaust manifold of automotive vehicles and at the same time, to maximize the thermodynamic efficiency of the real cycle, together with the reduction of air pumping losses and inertia forces.

Numerical optimization techniques comes the complicated problems, where the mathematical models are linear, not badly behaved and multidisciplinary numerically, these have formulation and appropriate characteristics to promote the development of geometric model of a mechanism of four bar linkage associated to a Stirling engine in beta configuration. The evolution of the mechanical system design from the point of view of his cinematic and dynamic behavior and taking into account their thermal characteristics can reduce the financial cost of the project or produces a significant increase in efficiency.

The NSGA algorithm II (non-dominated sorting genetic algorithm II) will be used to optimize this project. It is known that this algorithm is an extension of the NSGA developed by Deb et al. (2000). And, the NSGA is a well-known genetic algorithm is based on non domination, much used for optimizing multi-objective is a very efficient algorithm, but receives some criticism due to its computational complexity, lack of elitism and for choosing the ideal to share parameter value parameter. The elitism is an operator that maintains the best solutions found in previous generations in later generations, thus avoiding possible candidates to great are lost. The modified version, NSGA II developed to perform a land not elitist dominance and no sharing should be chosen. In the same way that conventional AGs Lobato (2008) the NSGA II works with a parent population P to generate the population daughter Q, as conventional AGs.

2. MATHEMATICAL REPRESENTATION – CONSTRUCTION OF THE OBJECTIVE FUNCTIONS

2.1 Mechanism of four bar

The four bars or articulated quadrilateral is simple and versatile. So, it will be used as a model to represent the geometry of the Stirling engine beta configuration, because this setting presents a better efficiency, Fig. 2.1 represents the four bar mechanism.

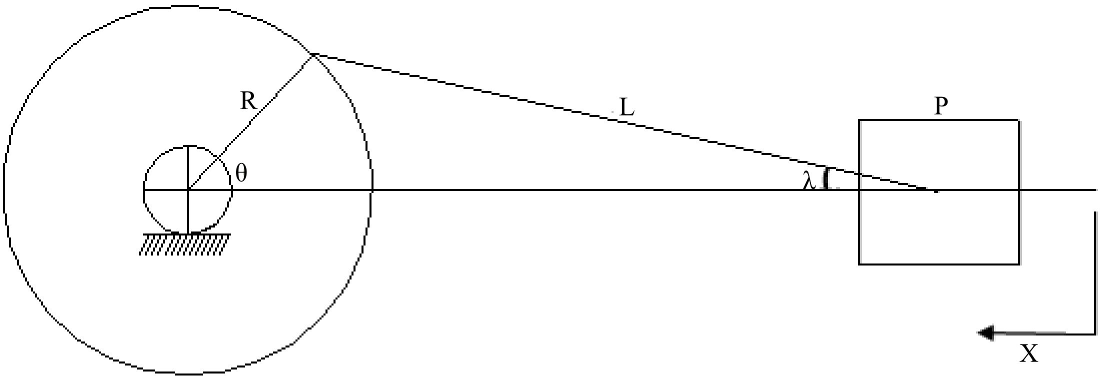

Figure 2.1 - Mechanism of four bars.

The four-bar mechanism is a bracket for fixing the device, the crank length R, the length L and the displacer rod P. Acts on the displacer gas pressure, the force is transmitted to the crank via the connecting rod. It can be observed that there are two dead points during the cycle, one in each extreme position of the piston stroke. To avoid stopping the engine in these dead spots is necessary to use a flywheel to crank supportive.

Figure 2.2 represents the geometry of the cylinder will be coupled to the four-bar mechanism.

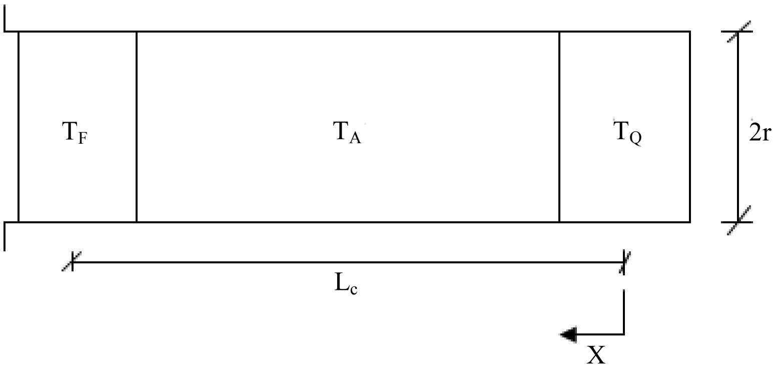

Figure 2.2 - Geometry of the cylinder.

Where:

$T_A \rightarrow$ Ambient temperature $\rightarrow$ Measure / established

$T_Q \rightarrow$ Temperature in the chamber hot of the cylinder $\rightarrow$ Admitted constant and originating from external heat source

$T_F \rightarrow$ Temperature in the chamber cold of the cylinder $\rightarrow$ Constant admitted by the hypothesis that the cylinder as a whole behaves like a cylindrical a flap of in continuous operation. So is the result of the temperature profile developed along the length of the cylinder (flap), with:

$L_c \rightarrow$ Distance that separates the center of the hot and cold chambers of the cylinder

$R \rightarrow$ Length of the crank $=$ Half of the piston stroke

$L \rightarrow$ Length of the conrod

$\theta \rightarrow$ Angular position of the crank

$X \rightarrow$ Piston displacement starting from the PMS toward PMI

$r \rightarrow$ Outer radius of the cylinder

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2.3 Formulation of the optimization problem

2.3.1 Objectives

• Maximize the work carried out by the motor in a cycle → Maximize thermodynamic efficiency
• Minimizing energy consumption with the movement of the displacer
• Minimize energy consumption with the displacement of the air between the hot and cold chambers

2.3.2 Considerations on the part thermal of the problem

Considering the fin as a cylindrical cylinder, the temperature profile which develops from the hot chamber and along its length which has:

$$\frac{T_F - T_A}{T_Q - T_A} = \frac{\cos h\left[m\left(L_C - X\right)\right]}{\cos h(m L_C)} \tag{2.1}$$

Where:

T_A and T_Q are laid down

$L_C \rightarrow$ Overall length of the flap = Distance that separates center of the hot and cold chambers

$X \rightarrow$ Position where you want to calculate the temperature along the flap (cylinder wall) as from the hot chamber equal piston displacement

For the calculation of: $T_F \rightarrow T(X)$ being, $X = L_C$.

So for the calculation of T_F:

$$\frac{T_F - T_A}{T_Q - T_A} = \frac{\cos h\left[m\left(L_C - X\right)\right]}{\cos h\left(m L_C\right)} \rightarrow \frac{T_F - T_A}{T_Q - T_A} = \frac{1}{\cos h\left(m L_C\right)} \tag{2.2}$$

$$T_F - T_A = \frac{T_Q - T_A}{\cos h\left(m L_C\right)} \rightarrow T_F = \frac{T_Q - T_A}{\cos h\left(m L_C\right)} + T_A \tag{2.3}$$

Being that:

$$m = \sqrt{\frac{hp}{Ak}} = \sqrt{\frac{2h}{kr}} \tag{2.4}$$

Where:

$h \rightarrow$ Coefficient of heat transfer by convection
$p \rightarrow$ Outermost perimeter of the flap (cylinder)
$A \rightarrow$ Cross sectional area of flap (cylinder)
$k \rightarrow$ Thermal conductivity of cylinder material
$r \rightarrow$ Outer radius of the cylinder

We know that the greater the length of the cylinder L_C (and hence of the displacer) is lower T_F for the same value of T_Q e T_A, ie better will be thermodynamic efficiency of the cycle theoretical the air $T_F = f\left(T_Q, L_C, T_A\right)$ e $\eta = f\left(T_F, T_Q\right)$.

However, the higher L_C, greater the inertia of the displacer, and consequently, energy expenditure to its displacement will also increase. Besides this, the work consumed in the displacement (pumping) air (operant fluid) between the hot and cold chambers will be large due to the greater distance to be traveled by the air in the same time interval.

2.4 Considerations on the objective 1

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As from the equation of the theoretical yield of cycle thermodynamic of Carnot into the air $\eta = \left(1 - \frac{T_F}{T_Q}\right)$, obtain an equation equivalent for the actual Stirling thermodynamic cycle, taking into account the phase angle $(\varnothing)$ between the cranks of the piston of the displacer.

$$F_1 = \eta = f\left(T_F, T_Q, \varnothing\right) \tag{2.5}$$

Being that: $T_F = f\left(L_C, T_Q, T_A\right)$ (2.6)

Admitting two indices of performance that will be multiplied by the yield theoretical ideal and each of these performance indices will be related to approach or spacing between the ideal conditions cycle where the real air circulation between the hot and cold chambers is made through a displacer and its phase angle with respect to crank of the cursor.

2.4.1 Kinematics of mechanisms - Optimizing a four-bar mechanism: connecting rod, crank and cursor

We have a mechanism in Fig 2.1 cursor-crank-connecting rod (MCMB) which will be used as a benchmark to arrive in an objective function that represents the mechanism is important salutary that it is about a model with its due simplifications.

$$X = R + L - R\cos\theta - L\cos\gamma \tag{2.7}$$

$$X = R\left(1-\cos\theta\right) + L(1 - L\cos\gamma) \tag{2.8}$$

$$X = R\left(1-\cos\theta\right) + L\left[1 - \sqrt{1-(R/L)^2\sin^2\theta}\right] \tag{2.9}$$

Simplifying eq. (2.9) the radical can be approximated replacing it with according to the series

$$\left(1 \pm B^2\right)^{\frac{1}{2}} = 1 \pm \frac{1}{2}B^2 - \frac{B^4}{2.4} \pm \frac{1.3B^6}{2.4.6} - \frac{1.3.B^8}{2.4.6.8} \pm \ldots \tag{2.10}$$

Where, $B = (R/L)\sin\theta$.

In general the use of the two first terms of the series already provides sufficient accuracy. So:

$$\sim \sqrt{1-\left(\frac{R}{L}\right)^2 \sin^2\theta} = 1 - \frac{1}{2}\left(\frac{R}{L}\right)^2 \sin^2\theta \tag{2.11}$$

$$X = R\left(1-\cos\theta\right) + \frac{R^2}{2L}\sin^2\theta \tag{2.12}$$

$$X_{min} = 0 \text{ to } \theta = 0$$

$$X_{max} = 2R \text{ to } \theta = \pi = 180^0$$

Considering the constant speed of rotation we have: $\theta = \omega t$ because ω it is constant, therefore $\omega = \dot{\theta} = $ constant.

$$V = \frac{dx}{dt} = \dot{X} = R\omega\left(\operatorname{sen}\theta + \frac{R}{2L}\operatorname{sen}2\theta\right) \tag{2.13}$$

$$A = \frac{d^2x}{dt^2} = \ddot{X} = R\omega^2\left(\cos\theta + \frac{R}{L}\cos 2\theta\right) \tag{2.14}$$

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Defined L, R and $\omega \rightarrow X = f(\theta) \rightarrow \dot{X} = \dot{f}(\theta) \rightarrow \ddot{X} = \ddot{f}(\theta)$

For the piston $\rightarrow \begin{cases} X_p = X_{piston} = f(\theta) \\ \dot{X}_p = \dot{X}_{piston} = \dot{f}(\theta) \\ \ddot{X}_p = \ddot{X}_{piston} = \ddot{f}(\theta) \end{cases}$

For the displacer $\rightarrow \begin{cases} X_D = X_{displacer} = f(\theta+\varnothing) \\ \dot{X}_D = \dot{X}_{displacer} = \dot{f}(\theta+\varnothing) \\ \ddot{X}_D = \ddot{X}_{displacer} = \ddot{f}(\theta+\varnothing) \end{cases}$

Where $\varnothing$ phase angle between cranks the piston and displacer in the configuration β of Stirling engine, being imposed a side constraint for $\varnothing$ such that $45^0 \leq \varnothing \leq 135^0$, aiming to find the best lag.

2.4.2. Calculation of the performance index related to the expansion

- During the expansion is desired that the air stay in the hot runner whole, ie that displacer stay close piston.
- To expansion $0 \leq \theta \leq 180^0$.
- Trace $X_p = f(\theta)$ to $\varnothing$ varying of 1º between 0º e 360º (store the vector).
- Trace $X_D = f(\theta+\varnothing)$ to θ varying of 1º between 0º e 360º (store the vector).
- $X_D < X_p \forall \theta\,(0^o$ to $360^o)$, otherwise, $X_D > X_p$ means that the displacer collided with the piston (or the has exceeded) what can not happen.
- Calculate the vector $\Delta X = X_D - X_p$.
- Determine the biggest positive value for $\Delta X = \Delta X_{Max}$.
- Calculate the new vector $X_{DN}\left(1^o \text{ to } 360^o\right) = X_D - \Delta X_{Max}$.
- Calculate the performance index in the expansion in the range of $0 \leq \theta \leq 180^0$ how:

$$I_E = \sum_{\theta=1^0}^{180^0} 1 - \left(\frac{\boldsymbol{X}_P - \boldsymbol{X}_{DN}}{\boldsymbol{X}_P}\right) \tag{2.15}$$

Physical meaning: evaluates the percentage of the volume of air that is in the hot runner as a function of the relative position of the displacer piston in relation to the phase angle due to the $\varnothing$, is shown in Fig. 2.3.

For preview only the correction of the displacer it is assumed that R = 0.0416 m and L = 0.0834 m. Because, so far there not are known the values of R and L. The graph associated with the correction of the displacer is shown in Fig. 2.3.

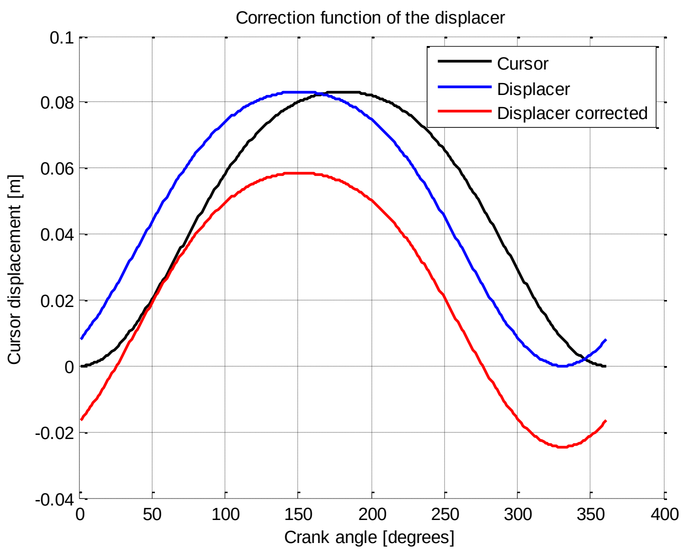

Figure 2.3 – Correction function of the displacer.

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2.4.3 The calculation of the performance index related to the compression

During compression, it is desired that the air remain completely in the cold chamber, ie, that displacer remain far from piston (ideally X_D = constant = 0), for compression $180^0 \leq \theta \leq 360^0$. From X_p and X_{DN} determined at the previous step the compression performance index where:

$$I_C = \sum_{\theta=180^0}^{360^0} 1 - \left(\frac{\boldsymbol{X}_{DN}}{\boldsymbol{X}_P} \right) \tag{2.16}$$

Physical meaning: Measures the position of each θ evaluated the percentage of the volume of air that is in the cold chamber a function of position of displacer piston in relation to the phase angle due to $\varnothing$.

Approximation of the objective function F_1, by:

$$F_1 = \eta = f\left(I_E + I_C\right) = \left(1 - \frac{T_F}{T_Q}\right)\left(I_E + I_C\right) \tag{2.17}$$

2.5 Considerations on the objective 2

We know that L_C (maximum length of the hot and cold chambers (established)) large, greater the inertia of the displacer and consequently, the energy expenditure for its displacement will also increase. Therefore was established a function related to the work carried out to movement of the displacer.

$$F_2 = f\left(m_d, a_d, 2R\right) \tag{2.18}$$

Where:

$$\left.\begin{matrix} m_d \rightarrow \text{Mass of thedisplacer} \\ a_d \rightarrow \text{Acceleration of thedisplacer} \end{matrix}\right\} F_d \rightarrow \text{Force acting on the displacer.}$$

To calculate the work carried out by this force should be made at in full of $\vec{F_d} \cdot \vec{d_X}$ over a full cycle, estimating the mass of the displacer due to the amount length of the cylinder (L_{TC}) and the distance that separates the center of the hot and cold chambers of the cylinder (L_{CM}). Where: $s = L_{TC} - L_{CM}$, the wall of the displacer u = 0,002 m and the material density of displacer $\rho = 2,7$.

So:

$$m_d = \rho\pi\left\{\left(\left(0,9r\right)^2 s\right) - \pi\left[\left(0,9r\right) - u\right]^2 \left[s - \frac{s}{2} - u\right]\right\} \tag{2.19}$$

Utilizing an alternative suggestion for implementation of this function can approximate the force that is acting on the displacer by the maximum value of the acceleration. Then:

$$V = \frac{dx}{dt} = R\omega\left[\text{sen}\theta + \frac{R}{2L}\text{sen}2\theta\right] \tag{2.20}$$

$$a_d = \frac{d^2x}{dt^2} = R\omega^2\left(\cos\theta + \frac{R}{L}\cos 2\theta\right) \tag{2.21}$$

To $a_{d_{máx}} \rightarrow \frac{d_a}{d_\theta} = 0.$ Where:

$$\frac{d_a}{d_\theta} = R\omega^2\left(\frac{-\text{sen}\theta - R}{L\,\text{sen}\,2\theta.2}\right) = 0 \tag{2.22}$$

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$$\frac{d_a}{d_\theta} = -R\omega^2\left(\operatorname{sen}\theta + \frac{2R}{L}\operatorname{sen}2\theta\right) = 0 \tag{2.23}$$

Once $R = 0$ or $\omega = 0$ implies in displacement mechanism no longer exists or may stand still, this does not represent a viable solution, then:

$$a_{max} \rightarrow \sin\theta + \frac{2R}{L}\sin 2\theta = 0 \tag{2.24}$$

The eq. (2.24) has solution for $\theta = 0$. Therefore:

$$a_{max} = R\omega^2\left(\cos 0 + \frac{R}{L}\cos 0\right) \tag{2.25}$$

$$a_{max} = R\omega^2\left(1 + \frac{R}{L}\right) \tag{2.26}$$

So, the approximation of the objective function F_2, will be:

$$F_2 = m_d\, a_{máx}\, 2R \tag{2.27}$$

Breaking apart the objective function F_2**,** it follows that:

$$F_2 = \rho\pi\left\{(0,9r)^2\, s - (0,9r - u)^2\left[s - \frac{s}{2} - u\right]\right\}R\omega^2\left(1 + \frac{R}{L}\right)2R \tag{2.28}$$

2.6. Considerations on the objective **3**

Establishing a function related to work carried out against the drag force (frictional viscous fluid dynamic) necessary for the displacement of the air between the hot and cold chambers.

$$F_3 = f\left(c_d, L_C, \dot{X}\right) \tag{2.29}$$

To: $c_d \rightarrow$ Aerodynamic coefficient of drag of the displacer to move against air into the cylinder (operant fluid).

To calculate the work done by this force should be made an integral of $\vec{F}_{ar} \cdot \vec{d}_X$ over a complete cycle. The alternative way adopted to represent the objective function was to estimate the drag coefficient c_d (dimensionless), and program the function towards closer the drag force that is acting on the displacer by the maximum value of the speed.
Thus:

$$V = R\omega\left(\sin\theta + \frac{R}{2L}\sin 2\theta\right) \tag{2.30}$$

Where $V_{max} \rightarrow \frac{d_v}{d_\theta} = 0$ it follows that:

$$\frac{d_v}{d_\theta} = R\omega\left(\cos\theta + \frac{R}{2L}\cos 2\theta(2)\right) = 0 \tag{2.31}$$

$$\frac{d_v}{d_\theta} = R\omega\left(\cos\theta + \frac{R}{L}\cos 2\theta\right) = 0 \tag{2.32}$$

Once $R = 0$ or $\omega = 0$ implies on displacement mechanism no longer exists or is stopped, this is not a viable solution, then:

$$V_{max} = \cos\theta + \frac{R}{L}\cos 2\theta = 0 \tag{2.33}$$

But, $\cos 2\theta = 2\cos\theta^2 - 1$. Therefore:

$$\frac{R}{L}\left(2\cos\theta^2 - 1\right) + \cos\theta = 0 \tag{2.34}$$

$$V_{max} \rightarrow \frac{2R}{L}\cos\theta^2 + \cos\theta - \frac{R}{L} = 0 \text{ Equation of the 2nd degree } \cos\theta. \tag{2.35}$$

Established R and L, resolves itself the eq. 2.35 to discover the value of θ wherein V_{max} occurs. To calculate V_{max} just replace this value θ in equation. The following equation represents the velocity vector of the displacer for each θ.

$$V_{max} = \dot{X} = R\omega\left(\sin\theta + \frac{R}{2L}\sin 2\theta\right) \tag{2.36}$$

Therefore, the objective function F_3 approximation will be:

$$F_3 = c_d (V_{máx})^2 2R\pi(0,9B(6))^2 \tag{2.37}$$

2.7 Multi-objective function

Normalize the values of the magnitudes for:

$$F = F_1 + F_2 + F_3 = F_{1n} + F_{2n} + F_{3n} \tag{2.38}$$

Establishing a "weight" for each of the functions to compose the multi objective to be minimized, it follows that:

$$F = c_1 F_{1n} + c_2 F_{2n} + c_3 F_{3n} \tag{2.39}$$

3. Multi-objective function

In real-world problems, like manufacturing of a freezer, in which such criteria as low power consumption, higher capacity to withdraw heat are desired there are more than an objective function to be optimized and these criteria are conflicting and have to be treated simultaneously. These are problems that involve simultaneous treatments and conflicting criteria are called multi-objective or multi that can be reached with optimization, there is no one solution, ie, there are numerous optimal solutions leads to the need to use the concept "multi collective "called Pareto optimal.

Though, the objective of this project is to optimize the geometric parameters in the construction of a beta configuration Stirling engine that was adapted in the future in the exhaust manifold of an automotive vehicle, in this work has implemented the three objective functions and six design variables with its restrictions.

The justification of applying of the methods NSGA II and SQP_{weight} is to validate the results of obtained after the execution of each of these methods, due to lack of mathematical calculation but rather a random selection.

3.1 Structure of the algorithm NSGA II

The algorithm NSGA II presents the following framework:

Input parameters:
Population father (P), Population daughter (Q), Fixed size for P of boundary j (F_j), Maximum number of generations (nMax) and Number of current generation (n).
(1) Generate the initial population P_0 and $Q_0 = \{\}$;
Use n = 0
(2) Perform selection, crossover and mutation to generate daughter Q_0. Use $R_n = P_n \cup Q_n$;
(3) Perform sorting by dominance in not R_n;
(4) $P_{n+1} = \{\}$;
(5) $\left|P_{n+1} + F_j\right| \leq N$, copy solutions of F_j in P_{n+1};
(6) Calculate the distances from the crowd at F_j, order F_j according to the distances d_j and copying the first $N-|P_{n+1}|$ solutions F_j to P_{n+1};

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(7) Apply selection, crossover and mutation into the new population Q_{n+1};
(8) If n > nMax so stop, otherwise assign n = n+1 and return to the second step.
Output: Solutions nondominated.
(Reprinted from Lobato (2008)).

It was used in the algorithm NSGA II the following design variables to maximize the thermodynamic efficiency, minimize energy consumption with the displacer movement and minimize losses by pumping air, ie, decrease the work performed by displacer against the the drag force (viscous friction fluid dynamics) necessary to displacement of the air between the hot and cold chambers:

B(1) = L $\rightarrow$ Length of the connecting rod
B(2) = B $\rightarrow$ Angle of lag
B(3) = R $\rightarrow$ Length of the crank
B(4) = Ltc $\rightarrow$ Full length of the cylinder
B(5) = Lcm $\rightarrow$ Distance between the center of the hot and cold chambers of the cylinder
B(6) = r $\rightarrow$ Outer radius cylinder

The limits used for the design variables are in the NSGA II:

$X_{min} = [0,11 \quad \pi/4 \quad 0,04 \quad 0,21 \quad 0,03 \quad 0,02] \rightarrow$ Lower limit of the design variables;

$X_{max} = [0,33 \quad 3\pi/4 \quad 0,15 \quad 0,65 \quad 0,15 \quad 0,10] \rightarrow$ Upper limit of the design variables;

From these data, computer code implemented in the environment MATLAB®, was possible to construct the graphs of Fig. 3.1 and 3.2 which highlights the geometrical behavior of F1/F2, F1/F3, F2/F3 and the behavior of three-dimensional geometric F1, F2 and F3 using the parameters found by the algorithm NSGA II. After, the acquisition of 150 optimal results for the project variables and its objective functions of each of the first results of 150 was chosen for the construction of prototype Stirling engine real. Thus, this contemplates the physical space in the existing automotive vehicle for coupling to the Stirling engine exhaust manifold. Being that there are other results that can be used in the prototype with different dimensions depending on the need of the designer.

Table 5.1 - Some solutions found using the algorithm NSGA II

F(1)	F(2)	F(3)	B(1)	B(2)	B(3)	B(4)	B(5)	B(6)
-357,31	1,83e-4	1,06e-5	0,299m	1,46 rad ou 83º39'27"	0,04m	0,392m	0,0497m	0,02m

Where:
Distance of crowd = 1,000
Solutions of extremity 3,229e-002

In the Fig. 3.3 are the graphics post-processing algorithm NSGA II These points were selected because the value of the total length of the cylinder contemplate the physical space existing in the automotive vehicle for the coupling Stirling engine to the exhaust manifold. The post-processing that highlights the optimum functions F1, F2 and F3 for two and three dimensional geometric behavior.

The Tab.3.2 presents the numeric values utilized for points A, B and C that represents F1, F2, F3, B(1), B(2), B(3), B(4), B(5) and B(6), respectively. Were used the points 1, 9 and18 to conduct post-processing. Being that these points were chosen from 150 optimal values for the objective function and the design variables of the NSGA II.

However, the points A and B cited the Tab.3 were excluded because the objective function that represents the thermal part had values below the point C.

Table 3.2 – Optimal solutions to the points A, B and C, respectively.

	F(1)	F(2)	F(3)	B(1)	B(2)	B(3)	B(4)	B(5)	B(6)
A	-316,30	9,83e-5	1,56e-5	1,10m	1,43 rad ou 81º44'53"	0,04m	0,21m	0,060m	0,02m
B	-346,98	1,46e-4	1,14e-5	2,18m	1,44 rad ou 82º40'46"	0,04m	0,34m	0,086m	0,02m
C	-357,31	1,83e-4	1,06e-5	0,30m	1,46 rad ou 83º39'27"	0,04m	0,39m	0,050m	0,02m

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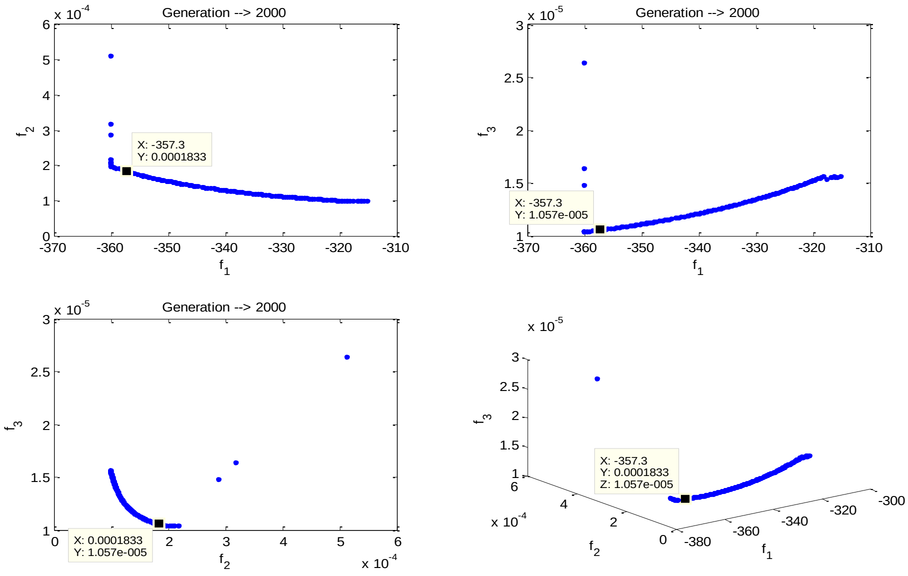

Figure 3.1 - Graph of f_1 versus f_2, f_1 versus f_3 e f_2 versus f_3.

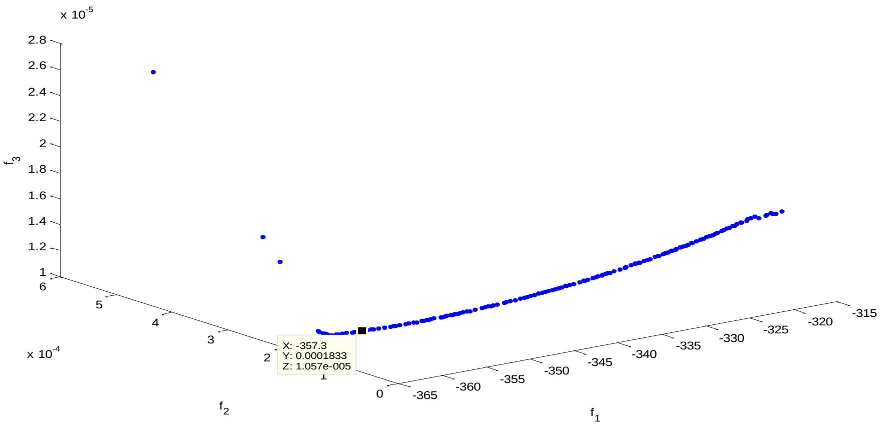

Figure 3.2 - Global solution Pareto frontier.

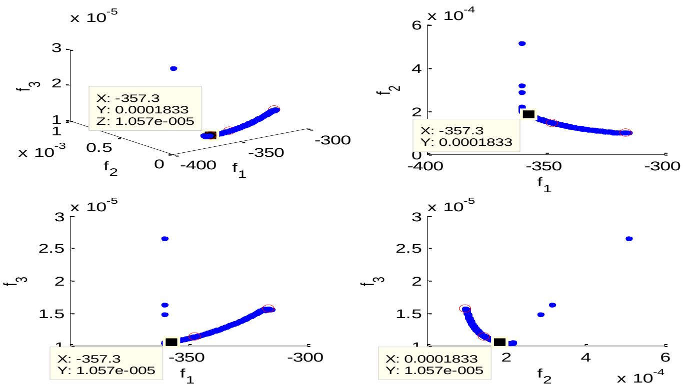

Figure 3.3 – Post-processing of results.

3.2 Multi-objective function using the method SQP_{weight}

The limits used for the design variables in the SQP_{weight} are the same NSGA II. The design variables are the same to find the best values of the objective functions to be prestaged separately in multi-objective function for the method SQP_{weight}. Thus, an estimate of match was held, i.e. used one hundred (100) points Random belonging to X_0 and the

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lower and upper bounds given. After each execution got certain amount of iterations with acquiring one thousand (1000) optimal results for the objective function and the design variables.

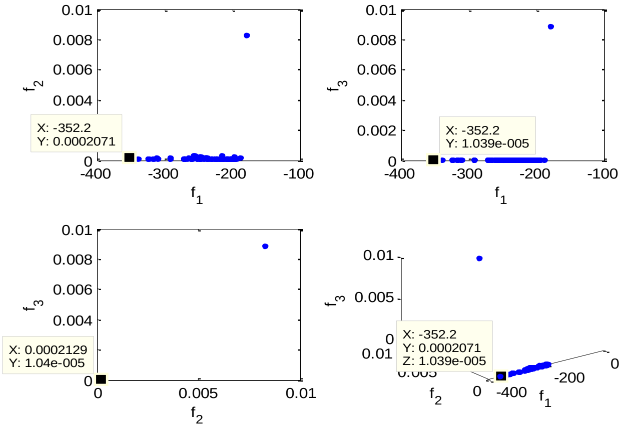

Figure 3.4 – Graphs of the results of the convergence of multi-objective function in the SQP_{weight}.

The Tab.3.3 shows the best results found by the SQP_{weight} for each of the objective functions with their respective weights (P) and the six design variables.

Tabela 3.3 – Best results found by the SQP_{weight} for the multi-objective.

P F(1)	P F(2)	P F(3)	F(1)	F(2)	F(3)	B (1)	B (2)	B (3)	B (4)	B (5)	B(6)
0,00	0,00	1,00	-352,2	2,07e-4	1,04e-5	0,33*m*	1,702rad ou 97º31'27"	0,04*m*	0,44*m*	0,056*m*	0,02*m*

3.2. Comparing methods **NSGA II *e* SQP_{weight} para função multi objetivo**

Comparing the methods for the multi-objective function, and they SQP_{weight} and NSGA II the results remained very close and assuming the data generated and set as standard showed little difference as indicated in Tab. (3.4 e 3.5) .

Table 3.4 – Results of different techniques for the objective functions.

	F(1)	*F(1)*	*F(1)*
SQP_{weight}	-352,2	2,0708e-4	1,039e-5
NSGA II	*-357,3*	*1,83e-4*	*1,06e-5*

Table 3.5 – Results of different techniques for the design variables.

	B(1)	*B(2)*	*B(3)*	*B(4)*	*B(5)*	*B(6)*
SQP_{weight}	0,33*m*	*1,702*rad ou *97°31'27"*	0,04*m*	0,44*m*	0,056*m*	0,02*m*
NSGA II	*0,299m*	*1,46 rad ou 83º39'27"*	*0,04m*	*0,392m*	*0,050m*	*0,02m*