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TOWARDS THE DESIGN OF COMPOSITE REINFORCED PANELS USING PARAMETRIC AND TOPOLOGY OPTIMIZATION

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Abstract. *This work investigates optimization techniques applied to composite panels with co-cured stiffeners, among them, topological and parametric techniques are investigated. Reinforced panels are widely used in the aircraft industry to confer resistance under in-plane and out-of-plane loads for lightweight structural elements that are employed in typical aircraft structures such as fuselages. Through parametric optimization, stringers geometric dimensions, positioning, and also the stacking sequence of laminated composite material employed for the panel and stringers layups are optimized. The optimization problem is formulated as the maximization of the panel buckling load obtained through commercial Finite Element software (Abaqus), subjected to constraints such as mass, maximum allowable strains, and stacking order of the laminate. As a contribution, the Discrete Material Optimization (DMO) method is used to parameterize the laminate orientation variables to reduce the occurrence of local minima in the solution found by the optimizer, the Method of Moving Assimptotes (MMA) algorithm. The results obtained through the proposed methodology are compared with those of a representative reinforced panel with typical stacking sequence and topology, and then the advantages and disadvantages of this methodology is presented.*

Keywords: *Parametric optimization, Buckling optimization, Composite structures, Discrete Material Optimization, Topology Optimization, Reinforced panel*

1. INTRODUCTION

Mechanical structures that requires lightweight constructions are a great field of application of unstiffened and stiffened shell structures made of fiber composite materials. The high stiffness and strength to weight ratios, high design flexibility and increasing manufacturing techniques availability at the same time that enables the wide use of fiber composite materials, add a complexity degree in the design phase of the structure. The variable number increases and then, a systematic approach to design this kind of structure helps to reduce the weight while respecting minimum performance requirements. This work explores different approaches and presents a methodology for the design of composite reinforced panels subjected to buckling while respecting manufacturing and failure criteria constraints.

Several authors have been working with optimization of composite material structures. Seibel *et al.* (1998) presents the optimization and experimental characterization of carbon fiber reinforced panels (CFRP) axially reinforced subjected to buckling loads. Potgieter and Stander (1998) analyze and compare the use of evolutionary optimization algorithms such as Genetic Algorithm for the stiffness maximization of laminated plates. Liu *et al.* (2004) discuss the buckling load maximization of composite panels without reinforcements using flexural lamination parameters, in an approach that evaluates the buckling load and laminated parameters analytically, comparing the results with those obtained from the genetic algorithm method. In Liu *et al.* (2008) a two-level optimization approach is applied to reinforced composite panels subjected to compression and lateral pressure.

The application of optimization involving laminated composite materials inside multi-disciplinary fields such as, for an example, aircraft industry, is very large. Neufeld *et al.* (2010) apply the optimization to the design of composite aircraft wing-box considering aerodynamic models to maximize the Lift to Drag ratio of the coupled aerodynamic and structural problem. Gasbarri *et al.* (2009) decompose the optimization in different levels, in order to evaluate a greater global problem (for an example, the wing flutter velocity) with different detailing levels considering smaller structure variables such as local fiber orientations. Other works such as the one written by Kaufmann *et al.* (2009) explores the manufacturing and operational costs together with the structural design of composite aircraft structures.

This paper is organized as follows. In Section 2, the finite element formulation applied to the structure is introduced.

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In Section 3, the optimization problem formulation is presented with its numerical implementation described in section 4. Section 5 shows preliminary results of numerical examples of the proposed methodology for design of composite reinforced panels. Section 6 presents the concluding remarks of this work.

2. FINITE ELEMENT MODELING

2.1 Parametric Optimization

In order to evaluate the structural behavior of composite panels under combined pre-load and buckling loads, it is employed the commercial finite element software Abaqus. The choice of a commercial solver instead of a “in-house" one represents an advantage in terms of analysis quality, flexibility and reliability, expanding the application of the methodology proposed by this work. However, as a trade-off, the numerical implementation is more restricted and relies on the use of the finite differences method for sensitivity calculations. For the parametric optimization case, this is a trade-off that it’s possible to accept, and then the Abaqus solver is selected for this work.

It was employed a shell element finite element mesh (see Fig. 5 for an example) that is controlled by the solver input card, created inside Matlab environment. As the laminate theory considered in this work was the Classic Lamination Theory, the four-node elements S4 with full integration was used, avoiding an unnecessary computational cost by the use of eight-node finite elements.

3. OPTIMIZATION PROBLEM FORMULATION

3.1 Parametric Optimization

The design variables chosen for the parametric optimization are shown in Fig. 1. The stringer topology is fixed, however, it may be changed with minor modifications during the optimization, through a different geometry parametrization. In the example cases for this work, a “I-shape” topology is considered. The three main dimensions of the stringers (b_f, b_w, b_a) and their positioning inside the panel (p) are design variables. The material lay-up of the panel and stringers is also subject of optimization. In this case, the layers fiber orientation and also the layer number in the lay-up are optimized.

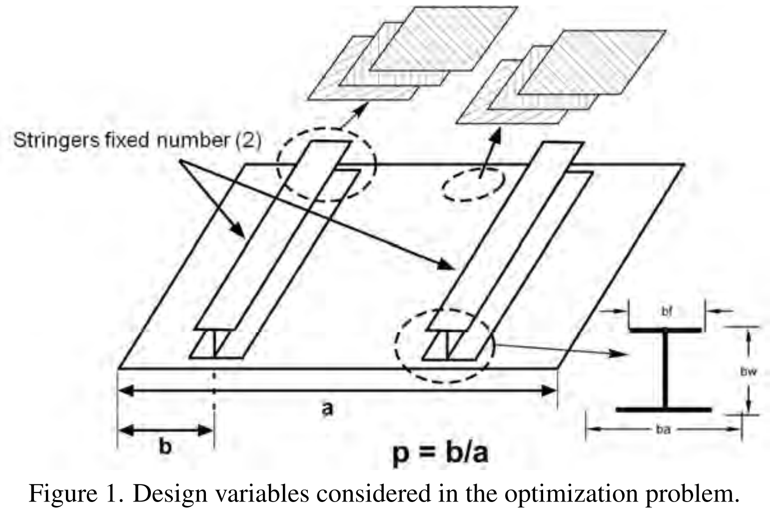

Figure 1. Design variables considered in the optimization problem.

Many different pre-load and buckling loads can be considered in the optimization. In this work, compression, shear and pressure loads may be applied, as also load combinations of these three load types. This is an advantage of employing a commercial finite element solver for the analysis, since it is more versatile and it can support a more wide range of load possibilities than “in-house” solvers, helping the application of this optimization methodology proposed in out-of-academy cases. Figure 2 presents the load types that may be used in this example optimization use.

As the most of reinforced panel structures are subjected to buckling due its geometry, the objective function in this work aims to reduce the mass of the reinforced panel subjected to buckling. This is achieved by using a well-posed optimization problem of buckling load (eigenvalue) maximization subjected to a mass constraint. In order to take into account failure criteria in the optimization, a maximum allowable strain constraint is applied. It is chosen because the allowable strain is easier to obtain experimentally and it is a homogeneous criteria, avoiding problems such as load dependent results, achieved by the use of non-homogeneous criteria in optimization such as Tsai-Wu criteria. Groenwold and Haftka (2006) compare the application of different failure criteria in optimization and its consequences.

Some manufacturing considerations are applied directly in the optimization problem. First, only symmetric and balanced laminated sequences can be selected in the design domain, avoiding the coupled torsion-bending effect as well as

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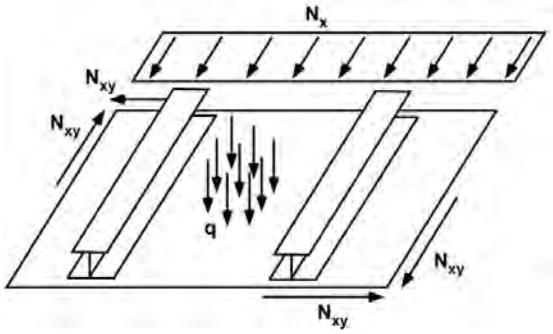

Figure 2. Loads that can be considered in the optimization problem.

the thermal expansion problem. Besides that, only the $0°, +45°, -45°, 90°$ fiber angles are considered feasible in the lay-up. It is important to observe that these design domain constraints are applied due to manufacturing requirements and it can be different depending on the manufacturing process.

Maximum tensile, compressive and shear strain constraints are inserted into the optimization problem, in order to avoid solutions that might lead to unfeasible designs due to failure criteria violation. The strain failure criteria is employed since its allowable values are easily obtained from public domain test results contained in Tomblin *et al.* (2002). Equations 1 through 3 present these strain constraints.

$$-\epsilon_{adm}^{c,1} < \epsilon_{11}^{r} < \epsilon_{adm}^{t,1} \quad (1)$$

$$-\epsilon_{adm}^{c,2} < \epsilon_{22}^{r} < \epsilon_{adm}^{t,2} \quad (2)$$

$$-\epsilon_{adm}^{12} < \epsilon_{12}^{r} < \epsilon_{adm}^{12} \quad (3)$$

where ϵ_{11}^r, ϵ_{22}^r, and ϵ_{12}^r are, respectively, the normal plane 11, 22, and shear plane strains observed in the sub domain r, $\epsilon_{adm}^{t,1}$, and $\epsilon_{adm}^{t,2}$ are the maximum admissible tensile strain in longitudinal and transverse directions, $\epsilon_{adm}^{c,1}$ and $\epsilon_{adm}^{c,2}$ are the same values, however, for compressive strains, and ϵ_{adm}^{12} the maximum admissible shear strain.

Additionally to these strain constraints, a mass constraint is added in order to allow the weight reduction of the design. The lay-up sequence also receive constraints. First, the thickness of each layer is held constant through the composite specified maximum layer quantity. However, the number of layers active in the final design is variable, since the Discrete Material Optimization method (explained in section 4.) can remove from the lay-up certain layers. This feature adds automatically an upper bound in the lay-up layer number.

Another lay-up sequence constraint used in this work is the inequality proposed by Zein *et al.* (2012) and summarized in Eq. 4. Based on manufacturing issues, it avoids that void layers could be inserted between two consecutive layers, such as the example shown in Fig. 3.

$$S_{u_i^r}^{0} + S_{u_i^r}^{45} + S_{u_i^r}^{-45} + S_{u_i^r}^{90} \leq S_{u_{i-1}^r}^{0} + S_{u_{i-1}^r}^{45} + S_{u_{i-1}^r}^{-45} + S_{u_{i-1}^r}^{90} \quad (4)$$

where $S_{u_i^r}^{\theta}$ is a binary variable (0 or 1 values are the only possible values) that represents the existence of a θ fiber orientation in layer i, inside a sub region r of the laminate.

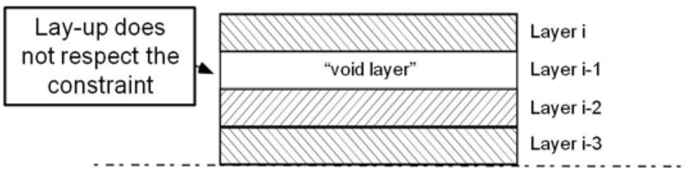

Figure 3. Example that does not respect the lay-up constraint.

In order to satisfy the optimization algorithm input vectors magnitude values and to avoid numerical instability issues, a normalization is necessary for the strain constraint values. Equation 5 exemplifies this normalization for the maximum longitudinal tensile strain, where the strains values are multiplied by 1000. After this procedure, all constraints independently of its absolute strain value (usually of 10^{-3} magnitude order), will be in the 1 to 100 range. This kind of representation is used for the results display in section 5of this paper.

$$\epsilon_{11}^{t,Norm}(x) = \epsilon_{11}^{r} - \epsilon_{adm}^{t,1} \quad (5)$$

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Concluding, the optimization problem formulation can be summarized as in Eq. 6. The optimization method adopted for the solution of this problem is the Method of Moving Assymptotes (Svanberg, 1987), (Svanberg, 2001).

$$\begin{aligned}
\underset{b_w,b_a,b_f,p,\theta_i^r,n^r}{\text{Maximize}} \quad & \lambda_{cr} \\
\text{subjected to} \quad & m \leq m_0 \\
& -\epsilon_{adm}^{c,1} < \epsilon_{11}^r < \epsilon_{adm}^{t,1} \\
& -\epsilon_{adm}^{c,2} < \epsilon_{22}^r < \epsilon_{adm}^{t,2} \\
& -\epsilon_{adm}^{12} < \epsilon_{12}^r < \epsilon_{adm}^{12} \\
& S_{u_i^r}^0 + S_{u_i^r}^{45} + S_{u_i^r}^{-45} + S_{u_i^r}^{90} \leq n_{max} \\
& S_{u_i^r}^0 + S_{u_i^r}^{45} + S_{u_i^r}^{-45} + S_{u_i^r}^{90} \leq S_{u_{i-1}^r}^0 + S_{u_{i-1}^r}^{45} + S_{u_{i-1}^r}^{-45} + S_{u_{i-1}^r}^{90}.
\end{aligned} \tag{6}$$

4. NUMERICAL IMPLEMENTATION

4.1 Discrete Material Optimization

The composite fiber angle optimization trough continuum variables optimization techniques creates a non-convex problem, therefore being susceptible to multiple local minimum, as shown by Stegmann and Lund (2005) and Lund (2009). These works propose a different optimization methodology, applicable to optimization of discrete values parameters, called Discrete Material Optimization (Stegmann, 2004).

This technique is based on the effective material property calculation, through the weighted sum of the different candidate materials properties. For the composite laminate materials, this may be done by the elastic property matrix C_b, that equals to the weighted sum of the constitutive matrices C_i, $i = 1, ..., n^c$, representing the same material, however, with fibers orientated in different discrete angles θ_i. This idea is applied in Eq. 7 and 8.

$$\mathbf{C}_b = \sum_{i=1}^{n^c} w_i \mathbf{C}_i = w_1 \mathbf{C}_1 + w_2 \mathbf{C}_2 + \ldots + w_{n^c} \mathbf{C}_{n^c} \tag{7}$$

$$\mathbf{C_i} = \mathbf{T}_{\theta_\mathbf{i}}^{\mathbf{T}} \mathbf{C_0} \mathbf{T}_{\theta_\mathbf{i}} \tag{8}$$

where $\mathbf{C}_0$ is the composite constitutive matrix in the local coordinate system, $\mathbf{C}_i$ is the constitutive matrix transformed by the fiber angle orientation θ_i, w_i are the weighting functions for the orientation design variable ϑ and n^c the total number of feasible fiber angles for the laminate.

The optimization algorithm searches for the weights w_i of each constitutive matrix that improves the objective function value. In the beginning of optimization, the $\mathbf{C}_b$ matrix is a mix of all allowed matrices, however, at the end of optimization, only one of these weights will be unitary and all the others will be null. So, only one orientation will be active, indicating the optimal fiber angle for each design sub-domain and layer. The weights w_i are calculated as in Eq. 9.

$$w_i = \frac{(\vartheta_i^e)^{p_\vartheta} \prod_{j=1, j\neq i}^{n_e} \left(1 - (\vartheta_j^e)^{p_\vartheta}\right)}{\sum_{k=1}^{n^e} \left[(\vartheta_i^e)^{p_\vartheta} \prod_{j=1, j\neq i}^{n_e} \left(1 - (\vartheta_j^e)^{p_\vartheta}\right) \right]} \tag{9}$$

where p_ϑ penalizes the intermediate values of orientation design variables ϑ, inducing the optimizer to choose between the 0 or 1 value for each ϑ. Beside this fact, when some of the ϑ_i value is increased, the $1 - (\vartheta_j^e)^{p_\vartheta}$ term in the product decreases the value of the other ϑ_i, helping to avoid intermediate values in all the orientation variables.

4.2 Parametric Optimization

Figure 4 shows the flowchart of the parametric optimization implemented. It starts with the parametric finite element model creation, with the analysis being made in Abaqus finite element commercial software. The solver input file is written through Matlab code (Fig. 5), being a function of the design variables specified. In the beginning of optimization, geometry variables are randomly specified and orientation variables are equally set to avoid orientation tendencies that might occur.

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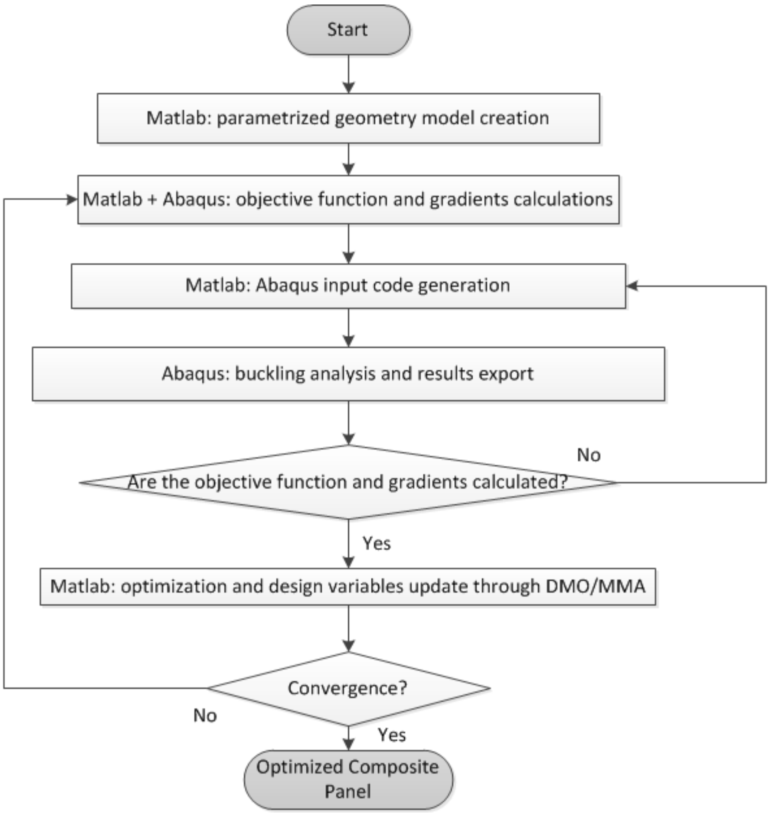

Figure 4. Implemented flowchart for the parametric optimization.

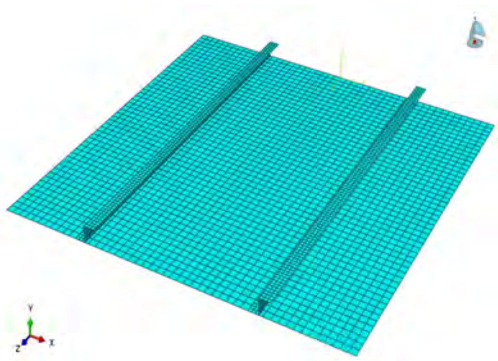

Figure 5. Finite element model parametrically created for the buckling analysis.

The optimization algorithm code that implements the Method of Moving Asymptotes (MMA), first published in Svanberg (1987) and Svanberg (2001), is provided by Professor Krister Svanberg. It requires as main inputs the design variables array, objective and constraints functions values at the current design point, as also the objective and constraint functions gradients with respect to the design variables. So, the next step of optimization code implemented is the gradients evaluation by the Finite Difference Method (eq. 10), where the Abaqus solver must be run consecutively under Matlab commands. The choice of this method is made because a commercial finite element solver is employed for the analysis. Although computationally expensive, it is an usual solution when the analysis code don't have access to the finite element stiffness matrices.

$$\frac{dF}{d\rho} \simeq \frac{F(\rho + \Delta\rho) - F(\rho - \Delta\rho)}{2\Delta\rho} + O(\Delta\rho^2) \tag{10}$$

After these calculations, the MMA algorithm computes the new design variables values and the optimization loop continues until the convergence criteria of DMO reaches a threshold value. A particular subset of the design domain is considered converged when the relation described in Eq. 11 is satisfied.

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$$w_i \geq \epsilon \sqrt{w_1^2 + w_2^2 + \ldots + w_{n^e}^2} \quad (11)$$

where ϵ is a threshold value for convergence criteria, equal for example, to 95% or 99%. The global convergence rate is evaluated as in Eq.12, where N_c^l is the number of subset domains converged and N^l the total number of subsets in the design domain. The optimization loop ends when h_ϵ reaches 95%.

$$h_\epsilon = \frac{N_c^l}{N^l} \quad (12)$$

5. NUMERICAL EXAMPLE

In this work, as a numerical example of the proposed methodology for the design of composite reinforced panels, the composite material chosen is the Carbon/Epoxy system Toray T700GC-12K-31E/#2510 (P707AG-15) Unidirectional Tape. The mechanical properties of this system and also the allowable strains are obtained from the test results B-Basis, published by NCAMP (National Center for Advanced Materials Performance) in Tomblin *et al.* (2002).

Table 1. Composite material mechanical properties considered in numerical example.

Tensile modulus, longitudinal (E_1^t)	125.50 GPa
Tensile strength, longitudinal (F_1^t)	1911.97 MPa
Tensile modulus, transverse (E_2^t)	8.41 GPa
Tensile strength, transverse (F_2^t)	42.80 MPa
Compressive modulus, longitudinal (E_1^c)	112.27 GPa
Compressive strength, longitudinal (F_1^c)	1280.95 MPa
Compressive modulus, transverse (E_2^c)	10.14 GPa
Compressive strength, transverse (F_2^c)	180.29 MPa
Major Poisson's ratio, tension (ν_{12})	0.31
In-plane shear modulus (G_{12})	4.23 GPa
In-plane shear strength (F_{12})	145.52 MPa

Table 2. Composite material allowables considered in numerical example.

Tensile strain allowable, longitudinal ($\epsilon_{adm}^{t,1}$)	1.52%
Tensile strain allowable, transverse ($\epsilon_{adm}^{t,2}$)	0.51%
Compressive strain allowable, longitudinal ($\epsilon_{adm}^{c,1}$)	1.14%
Compressive strain allowable, transverse ($\epsilon_{adm}^{c,2}$)	1.78%
In-plane shear allowable (ϵ_{adm}^{12})	5.00%

The results are compared with a baseline panel design (Fig. 6). It is considered a symmetric lay-up $[45° - 45°45° - 45°]_S$ for the panel and stringers laminate. The stringers geometry variables are set to 20 mm and the positioning variable equal to 0.25 what represents that the stringers are located at $1/4$ and $3/4$ of panel width. For all the cases the boundary conditions imposed are the 4 simply supported edges, however, the pre-load and the buckling load of the baseline are updated in according with the optimization case loading conditions.

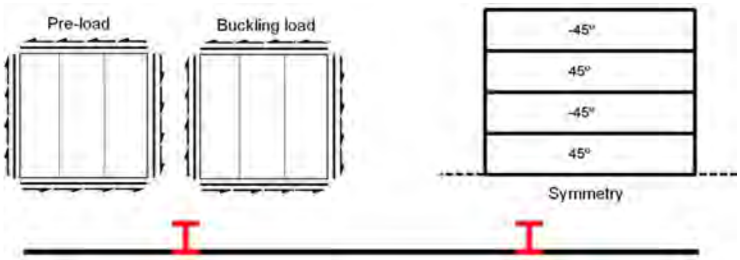

Figure 6. Baseline design for optimization results comparison.

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5.1 Parametric Optimization

Many different examples may be created by combining the pre-load and buckling load cases (shear, compression, pressure). In this section, 3 different cases results will be presented. All examples of parametric optimization shown in this paper consider 1 fiber orientation per layer due to manufacturing constraints. The material local axis definition employed for the results display is shown in Fig. 7 and the geometry variables interval considered feasible for the stringers are shown in Fig. 8.

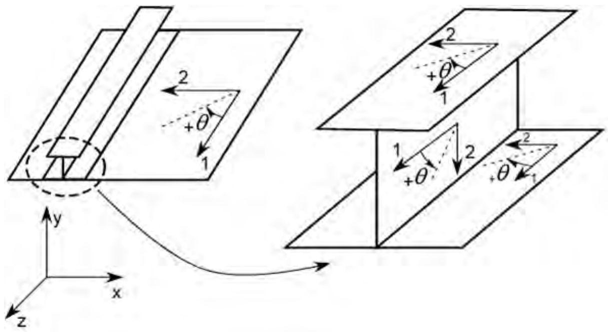

Figure 7. Material local axis definition employed for the results display.

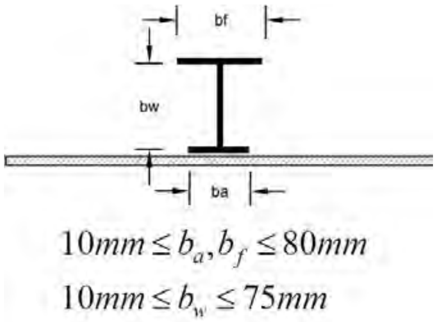

Figure 8. Stringer geometry variables feasible values considered.

Figures 9 through 11 show the results for 3 different load cases. In the objective function convergence curves, the red line indicates the objective function value for the baseline design, and the black line the objective function value for the optimized lay-up, with the constitutive matrices weights (w_i) rounded. In the weight convergence curves, the red line shows the weight constraint value.

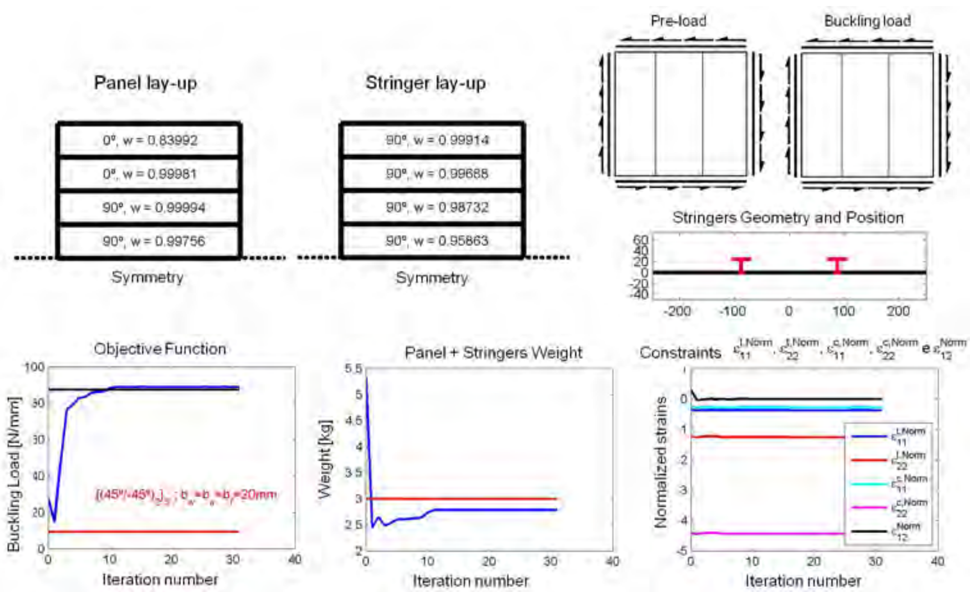

Figure 9. Results for case 1: shear pre-load and shear buckling load.

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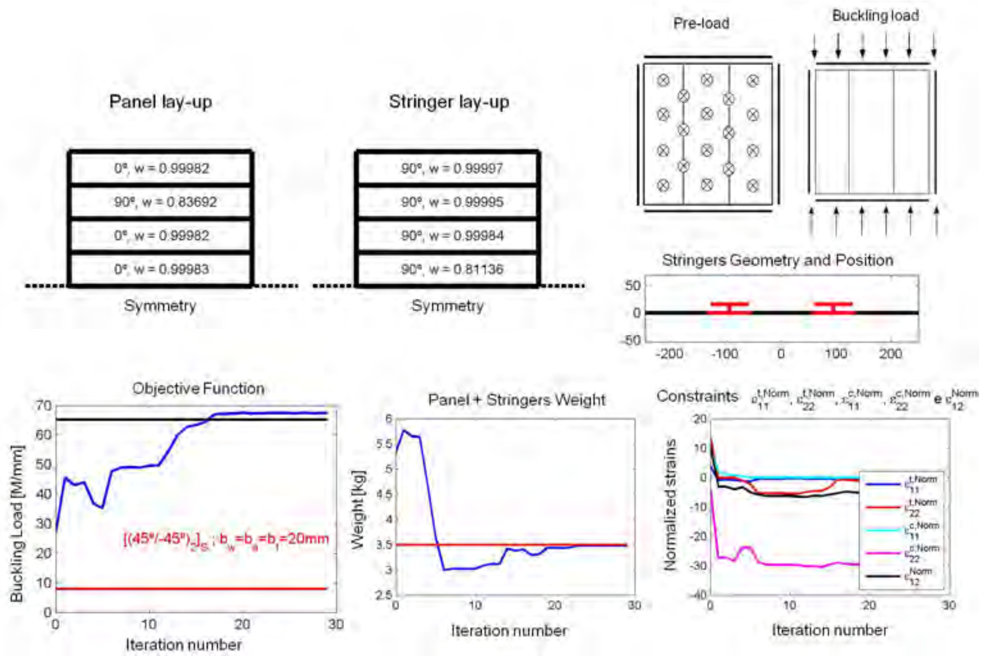

Figure 10. Results for case 2: pressure pre-load and compression buckling load.

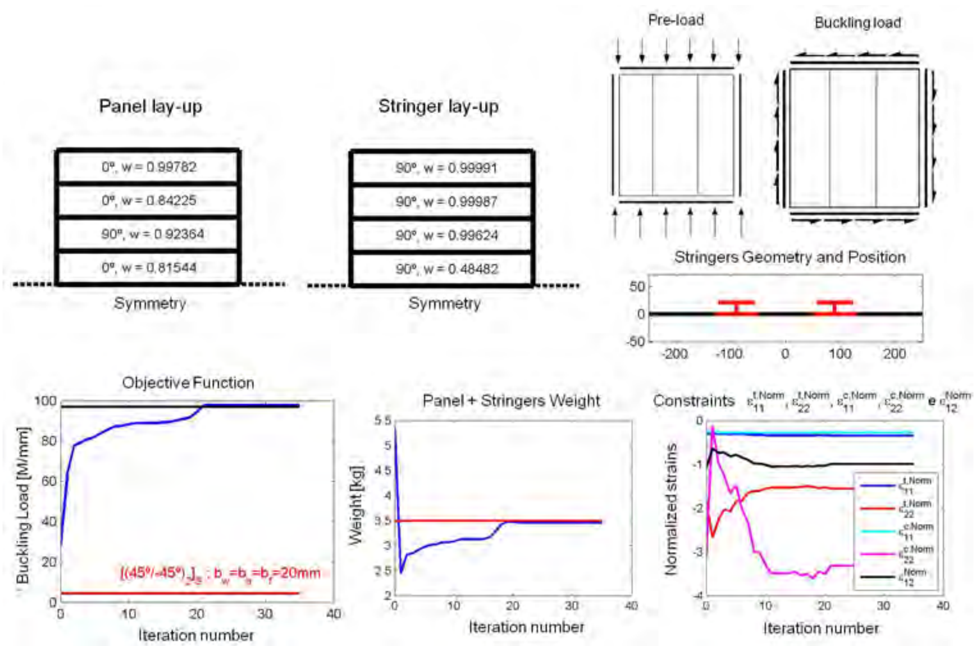

Figure 11. Results for case 3: compression pre-load and shear buckling load.

All cases results show an objective function increase when compared to the typical quasi-isotropic lay-up, even dealing with simple load cases and a small design domain such as those considered in the examples. The mass constraint remains active in the end of the three optimization cases, suggesting that the optimization algorithm tried to increase the lay-up size and stringers dimensions as much as possible, in order to maximize the panel buckling load through a cross-section inertia growth. At least one strain constraint remains active in the final result for all the three cases, indicating that the laminate strength is a constraint for the panel buckling load and, therefore, a failure criteria might not be disregarded.

6. CONCLUDING REMARKS

In this paper, the application of a parametric optimization methodology is studied in the case of composite reinforced panels design. The results obtained from parametric optimization employing the Discrete Material Optimization method confirms that the this methodology can be employed with commercial solvers and may bring very significant weight reductions for this kind of structure.

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A systematic approach for the composite laminate design is important due to the flexible design nature of this kind of material. Even in high constrained manufacturing methods such as the one considered in the numerical example presented in section 5, the weight reduction and buckling load increase are remarkable.

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9. RESPONSIBILITY NOTICE