



ANALYTICAL MODEL OF THE ACOUSTIC FIELD INSIDE A DUCT EXCITED BY SMART STRUCTURES FOR THE USE IN HIGHER ORDER NOISE ACTIVE CONTROL

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Abstract. *The best option for the active control of higher order noise in ducts was the use of axial splitters in the duct in order to prevent higher order mode propagation. It is possible to perform the active noise control in each splitter section by using a single channel control system. The use of smart structures takes advantage of the splitter plate and uses it as the control source. In order to evaluate the possibility of the noise control using smart structures, an analytical model of a thin plate with piezoelectric actuators was built. The use of the dynamic model of the plate for the development of an analytical model of the acoustic field inside the duct was the objective of this study. Then, a finite element model was built for the validation of the results. Both methods had the modeling of the acoustic field shape very similar but the amplitudes of pressure showed small differences, which is explained by the fact that the modeling of the contour have been made in different ways.*

Keywords: *Acoustic, Active Noise Control, Smart Structures, Higher Order Noise, Acoustic Duct*

1. INTRODUCTION

The noise level generated by machinery or equipment is an important issue to the design engineers. In the acoustic engineering, there are a lot of ways to solve this kind of problem.

Actually, the most used techniques are the passive noise control. Their application can be very effective in the reduction of the noise generated by an equipment, but, as the projects get more rigorous, the development of techniques even more effective is needed [1].

Therefore, the research in active noise control area has increased, being an alternative to the passive noise control that is very effective in the situations in which the passive techniques cannot be applied. An example of this situation is the control of low frequency noises [2].

It is known that in an industrial area, among other relevant equipment, the most important noise sources are the fans or exhausters [1]. An important part of the ventilation systems is the duct, which carries the noise, in addition to the air flow.

For these equipment, the existing passive noise control techniques are inefficient. In addition, there is a great concern about the low frequency noise propagation.

This work aims the development of a new technique of active noise control in ducts, in which the propagates higher order noises. In order to do that, numeric and analytical models of the splitted duct were created, which takes advantage of the splitter plate and uses it as the control source. This plate is excited by piezoelectric actuators. Following, the model will be simulated in the Ansys © environment to compare the analytical results with the Finite Element Method results.

2. DYNAMIC MODEL OF THE PLATE

First, for the development of the equations of motion, the free vibration of the plate will be studied. The most appropriated theory is the Kirchhoff's theory for thin plates [3].

This theory assumes the following hypothesis: the plate is very thin, and its thickness is h and its surfaces are defined by the $z = \pm h/2$ planes, just the transverse displacement will be considered (w), any stress in the z direction is null, the displacement on the x and y directions (u and v) are results of the rotation of the transverse planes.

Theses hypothesis makes relevant just the deformations on the direction x (ϵ_x) and y (ϵ_y) and the angular deformation (γ_{xy}).

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In the Kirchhoff theory development, contained in [4], this simplifications are used to obtain the plate's Kinect and deformation energies and then to determine its equations of motion using the variational principle of Hamilton, as shown by Eq. (1). In this equation, the external forces were not considered.

$$D \left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) + m\ddot{w} - p = 0 \quad (1)$$

Where m is the mass per area of the plate and p are the external pressures applied. The value of D is calculated by Eq. (2).

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad (2)$$

2.1 The plate's free vibration

The modeling will be developed to a simply supported beam in its edges. This kind of support allows the rotation around the edge's axis, but don't allow any displacement or other rotation.

The natural frequencies of the plate and its modal Eigen functions after the boundary conditions application are defined by Eq. (3) and Eq. (4), respectively.

$$\omega_{rs} = \beta^2 \sqrt{\frac{D}{m}} = \pi^2 \left[\left(\frac{r}{L_x} \right)^2 + \left(\frac{s}{L_y} \right)^2 \right] \sqrt{\frac{D}{m}} \quad (3)$$

$$\Phi_{rn}(x,y) = \text{sen} \left(\frac{r\pi x}{L_x} \right) \text{sen} \left(\frac{s\pi y}{L_y} \right) \quad (4)$$

In Eq. (3) and Eq. (4), L_x and L_y are the dimensions of the plate, and the r and s values indicate the plate's modes.

2.2 The dynamics of a Piezoelectric excited plate

In the dynamic formulation of the electrodynamic coupling, the installation of two piezoelectric actuator will be considered. Each actuator will be placed in one face of the plate and will be symmetric with respect to its medium plane. The actuators are activated by the application of tensions of same amplitude, but in phase opposition.

According to Santana, the actuators generate bending moments distributed around the x and y axis. The moments in both directions are the same and can be calculated by Eq (5).

$$m_x = m_y = C_0 \frac{d_{31}}{t} V [h(x - x_1) - h(x - x_2)] [h(y - y_1) - h(y - y_2)] \quad (5)$$

In Eq. (5), m_x is the bending moment around the x axis, d_{31} and t are the piezoelectric modulus and the thickness of the actuator, respectively, V is the amplitude of the applied tension and $h(\bullet)$ is the unity step function.

The C_0 value is given by Eq. (6), and the x_1 , x_2 , y_1 and y_2 values are the initial and final coordinates of the actuator in respect with and edge of the plate.

$$C_0 = -E_p \frac{(1 + \nu_{ps})}{(1 - \nu_p)} \frac{P}{[1 + \nu_p - (1 + \nu_{ps})]} \frac{2}{3} h^2 \quad (6)$$

The E_p value is the elasticity modulus of the plate and ν_p is its Poisson coefficient. This properties of the actuator material are given by E_{pe} and ν_{pe} respectively.

The value of P in Eq. (6) can be calculated as shown by Eq. (7).

$$P = -K \frac{E_{ps} (1 - \nu_p^2)}{E_p (1 - \nu_{ps}^2)} \quad (7)$$

where

$$K = \frac{3th(2h+t)}{2(h^3+t^3)+3ht^2} \quad (8)$$

According to Fuller et. al., the mass of the actuators can be neglected. Therefore, it is possible to write the plate's equation of motion considering the moments generated by the actuators applied to the equations developed with the Kirchhoff theory. The movement of the plate can be described by the Eq. (9).

$$D\Delta^4 w + m\ddot{w} = \frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2} \quad (9)$$

In order to obtain the solution of Eq. (9), it is used the modal expansion of the $w(x,y)$ response. using the modes calculated in Eq. (2) and the expansion theory, it is possible to write the response of the plate as in Eq. (10).

$$w(t, x, y) = \sum_r \sum_s W_{rs}(t) \Phi_{rs}(x, y) \quad (10)$$

Where W_{rs} is the modal amplitude of the plate, which is function of time. For a harmonic analysis, this value is an harmonic function with amplitude W_{rs} , that can be calculated by Eq. (11).

$$\bar{W}_{rs} = \frac{4C_0 V d_{31}}{tAm(\omega_{rs}^2 - \omega^2)} \left\{ - \left(\frac{\gamma_r^2 + \gamma_s^2}{\gamma_r \gamma_s} \right) [\cos(\gamma_r x_1) - \cos(\gamma_r x_2)] [\cos(\gamma_s y_1) - \cos(\gamma_s y_2)] \right\} \quad (11)$$

The A value in this Equation represents the area of the plate, calculated by $A = L_x \cdot L_y$.

The Eq. (11) shows that the sum of the modes is weighted by the frequency. When the excitation frequency is close to the natural frequencies, its respective mode is predominated.

The amplitude value is also influenced by the position and size of the actuator, represented in the second factor of the Eq. (11) by the sine and cosine. This factor can be simplified considering the symmetry with respect to the plate's axis.

As example, considering the symmetry with respect to the x axis, the second factor is zero for even values of n, but to odd values, it can be calculated by Eq. (12).

$$2\cos\left(\frac{s\pi}{L_y} y_1\right) \quad (12)$$

3. THE DUCT SOUND FIELD MODELING

3.1 The primary source

First, the obtainment of the sound field generated by the primary source is needed. This field will be considered the noise that needs to be controlled.

The model considers that the primary source is at the beginning of the duct, in the left side of the duct, which is closed. The end the duct is opened. In this situation, the reflections in the end of the duct will not be considered, only the direct field of the source.

It will be considered the propagation of the higher order noise. This modes will occur when the excitation frequency of the sound source is higher than the critical frequency, calculated by Eq. (13).

$$f_c = \frac{c_0}{2H} \quad (13)$$

Where H is the bigger dimension of the transverse section of the duct and c_0 is the sound velocity in the air. The higher is the value of H, the lower is the critical frequency of the duct, and more higher order modes will propagate in the duct. A higher order mode (m,n) will propagate if the condition of Eq. (14) is met, where ω is the excitation frequency in rad/s.

$$\left(\frac{\omega}{c_0}\right)^2 - \left(\frac{m\pi}{b}\right)^2 - \left(\frac{n\pi}{d}\right)^2 > 0 \quad (14)$$

In the situations that the excitation frequency is lower than the critical frequency, the propagation will be of plane waves.

The position of the primary source will be considered in the $(x_p, y_p, 0)$ coordinates. The x and y axis are contained in the plane of the transverse section of the duct.

For the application of this work, the primary source will be punctual. In this case, the equation of the acoustic field inside the duct is predicted by Eq. (15).

$$P_p(x, y, z) = \frac{\rho_0 \omega Q_p}{S} \sum_m \sum_n \frac{\Psi_{m,n}(x, y) \Psi_{m,n}(x_p, y_p)}{\Lambda_{mn} K_{mn}} e^{-jK_{mn}z} \quad (15)$$

where

$$\Psi_{mn}(x, y) = \cos\left(\frac{m\pi x}{b}\right) \cos\left(\frac{n\pi y}{d}\right) \quad (16)$$

$$\Lambda_{mn} = \frac{1}{S} \int \Psi_{mn}^2 dS \quad (17)$$

$$K_{mn} = \sqrt{\left(\frac{\omega}{c_0}\right)^2 - \left(\frac{\pi m}{b}\right)^2 - \left(\frac{\pi n}{d}\right)^2} \quad (18)$$

In these equations, j is the imaginary unit, ρ_0 is the density of the fluid contained in the duct, Q_p is the volume velocity of the sound source, S is the transverse section area of the duct ($S = b \cdot d$) and ω is the excitation frequency.

If the propagation is just of plane waves, the acoustic field inside the duct can be calculated by the Eq. (19).

$$P_p(z) = \frac{\rho_0 c_0}{S} Q_p e^{-jKz} \quad (19)$$

3.2 Control Source Modeling

In this section, the sound field generated by the a vibrating plate will be studied. In order to do that, a analysis of the sound field in an enclosure with different boundary conditions is needed. After that, it will be calculated the influence of a vibrating surface as the boundary condition.

This study presents an analysis of the acoustic response of a fluid inside an enclosure volume with the appropriated boundary conditions. This development was performed by Fahy and Gardonio.

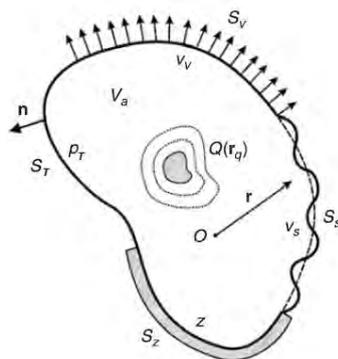


Figure 1. Acoustic enclosure with different boundary conditions.

In Fig. 1, the vector r represents one generic position inside the enclosure and Q represents a sound source in the r_q position. The vector n represents the normal direction with respect of the boundary surface of the volume.

The total acoustic field inside the enclosure is given by the sum of the sound field generated by the sources with the sound field generated by the boundary conditions, as shown in Eq. (20).

$$p(r) = P_p(r) + p_f(r) \quad (20)$$

Developing the formulation of the boundary of the enclosure, the sound pressure generated inside it by

one general condition is given by Eq. (21).

$$p(r) = \frac{1}{c(r)} \int_{S_a} \left(p(r_a) \frac{\partial G(r, r_a)}{\partial n} + j\rho_0 \omega G(r, r_a) v_n(r_a) \right) dS_a + \frac{\rho_0 \omega Q_p}{S} \sum_m \sum_n \frac{\Psi_{m,n}(x, y) \Psi_{m,n}(x_p, y_p)}{\Lambda_{mn} K_{mn}} e^{-jK_{mn}z} \quad (21)$$

where $c(r) = 1/2$ and $G(r, r_a)$ is the Green function that relates the pressure to the velocity of a particle in any Enclosure.

The development of the anterior section will be applied to the work's problem, which considers the duct as the enclosure with a piezoelectric excited flexible surface as its boundary. The obtained equations of the motion will be used to calculate de velocity of the boundary of the enclosure.

Combining the formulation of the acoustic field of the duct with the formulation of the plate vibration, the Eq. (22) is obtained. This equation predicts the desired acoustic field.

$$P(x, y, z) = \frac{j\rho_0 \omega^2}{S} \sum_r \sum_s \sum_m \sum_n \left[\frac{\Psi_{m,n}(x, y)}{\Lambda_{mn} K_{mn}} INT_1 INT_2 \right] \quad (22)$$

where

$$INT_1 = \int_{x_i}^{x_f} \bar{W}_{r,s} \text{sen}(\gamma_r x_0) \text{sen}(\gamma_s z_0) dx_0 = \frac{b^2 \gamma_r}{b^2 \gamma_r^2 - m^2 \pi^2} \left[b \cos\left(\frac{m\pi x_i}{b}\right) - (-1)^r \cos\left(\frac{m\pi x_r}{b}\right) \right] \quad (23)$$

$$INT_2 = \int_{z_i}^{z_f} \cos\left(\frac{m\pi x_0}{b}\right) e^{-jK_{mn}|z-z_0|} dz_0 = \frac{\gamma_s}{\gamma_s^2 - K_{mn}^2} \left[e^{-jK_{mn}|z-z_i|} - (-1)^r e^{-jK_{mn}|z-z_f|} \right] \quad (24)$$

In the Eq. (23) and Eq. (24), the values of x_i , x_f , z_i and z_f are the initial and final coordinates of the plate inside the duct.

4. FINITE ELEMENT MODEL OF THE DUCT

To simulate the duct by the Finite Element Method the ANSYS ® environment was used. The scheme of the model constructed model is shown in Fig. 2.

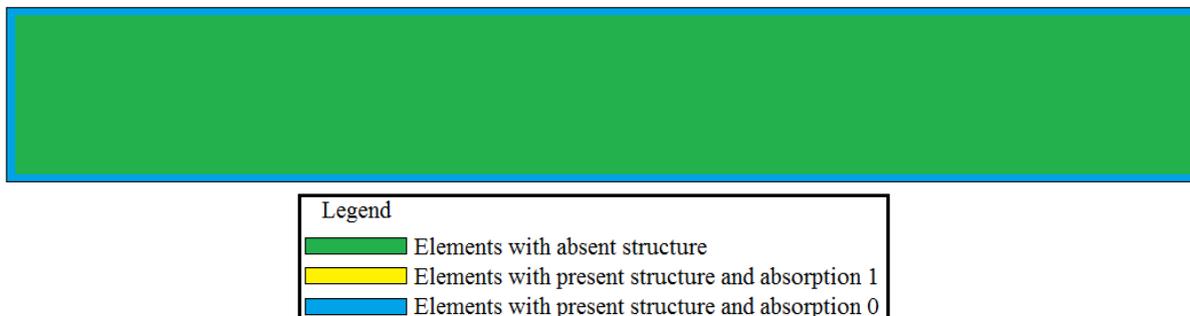


Figure 2. Specification of the elements used in Finite Elements Method model.

The element used for the simulation was Fluid 30, which is available in the software. This element is suitable for simulating three-dimensional sound field. At the left end of duct elements with total absorption was used to simulate a situation of open wall, while the rest of the contour the absorption was 0, which means a perfectly rigid wall. For all the volume the properties of air were used.

5. RESULTS

In this section we will evaluate the reliability of the analytical equations of the duct. For this purpose the duct was considered to have the geometry shown in Fig. 3.

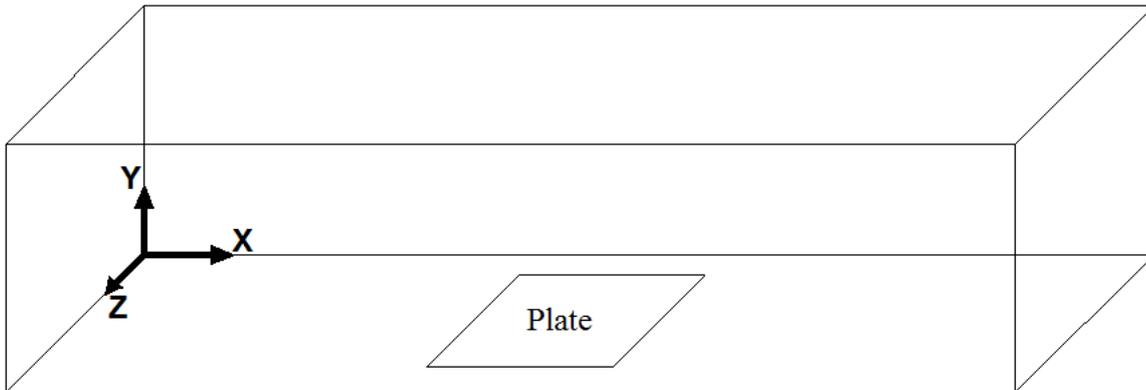


Figure 3. Model of a duct excited by a vibrating plate.

The frequencies were chosen so that the three excitations could generate plane-wave propagation and another five for the propagation of higher order modes. For a duct with cross-sectional dimension equal to 1 meter, the cutoff frequency is 171.5 Hz. Thus, the frequencies chosen were 80 Hz, 100 Hz, 150 Hz, 180 Hz, 210 Hz, 250 Hz, 280 Hz and 300 Hz. The unit used for the visualization of acoustic fields was Pascal.

The results of the first frequency are shown in the first frequency by Fig. 4 and Fig. 5. As this frequency is 80 Hz, the plane wave propagation is expected.

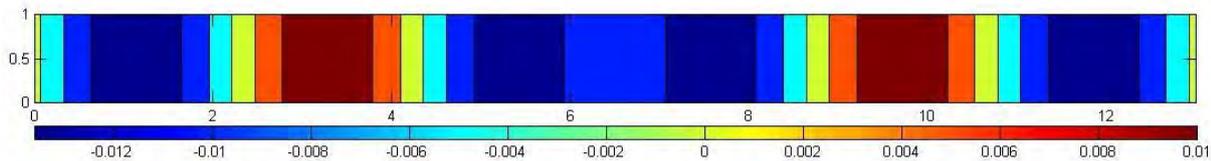


Figure 4. Analytical acoustic field in a semi-infinite duct analytical semi-infinite with a plate excited by a piezoelectric wafer instrumented with the frequency of 80 Hz.

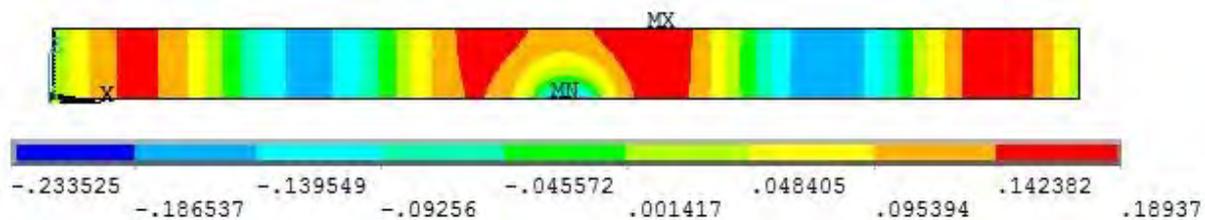


Figure 5. Acoustic field of a semi-infinite duct excited by a piezoelectric wafer plate instrumented with the frequency of 80 Hz obtained via MEF.

Observing the Fig. 4 and Fig. 5, we note that the shape of the acoustic field is similar, except near the plate. This is because the analytical model does not have the direction cosines of the y-axis in situations of plane wave propagation. Thus, the pressure variation is explicit only in the axial direction of the duct.

Regarding the pressure amplitude of the acoustic field, there is a strong correlation, which was expected since the modeling of the contour shape of the duct in the two methods is different, and the resolution of the finite elements used a generic value for absorption for the rigid walls in order not to occur damping of the sound wave over duct. It is

noteworthy, too, that the absorption vary with frequency. Thus, only the shape of the acoustic field was considered for validation.

Then, in the Fig. 6 and Fig. 7 are exposed analytical and numerical results, respectively, to the excitation of 100 Hz.

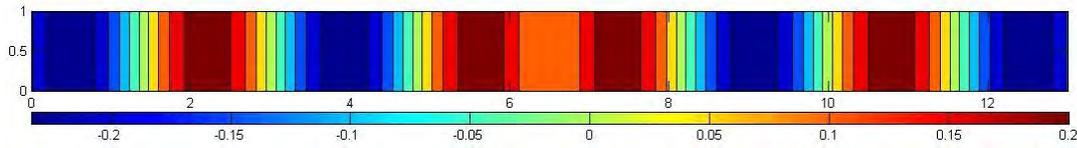


Figure 6. Analytical acoustic field in a semi-infinite duct analytical semi-infinite with a plate excited by a piezoelectric wafer instrumented with the frequency of 100 Hz.

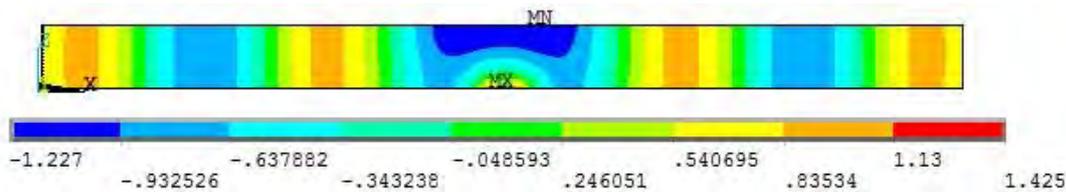


Figure 7. Acoustic field of a semi-infinite duct excited by a piezoelectric wafer plate instrumented with the frequency of 100 Hz obtained via MEF.

In the frequency of 100 Hz it is also possible note that the acoustic field near the plate on the analytical model lacks variations in y-axis direction, a fact previously explained. Moreover the shape of the acoustic field has good correlation.

In Fig. 8 and Fig. 9 shows the results obtained from the simulation frequency of 150 Hz.

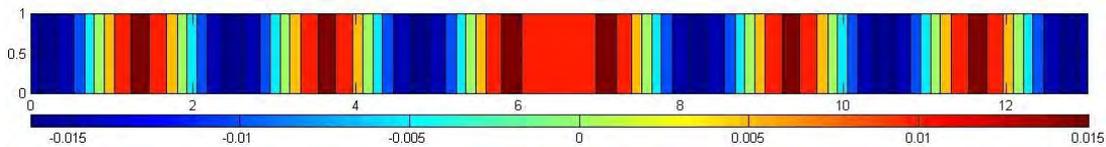


Figure 8. Analytical acoustic field in a semi-infinite duct analytical semi-infinite with a plate excited by a piezoelectric wafer instrumented with the frequency of 150 Hz..

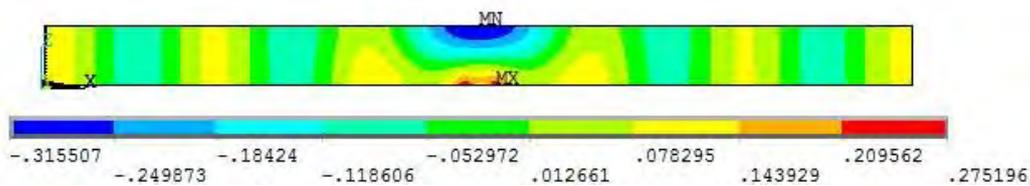


Figure 9. Acoustic field of a semi-infinite duct excited by a piezoelectric wafer plate instrumented with the frequency of 150 Hz obtained via MEF.

As this frequency is already close to the cut-off frequency of the duct, there are small variations of pressure in the plane of the wave in the finite element model. Despite this, the plane-wave propagation is still evident and the position of peaks and valleys in the two models are quite similar.

Leaving now for excitations at frequencies above the cutoff frequency of the duct, the first simulation was performed at a frequency of 180 Hz, the results of which are shown in Fig. 10 and Fig. 11.

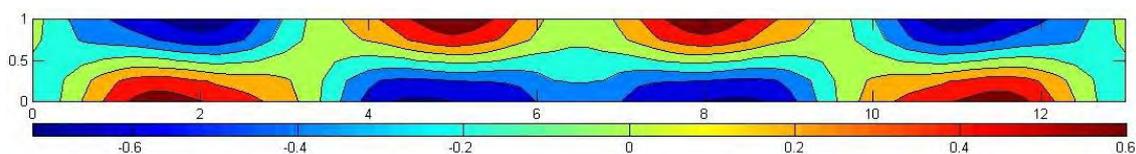


Figure 10. Analytical acoustic field in a semi-infinite duct analytical semi-infinite with a plate excited by a piezoelectric wafer instrumented with the frequency of 180 Hz.

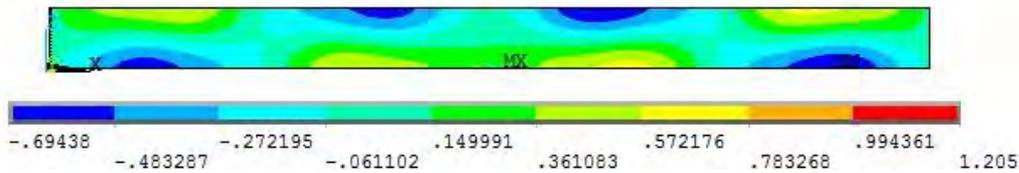


Figure 11. Acoustic field of a semi-infinite duct excited by a piezoelectric wafer plate instrumented with the frequency of 180 Hz obtained via MEF.

Noting the sound fields of Figures 10 and 11, there is a strong correlation in forms obtained by both methods. In this case, as the high-order modes propagation already occurs, the analytical model already has cosines to model the variation of pressure in the direction of the vertical axis, providing great accuracy throughout the sound field, including nearby plate.

Following with simulations of high-order modes, the excitation was now at the frequency of 210 Hz, with the results in Fig. 12 and Fig. 13.

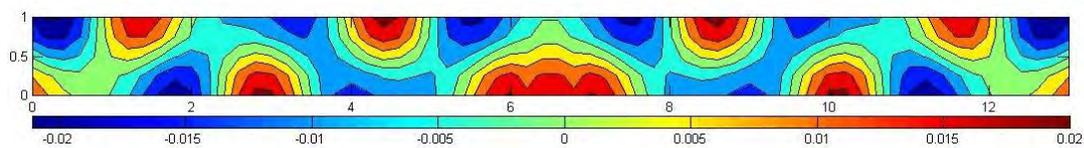


Figure 12. Analytical acoustic field in a semi-infinite duct analytical semi-infinite with a plate excited by a piezoelectric wafer instrumented with the frequency of 210 Hz.

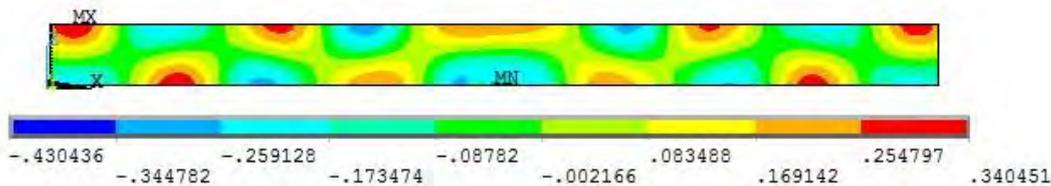


Figure 13. Acoustic field of a semi-infinite duct excited by a piezoelectric wafer plate instrumented with the frequency of 210 Hz obtained via MEF.

Noting Fig. 12 and Fig. 13, it is possible to confirm that the analytical is satisfactory on frequencies that excite high order modes, which are most evident at this frequency.

In sequence, the excitation frequency of 250 Hz was simulated. The results are shown in Fig. 14 and Fig. 15.

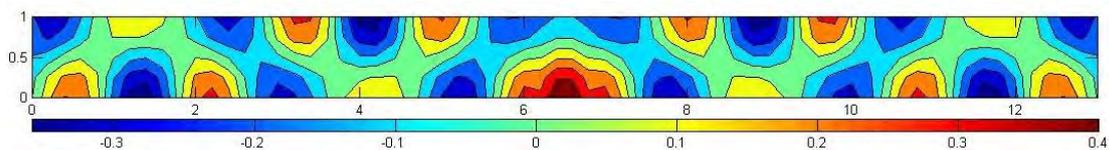


Figure 14. Analytical acoustic field in a semi-infinite duct analytical semi-infinite with a plate excited by a piezoelectric wafer instrumented with the frequency of 250 Hz.

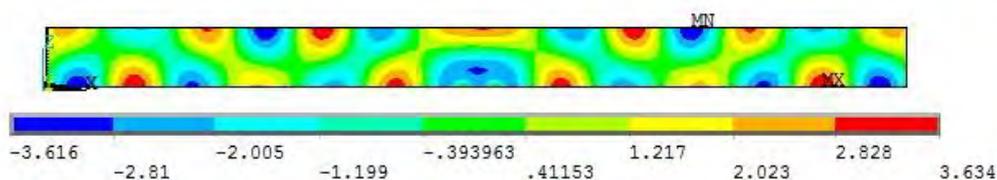


Figure 15. Acoustic field of a semi-infinite duct excited by a piezoelectric wafer plate instrumented with the frequency of 250 Hz obtained via MEF.

The results shown for the frequency of 250 Hz were also satisfactory, and observing the position of peaks and valleys, there is a strong correlation between the two models on this frequency.

Continuing the Fig. 16 and Fig. 17 expose the results obtained with both methods for the excitation frequency of 280 Hz.

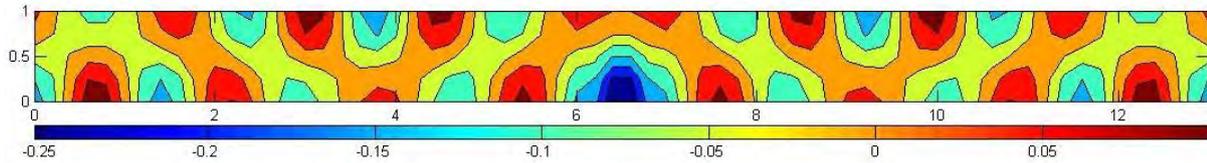


Figure 16. Analytical acoustic field in a semi-infinite duct analytical semi-infinite with a plate excited by a piezoelectric wafer instrumented with the frequency of 280 Hz.

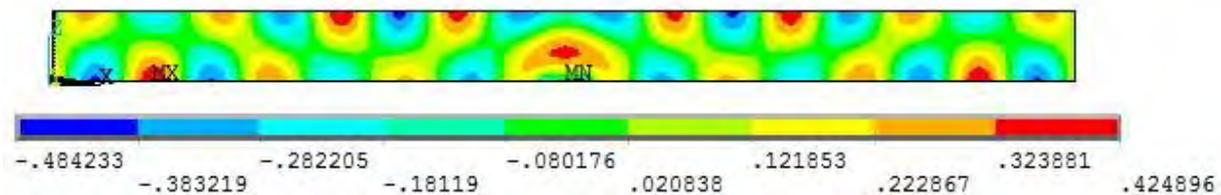


Figure 17. Acoustic field of a semi-infinite duct excited by a piezoelectric wafer plate instrumented with the frequency of 280 Hz obtained via MEF.

It is noted that, according to whether the frequency increases, it becomes more obvious the pressure variation along the duct sound field in the analytical model. As the frequency is higher, it excites a larger number of high order modes inside the duct, providing in this way, more arguments for the modeling of Equation (16). However, it is also possible to observe that forms from the frequency of 210 Hz does not suffer many modifications, except for the fact that the wavelength decrease. This already occurs because dimensions of the duct limit the amount of high-order modes that can propagate.

The last simulated frequency was 300 Hz, and their responses on the analytical model and the finite element model are displayed in Fig. 18 and Fig. 19.

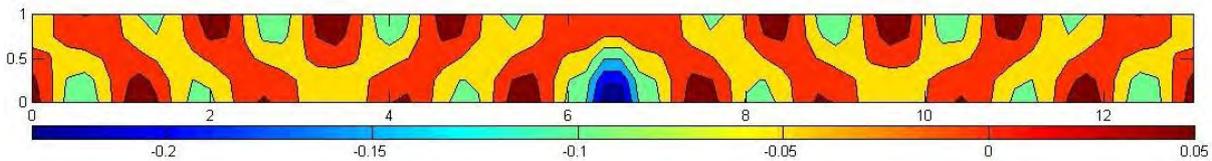


Figure 18. Analytical acoustic field in a semi-infinite duct analytical semi-infinite with a plate excited by a piezoelectric wafer instrumented with the frequency of 300 Hz.

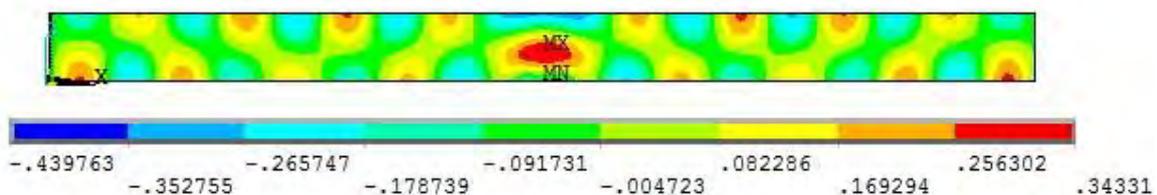


Figure 19. Acoustic field of a semi-infinite duct excited by a piezoelectric wafer plate instrumented with the frequency of 300 Hz obtained via MEF.

In this frequency is already observed that the analytical model, the discretization of the acoustic field begins to be insufficient. Nevertheless, the forms of these acoustic fields are still correlated, with similar positioning of peaks and valleys.

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In a general analysis, despite having amplitudes errors, there is, by the scales of the fields, the frequency-dependent behavior is similar. But these are not proportional. This means that the excitation of modes and natural frequencies in both models is similar.

5. CONCLUSIONS

In this paper the use of smart structures for obtaining the active noise control in a partitioned duct partitioned was proposed. In order to evaluate this technique, it was necessary to understand the dynamic behavior of structures excited by piezoelectric actuators and develop equations for obtaining the acoustic field inside a duct generated by these structures. Subsequently, the same model was developed in the finite elements method to validate the equations analytically

Knowing the dynamic behavior of a thin plate as a smart structure, the development of equations to obtain the sound pressure generated at any point of the duct by a vibrating surface is possible. Analyzing what was developed, it was possible to draw the following conclusions:

- the most feasible way for the modeling of the acoustic field was to consider the calculation of sound pressure within a closed volume due to different boundary conditions;
- the most appropriate boundary condition is the one in which the particle velocity of the fluid contained in the duct is equal to the surface speed at which these particles are in contact;
- with the displacement field of the plate generated by the piezoelectric actuators inserted in the acoustic equations, it was possible to obtain an expression that predicts the pressure at any point in the duct generated by this fluid-structure interaction.

Based on the above results, the analytical model developed can be considered the validated. During the simulations and calculations, it was noted advantage of this model in the sense of computational cost, since it is embedded in your equation considerations about the contour of the duct. Nevertheless, this model do not considered the duct damping, and with the finite element method it has just to be modified the absorption limit of the material present in the duct volume.

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8. RESPONSIBILITY NOTICE

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