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THE SOIL EFFECT ON THE DYNAMIC RESPONSE OF BUILDINGS: A BEM-FEM COUPLED PROCEDURE

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Abstract. *The vibration of structures and the modal analysis has become more relevant by the use of less mass and more flexible structural elements. The stiffness of the connections also affects the dynamic parameters. It happens because less or more flexible links can change all the structural mass, global stiffness and damping. These aspects are present in several structures of buildings and can affect their static and dynamic behaviors. This paper presents a BEM-FEM (Boundary Element Method – Finite Element Method) procedure to obtain the dynamic response of a flat frame building structure subjected to two different kinds of constrains: in the first case, its bases are clamped (zero displacement) and in the second one, they are coupled to a soil-foundation system. To performance these evaluations, the frame structure is modeled by FEM and a direct version of BEM is applied to synthesize the dynamic compliance matrix of a rigid and massless foundation interacting with unbounded soil. The foundation compliance matrix is coupled to a frame, leading to the dynamic response of a coupled frame-foundation-soil system. The article describes the methodology applied to couple rigid bodies with the BEM mesh. This strategy allows performing frequency domain analysis of a frame-foundation-soil system. These analyzes are related to natural frequencies comparing a clamped frame and a frame-foundation-soil system under wind effects.*

Keywords: *boundary element method, finite element method, soil-structure interaction*

1. INTRODUCTION

A classical way of performing dynamic analysis of structures is called modal analysis which is a method to obtain enough information of systems or structures that reproduce their dynamics (Clough and Penzien, 1975). In the classical modal analysis these information are related to the natural frequencies of the system (eigenvalues) and the modes of vibration (eigenvectors) (Meirovitch, 1975). This kind of analysis allows uncoupling the system of equations and the dynamic solution becomes the weighted superposition of the uncoupled solutions. However, there are some pre-requisites to perform the classical modal analysis. The system of equations that describes the structure must be linear and has constant coefficients. Another aspect is related to the damping which must be proportional to a combination of mass and stiffness of the system (Coughey, 1960).

A typical structure with constant coefficients and proportional damping can be seen in Fig. 1a. The structure is composed by three masses m_i , three measures of stiffness k_i and damping with three coefficients c_i *(i = 1, 2, 3)*. The structure is supported by a fixed foundation with mass m_f . This model states that the foundation has a displacement of zero, regardless the effort acting on it. Under many circumstances this assumption is reasonable but for many other conditions this is not a good description of reality. In fact the foundations are supported by some means, for example, soils. Fig. 1b shows the foundation bonded to a half space which reproduces reasonably homogeneous soil and with great depth.

It is important to notice that the soil interacts with the foundation, which has inertia and stiffness and, therefore, also deforms or moves. If soil deforms or moves under the action of efforts, the foundation bonded to it also moves. Considering this, the hypothesis shown in Fig. 1a, which describes a fixed foundation, is no longer reasonable.

The soil usually has an unlimited dimension which excludes the classical modal analysis. The dynamics of soil is characterized by the wave propagation towards the unlimited dimension without reflections of them. These waves carry energy that are removed from the system. This effect is equivalent to a damping and is named geometric damping (Richart, *et al.*, 1970; Hall and Olivetto, 2003).

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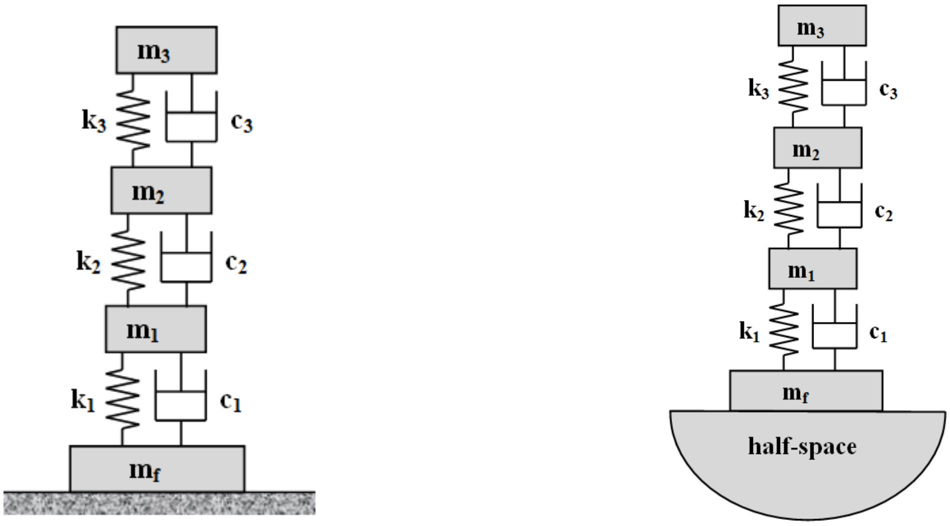

Figure 1. a) Dynamic system supported by a rigid base; b) Dynamic system supported by a half space.

An interesting aspect of the dynamics of soil can be illustrated in the case that a half-space is subjected to a vertical excitation, as shown in Fig. 2a which shows a massless rigid foundation bonded to a half-space. On this foundation acts a vector of generalized forces $\{F_s\}$and as a result the foundation has generalized displacements characterized by its vector $\{U_s\}$. Analyzing the relationship between $\{U_s\}$ and $\{F_s\}$ it is possible to associate the dynamic behavior of a soil spring $k_{zz}(\omega)$ and a damping $c_{zz}(\omega)$ that depend on the frequency, ω.

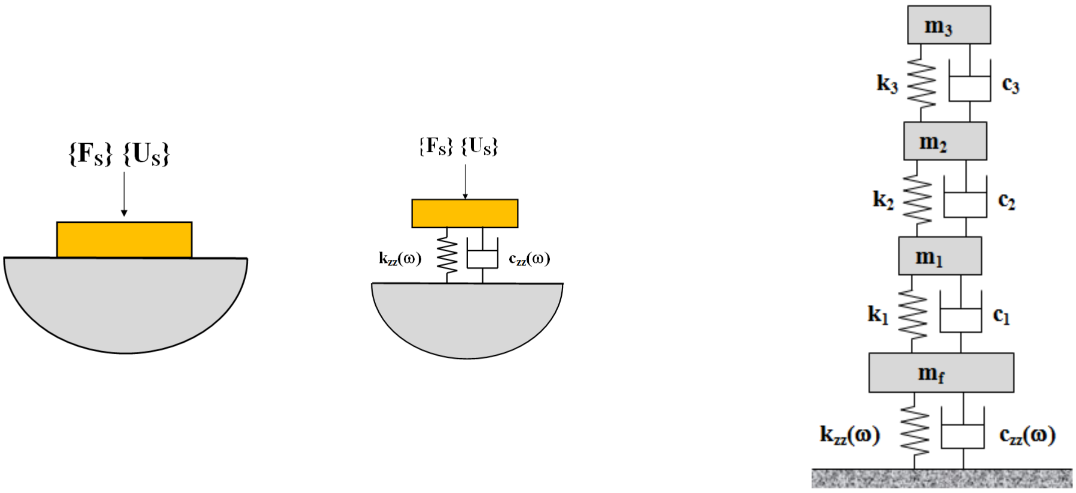

Figure 2. a) Dynamic system supported by a half space; b) Lumped parameters soil model.

Thus one can replace the vertical dynamic soil response as a set of springs and dashpots that are frequency function. The soil model of Fig. 1b was replaced by the equivalent lumped parameters. The result can be seen in Fig. 2b.

In general, in 2D or 3D problems, it is possible to relate the vectors generalized force $\{F_s\}$ and generalized displacement $\{U_s\}$ by the dynamic compliance matrix $[S(\omega)]$ or its inverse, dynamic flexibility $[N(\omega)] = [S(\omega)]^{-1}$

$$[S(\omega)]\{U_s\} = \{F_S\} \text{ and } \qquad [N(\omega)]\{F_S\} = \{U_s\} \tag{1}$$

It was shown that the soil dynamics implies force and displacement relations that depend on frequency. Analysis of Fig. 2b shows that the coefficients of the system of equations are not constant but functions of frequency. This also means that one cannot perform the classical modal analysis of dynamical systems where the soil is included.

It is possible to evaluate the coupling of structures dynamics with soil dynamics to describe the dynamic soil-structure interaction in the frequency domain (Hall and Olivetto, 2003; Mesquita, 1989). However, for this complete analysis, is necessary to solve the whole system of equations for each frequency.

2. BOUNDARY ELEMENT METHOD (BEM) AND THE MODELING OF DYNAMIC SOIL-FOUNDATION INTERACTION

The direct version of the BEM is used to synthesize the dynamic stationary soil response. The soil is an isotropic, viscoelastic continuum, presenting shear modulus G_s, density ρ_s, Poisson ration ν and internal damping coefficient η_s. The soil-foundation interface is Γ_f and the remaining boundary of the soil domain is Γ_s, like in Fig. 3a. With this definition the boundary displacements and tractions at the soil foundation interface and on the remaining soil surface are, respectively u_f t_f and u_s t_s. The algebraic BE system for the soil response may be written as:

$$\begin{bmatrix} H_{ff} & H_{fs} \\ H_{sf} & H_{ss} \end{bmatrix} \begin{Bmatrix} u_f \\ u_s \end{Bmatrix} = \begin{bmatrix} G_{ff} & G_{fs} \\ G_{sf} & G_{ss} \end{bmatrix} \begin{Bmatrix} t_f \\ t_s \end{Bmatrix} \tag{2}$$

The rigid foundation response is obtained by imposing kinematic compatibility and equilibrium at the soil-foundation interface Γ_f. In this article the basis of the 3D foundation has dimensions 2a × 2a (see Fig. 3a). Considering the vector of the 3D rigid foundation degrees of freedom (DOF) $U_s = \left(u_x, u_y, u_z, \phi_x, \phi_y, \phi_z\right)^T$ and the soil displacements u_f at the interface Γ_f, a relation may be established between them, by means of the kinematic compatibility matrix $[CC]$:

$$\{u_f\} = [CC]\{U_s\} \tag{3}$$

Equilibrium relations between the interface tractions t_f and the vector of external forces applied to the rigid (and massless) foundation $F_s = \left(F_x, F_y, F_z, M_x, M_y, M_z\right)^T$ may be written using a matrix $[D]$ (see Fig. 3a):

$$\{F_s\} = [D]\{t_f\} \tag{4}$$

The solution of equations (2) to (4) lead to a frequency dependent rigid foundation dynamic flexibility matrix $N(\omega)$ (or its inverse, dynamic compliance matrix $[S(\omega)]$) relating the external forces F_s and the rigid foundation DOFs U_s:

$$\{U_s\} = \frac{1}{a\,G_s}[N(\omega)]\{F_s\} \quad \text{or} \quad a\,G_s[S(\omega)]\{U_s\} = \{F_s\} \tag{5}$$

This procedure has been applied to obtain the dynamic flexibility matrices for soil profiles. Although the synthesis of the soil response is three-dimensional, in the remaining of this article only the degrees of freedom and external forces on the $x-z$ plane will be described. Figure 3b shows the structure of the rigid foundation dynamic flexibility matrix, considering the $x-z$ plane as well a possible interpretation of it. The vertical excitation and vertical degree of freedom, F_z and u_z, are uncoupled from the horizontal and rocking DOFs.

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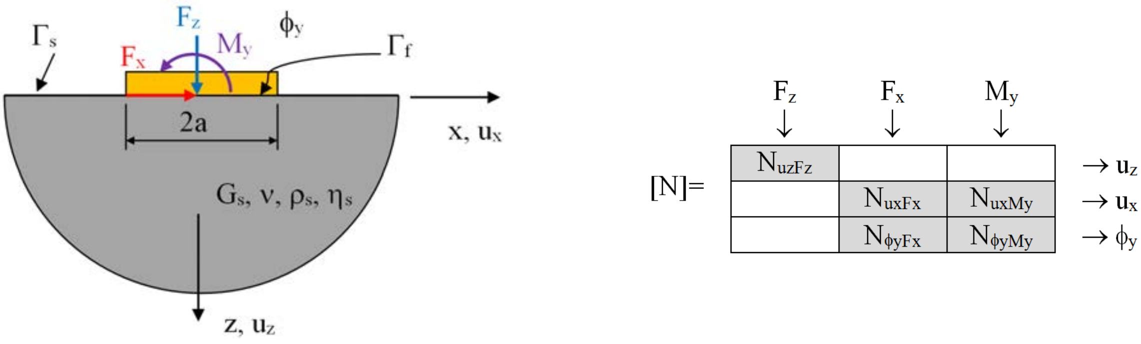

Figure 3. a) Foundation over half-space – definitions; b) Structure of the dynamic flexibility matrix.

3. STRUCTURAL ANALYSIS OF FRAME BY THE FINITE ELEMENT METHOD (FEM)

3.1. General 2-D Beam Element – Stiffness matrix

A well known concept used in FEM is the stiffness matrix of an element which is used to relate the external forces applied to the nodes of the structural element to its nodal displacements. The general 2D beam element used in the frames discussed in this article is shown in Fig. 4:

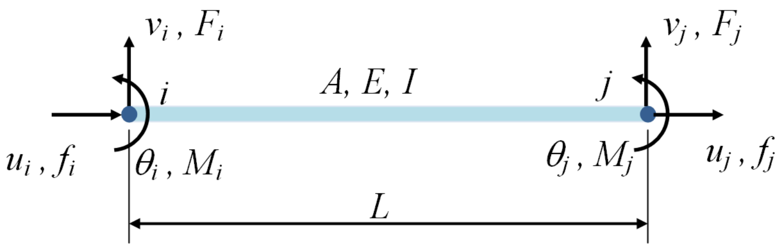

Figure 4. Ggeneral 2D beam finite element.

The element equilibrium equation, and consequently the stiffness matrix, can be written as:

$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ \theta_i \\ u_j \\ v_j \\ \theta_j \end{Bmatrix} = \begin{Bmatrix} f_i \\ F_i \\ M_i \\ f_j \\ F_j \\ M_j \end{Bmatrix} \tag{6}$$

where:

$L \rightarrow$ length
$A \rightarrow$ cross-sectional area
$E \rightarrow$ elastic modulus
$I \rightarrow$ moment of inertia of the cross-sectional area

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$u = u(x) \rightarrow$ displacement
$f = f(x) \rightarrow$ axial force
$F = F(x) \rightarrow$ shear force
$M = M(x) \rightarrow$ moment about z-axis
$v = v(x) \rightarrow$ deflection (lateral displacement) of the neutral axis
$\theta = \dfrac{dv}{dx} \rightarrow$ rotation about the z-axis

3.2. General 2-D Beam Element – Mass matrix

The lumped mass matrix for the general 2-D beam element mentioned above is based on the idea the mass is equally divided on the nodes like the Fig. 5:

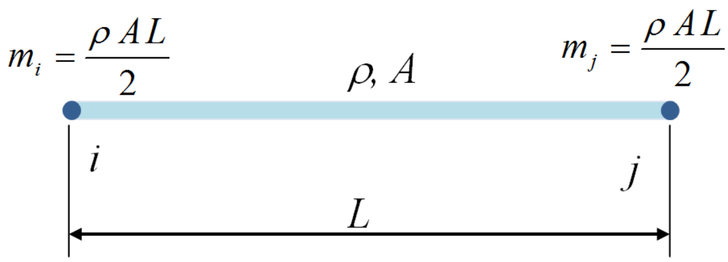

Figure 5. Lumped mass element.

and it is represented as:

$$[M_{LUMPED}]\begin{bmatrix} \frac{\rho AL}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\rho AL}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\rho AL}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\rho AL}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

where:

$\rho \rightarrow$ mass density

In this work, the mass matrix $[M]$ is considered to be the lumped one, where the mass is equally divided on the nodes and for assembling the damping matrix $[C]$, it will be used a simple and widely applicable model of damping, called proportional damping, defined as the linear combination of matrices $[M]$ and $[K]$, i.e.:

$$[C] = \alpha[K] + \beta[M] \quad (8)$$

where α and β are defined constants.

4. SOIL EFFECTS INCORPORATION

As it was mentioned in the introduction and showed in Fig. 2a, it is possible to replace the vertical dynamic soil response as a set of springs and dashpots that are frequency function. In order to model the soil effect, it is considered the Fig. 6 below.

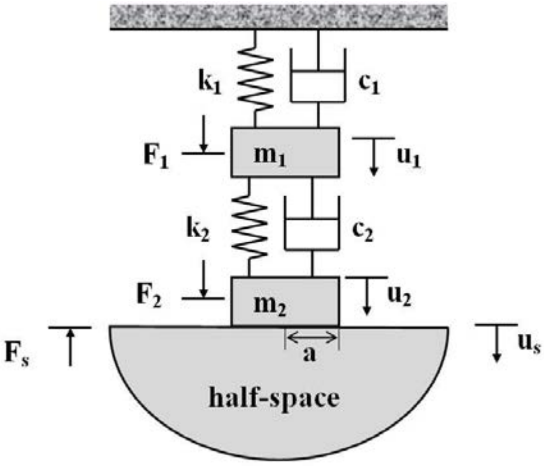

Figure 6. Soil-structure interaction model.

The system shown in Fig. 6 leads to the following motion equation:

$$
\begin{aligned}
& m_1\,\ddot{u}_1 + (c_1 + c_2)\dot{u}_1 - c_2\,\dot{u}_2 + (k_1 + k_2)u_1 - k_2\,u_2 = F_1 \\
& m_2\,\ddot{u}_2 - c_2\,\dot{u}_1 + c_2\,\dot{u}_2 - k_2\,u_1 + k_2\,u_2 = F_2 - F_s
\end{aligned}
\qquad (9)
$$

Remembering that:

$$
a\;G_s\left[\,S\left(\omega\right)\right]\;u_s = F_s \qquad \text{and} \qquad u_2 = u_s \qquad (10)
$$

and substituting eqs. (10) into eq. (9), one can conclude that:

$$
\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & k_2 + a\,G_s[S(\omega)] \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \qquad (11)
$$

Based on Eq. (11), it is possible to conclude that the soil affects the structure stiffness and it can be seen by the soil compliance added to the structure DOF in contact to it in the global stiffness matrix.

5. RESULTS

5.1. Validation example: bar-beam element analysis

In order to validate the BEM-FEM code developed, one example is analyzed. Fig. 7 shows a cantilever beam, whose mesh contains 20 finite elements, supported by a foundation bonded to the soil with all theirs geometrical characteristics and material properties.

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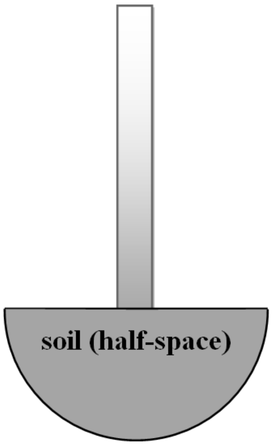


Cantilever properties
$L = 2$ m
$A = 0.25$ m^2
$I = 5.208\ 10^{-3}$ m^4
$E = 210.10^7$ Pa
$\rho = 7800$ kg/m^3

Foundation- soil properties
$a = 1$ m
$G_{soil} = 1$ Pa
$\nu_{soil} = 0.25$
$\rho_{soil} = 1$ kg/m^3
$\eta_{soil} = 0.05$

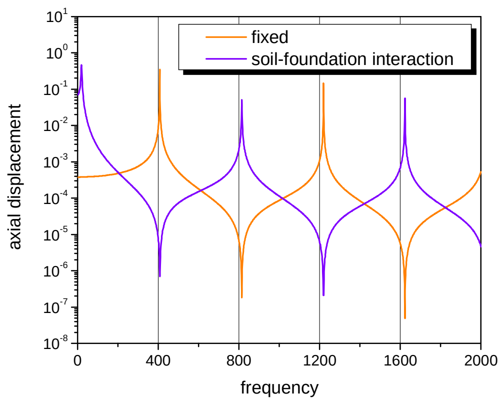
Figure 7. Cantilever beam supported by a foundation bonded to the soil.

Table 1 shows the first three analytical eigenfrequencies for the cantilever beam, without interacting with the soil, both in the axial and transversal direction:

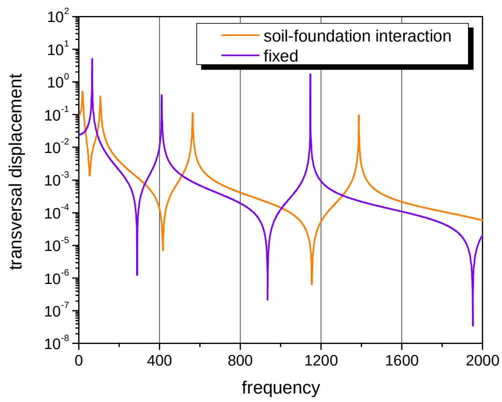
Table 1. Cantilever eigenfrequencies.

Axial	Transversal
$\omega_1 = 407.5\,rad/s$ $\omega_2 = 1222.5\,rad/s$ $\omega_3 = 2037.6\,rad/s$	$\omega_1 = 65.8\,rad/s$ $\omega_2 = 412.5\,rad/s$ $\omega_3 = 1155.2\,rad/s$

Figures. 8a and 8b show the Frequency Response Function (FRF) of the cantilever beam, both in the axial and transversal directions respectively. This structure also presents as the rigid-body axial eigenfrequency the analytical value $\omega_{rb} = 19.2\,rad/s$. This last value is used to determinate the cantilever beam first eigenfrequency under the soil-foundation interaction.



a)



b)

Figure 8. a) Cantilever beam FRF (axial); b) Cantilever beam FRF (transversal).

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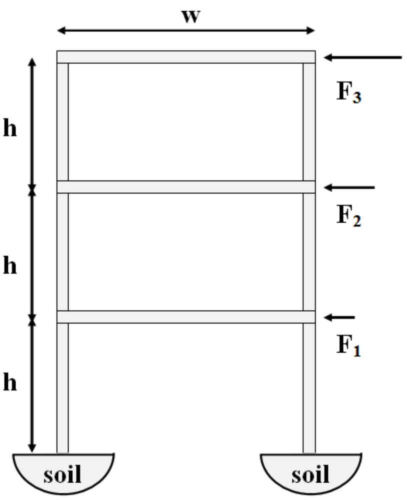
5.2. Flat Frame Building Plane Structure

Since the former results are according to the literature, it is possible to evaluate the dynamic behavior of a generic frame-foundation-soil system, like the flat frame building presented in Fig. 9, which simulates the wind effect.



Frame properties
$h = 2\ m$
$w = 3\ m$
$A = 0.00741\ m^2$
$I = 8.728E^{-5}\ m^4$
$E = 210.10^9\ Pa$
$\rho = 7800\ kg/m^3$
$F_3 = 300\ N$
$F_2 = 200\ N$
$F_1 = 100\ N$

Foundation-soil properties
$a = 1\ m$
$G_{soil} = 1\ Pa$
$\nu_{soil} = 0.25$
$\rho_{soil} = 1\ kg/m^3$
$\eta_{soil} = 0.05$

Figure 9. Flat frame building interacting with soil-foundation.

Figures. 10a and 10b show the frame FRF, for 18 and 27 elements for both cases where the structure is fixed over a rigid base as well interacting with the soil-foundation. The point considered to build the FRFs is located at the upper left corner

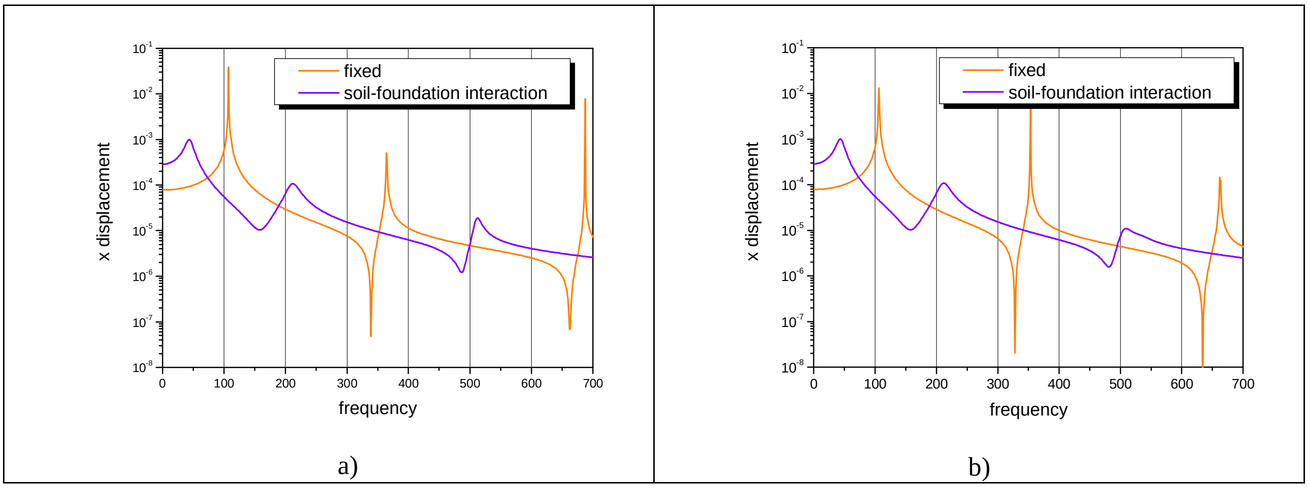


Figure 10. a) Frame FRF (18 elements); b) Frame FRF (27 elements).

By analyzing Fig. 10a and 10b, the first aspect to notice is related to meshes convergence: results for 18 and 27 elements are very close. The second aspect is related to the soil coupling: the peak and the value of eigenfrequencies decreased, which was expected according to literature.

6. CONCLUSIONS

The paper analyzes the influence of soil on the dynamic response of frame-foundation systems. In this study two distinct frame models have been considered. Initially, a frame with a known analytical behavior coupled to a rigid foundation bonded to a 3D half-space was considered just to validate the code. The second analysis deal with a flat frame building with the same coupling considered before simulating the wind effects.

Both analysis considered BE and FE convergence meshes as well the structures modal analysis without considering the soil effects. Finally the BEM-FEM coupling was developed to evaluate the frame-foundation-soil system behavior.

A more detailed analysis of the system response must be performed. However, the BEM has shown to be an efficient and versatile numerical tool to synthesize the dynamic response of unbounded domains.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE

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