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# A PRACTICAL INVESTIGATION OF $H_\infty$ LOOP SHAPING CONTROLLERS USING THE LMI FRAMEWORK FOR A HOVER SYSTEM

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***Abstract.*** *The design of a static parametric $H_\infty$ loop shaping controller using a linear matrix inequality (LMI) approach for a hover system is presented. The main steps to obtain such controller are explained. The parametric $H_\infty$ loop shaping technique explores more design flexibility by introducing a free parameter into the design process that ensures robust stabilization with regard to normalized coprime factor uncertainty of the shaped plant. The shaped plant is designed based on the closed-loop design specification by appropriately choosing the weighting transfer function matrices. Several controllers were designed using this methodology and were then evaluated with sets of specifications for robust stability and performance. This hover system is a laboratory system produced by Quanser Consulting and simulates typical behaviors of a VTOL ("vertical taking-off landing") aircraft, also known as X4-flyer. The dynamics of the hover can be described by a 6th order model taking as state variables the angles of roll, pitch, yaw and associated rates. Candidate controllers must solve the problem of tracking reference trajectories, ensure performance specifications and guarantee robust stability of the hover. Finally, we present experimental results obtained with the implementation of the designed controllers in the context of tracking problems of this hover system.*

***Keywords:*** *Robust control , $H_\infty$ loop shaping controllers, LMI, hover system.*

## 1. INTRODUCTION

In recent years, linear matrix inequalities (LMI) techniques, have become an essential tool for the analysis and synthesis of control systems in the area of robust control (Isidori and Astolfi, 1992). This is because LMI techniques offer the advantage of operational simplicity when compared to classical approaches which involve Riccati equations. Furthermore, based on interior-point algorithms, solving LMIs nowadays can be performed efficiently (S. Boyd and Balakrishnan, 1994), (Gahinet and Apkarian, 1994), (Gahinet, 1996). In the context of analysis and control synthesis of robust controllers, several performance indices could be used, but in this paper the focus will be on the use of the $H_\infty$ norm, which is useful to measure a system's capacity to reject energy bounded disturbances (V.F Montagner and Peres, 2005).

The strategies of robust controller design based on $H_\infty$ synthesis techniques result in advantages over classical design methods. This is due fundamentally to the ease of making trade-offs between performance and robustness to plant uncertainly. Mixing the concepts of classical control and $H_\infty$ optimization technique, the $H_\infty$ loop shaping method introduced by (McFarlane and Glover, 1992) has been successfully applied to several industrial and aeronautical problems (Skogestad and I.Postlewaite, 2005). The method consists basically in a two-stage design. First, loop shaping is used to shape the singular values of the nominal plant to give desired open-loop properties at high and low frequencies. Then, in a second step, a $H_\infty$ controller is calculated to guarantee robust stabilization.

Many techniques have been proposed exploiting the characteristics of the $H_\infty$ loop shaping method using LMI. A novel technique for design of a static $H_\infty$ loop shaping controller with high performance was reported in the literature (Prempain and Postlethwaite, 2005). For the existence of static $H_\infty$ loop shaping controllers, a set of sufficient conditions in LMI was derived. A $H_\infty$ loop shaping controller is introduced in cascade with weight functions that shape the singular values of the nominal plant in open-loop. Another important contribution using a LMI approach is given in (S. Patra and Ray, 2011b) where an alternative but simple technique for the design of a parametric $H_\infty$ loop shaping controller is derived from an observer-controller framework.

The purpose of this paper consists in extending the procedure of static output feedback $H_\infty$ loop shaping controller proposed by (Prempain and Postlethwaite, 2005) using a free parameter to explore more design flexibility of the method in order to increase the stability margins and consequently the robustness of controlled systems. The effectiveness of the design method was evaluated using a 3DOF Hover didactic plant.

This paper is organized as follow. Section 2 contains the case-study system description. Section 3 presents the robust control methodology. Section 4 describes the design of static parametric $H_\infty$ loop shaping controllers. Section 5 shows the results and discussions. Finally, section 6 contains the conclusion.

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## 2. DESCRIPTION OF THE 3DOF HOVER

The 3DOF Hover didactic plant, presented in Fig. 1, is formed by a frame with four propellers. The system is assembled on a pivot joint that enables rotations about the yaw, roll and pitch axes. The plant base is fixed to the workbench, having sliprings which allow the free movement on the yaw axis with low friction. Each propeller generates a lift force which is used to control the roll and pitch angles.

Figure 1. 3DOF Hover didactic plant.

Consequently, the torque resulting from propeller rotation causes the movement of the structure around the yaw axis. In the case of a controlled environment, with the four forces balanced, the total torque is matched. In this study the model from the manufacturer's manual is used (Quanser, 2009).

When a positive voltage is applied to any motor, a lift force is generated causing a lifting of the propulsion system. The group formed by front and back motors (supply voltages given by $v_f$ and $v_b$) causes the movement on the pitch and yaw axes, while the lateral motors (analogously $v_r$ and $v_l$) entail movement about the roll and yaw axes (Quanser, 2009).

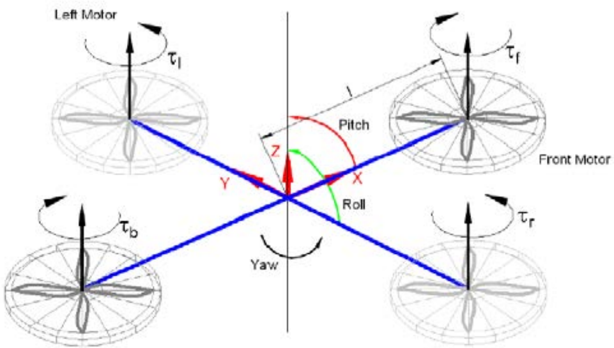


Figure 2. 3DOF Hover dynamic system.

The system has three encoders which measure the angular displacements in the three freedom axes of the plant from an initial position. Assuming a linear model (with the equilibrium point in which the propellers are aligned with the axes $X$, $Y$ and $Z$, Fig. 2), the pitch movement can be described as:

$$J_p \frac{\partial^2 p}{\partial t^2} = l K_f \left( v_f - v_b \right), \tag{1}$$

where, $J_p$ is the equivalent moment of inertia about the pitch axis, $p$ is the pitch angle, $l$ is the distance from pivot to each motor and $K_f$ is the propeller force-thrust constant (Cavalca and Kienitz, 2009). In a similar manner, for the roll movement:

$$J_r \frac{\partial^2 r}{\partial t^2} = l K_f \left( v_r - v_l \right), \tag{2}$$

in which, $J_r$ is equivalent moment of inertia about the roll axis and $r$ is the roll angle. The torques generated by the front and back propellers are called $\tau_f$ and $\tau_b$, and similarly, the torques generated by the right and left propellers are $\tau_r$ and $\tau_l$.

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As shown in Fig. 2, the torque generated by lateral propellers has a reverse direction compared to the torque generated by front and back propellants. The yaw movement is given by:

$$J_y \frac{\partial^2 y}{\partial t^2} = \tau_f + \tau_b + \tau_r + \tau_l \tag{3}$$

$$J_y \frac{\partial^2 y}{\partial t^2} = K_{t,c}(v_f + v_b) + K_{t,n}\,(v_r + v_l) \tag{4}$$

in which $J_y$ is the equivalent moment of inertia about the yaw axis, $y$ is the yaw angle, $K_{t,n}$ and $K_{t,c}$ are torque constants that relate generated torque and voltage applied to the motor (Cavalca and Kienitz, 2009). Finally, the linear model of the 3DOF Hover is,

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{K_{t,c}}{J_y} & \frac{K_{t,c}}{J_y} & \frac{K_{t,n}}{J_y} & \frac{K_{t,n}}{J_y} \\ l\frac{K_f}{J_p} & -l\frac{K_f}{J_p} & 0 & 0 \\ 0 & 0 & l\frac{K_f}{J_r} & -l\frac{K_f}{J_r} \end{bmatrix} \tag{6}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

The 3DOF Hover plant parameters are presented in Table 1 (Quanser, 2009).

Table 1. 3DOF Hover didactic plant parameters.

| Symbol | Value | Unit |
|---|---|---|
| $K_{t,n}$ | 0.0036 | $N.m/V$ |
| $K_{t,c}$ | -0.0036 | $N.m/V$ |
| $K_f$ | 0.1188 | $N/V$ |
| $l$ | 0.197 | $m$ |
| $J_{yh}$ | 0.110 | $kg.m^2$ |
| $J_{ph}$ | 0.0552 | $kg.m^2$ |
| $J_{rh}$ | 0.0552 | $kg.m^2$ |

## 3. CONTROL DESIGN METHODOLOGY: $H_\infty$ LOOP SHAPING TECHNIQUES

The loop-shaping design procedure described in this section is based on $H_\infty$ robust stabilization combined with classical loop shaping, as proposed by (McFarlane and Glover, 1992).

It is essentially a two-stage design process. First, the open-loop plant is augmented by pre ($W_1$) and post-compensators ($W_2$) to give a desired shape to the singular values of the open-loop frequency response. In the second step, we calculate a $H_\infty$ controller $K$ which guarantees,

$$\left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - G_s K)^{-1} M^{-1} \right\|_\infty \leq \gamma, \qquad G_s = W_2 G W_1, \tag{9}$$

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where $G$ is the nominal plant and $G_s = M^{-1}N$ is a normalized left coprime factorization of the shaped plant $G_s$.

The controller $K$ is synthesized by solving the robust stabilization problem of Eq. (9). The feedback controller for the plant $G$ is then $K_\infty = W_1 K W_2$. The Linear Matrix Inequality formulation of the $H_\infty$ control problem allows one to take into account the order of the controller in the synthesis procedure.

Here a static parametric controller is considered. Let $G_s$ be a proper plant of order $n$ having a stabilizable and detectable realization (10) with $A \in \Re^{n\times n}$, $B \in \Re^{n\times n_u}$ and $C \in \Re^{n_y\times n}$.

$$G_s = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \tag{10}$$

The factors of normalized left coprime factorization of $G_s = M^{-1}N$, have realizations given by

$$\left[\begin{array}{cc} N & M \end{array}\right] = \left[\begin{array}{c|cc} A+HC & B+HD & H \\ \hline E^{-\frac{1}{2}}C & E^{-\frac{1}{2}}D & E^{-\frac{1}{2}} \end{array}\right] \tag{11}$$

with $H = -\left(BD^T + XC^T\right)E^{-1}$, $E = \left(I + DD^T\right)$ and the matrix $X$ being the unique symmetric positive semi-definite solution to the Algebraic Riccati Equation (ARE),

$$\begin{array}{l} \left(A - BF^{-1}D^TC\right)X + X\left(A - BF^{-1}D^TC\right)^T \\ \quad -XC^TE^{-1}CX + BF^{-1}B^T = 0 \end{array} \tag{12}$$

with $F = I + D^TD$. From the normalized left coprime factorization of $G_s$, the generalized plant corresponding to parametric $H_\infty$ loop shaping procedure is given by

$$P = \left[\begin{array}{c|c|c} A & -HE^{1/2} & \frac{B}{\beta} \\ \hline C & E^{1/2} & D \\ 0 & 0 & I_{n_u} \\ \hline C & E^{1/2} & D \end{array}\right] \tag{13}$$

Making $\left\|\left[\begin{array}{cc} \beta^{-1}\Delta_N & \Delta_M \end{array}\right]\right\|_\infty \le 1/\gamma$ in the normalized coprime factor robust stabilization framework, the static controller $K$ will stabilize the system in Fig. 3 and is synthesized satisfying,

$$\left\|\left[\begin{array}{c} \beta K \\ I \end{array}\right](I - G_s\beta K)^{-1}M^{-1}\right\|_\infty \le \frac{1}{\varepsilon_{\max}} = \gamma, \tag{14}$$

where $\varepsilon_{\max}$ is the maximum achievable robust stability margin (Skogestad and I.Postlewaite, 2005). It is observed that for different values $\beta$, different normalized coprime factors can be obtained. Thus $\beta$ can be made a parameter to increase the flexibility of the design. For a detailed presentation of the method refer to (S. Patra and Ray, 2011b).

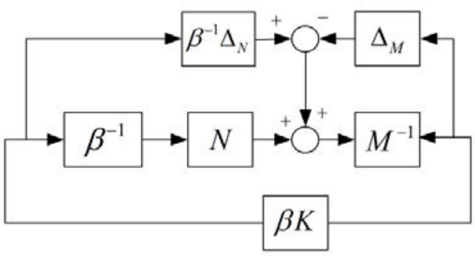


Figure 3. Block diagram for the parametric $H_\infty$ loop shaping controller.

For the generalized plant $P$, synthesis conditions are now presented to design a static parametric $H_\infty$ loop shaping controller. We have the following result which is a generalization of a theorem by (Prempain and Postlethwaite, 2005):

*Theorem 1*: For given values $\beta > 0$, if there exists a positive definite symmetric solution $R$ for inequalities (15) and (16), then there exists a stabilizing static parametric controller $K$ that satisfies (14):

$$(A + HC)R + R(A + HC)^T < 0 \tag{15}$$

$$\left(\begin{array}{ccc} AR + RA^T - \gamma\beta^{-2}BB^T & RC^T - \gamma\beta^{-1}BD^T & -HE^{1/2} \\ RC - \gamma D\beta^{-1}B^T & -\gamma E & E^{1/2} \\ -E^{1/2}H^T & E^{1/2} & -\gamma I_{n_y} \end{array}\right) < 0 \tag{16}$$

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*Proof:* The proof of this theorem follows the proof for the existence of parametric loop-shaping controller without $\beta$ presented in (S. Patra and Ray, 2011b) and uses the following definitions and considerations. Consider a generalized plant $P$ given by the state-space realization in (Prempain and Postlethwaite, 2005)

$$P = \left[\begin{array}{c|cc} A & B_w & B \\ \hline C_z & D_{zw} & D_z \\ C & D_w & 0 \end{array}\right] \tag{17}$$

There exists a $H_\infty$ stabilizing controller such that $\|T_{zw}\|_\infty \leq \gamma$, where $\gamma > 0$, if and only if, there exist $R > 0$, $S > 0$ which satisfy the following inequalities (S. Boyd and Balakrishnan, 1994), (Iwasaki and Skelton, 1994):

$$\begin{pmatrix} N_z & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} AR + RA^T & RC_z^T & B_w \\ C_zR & -\gamma I & D_{zw} \\ B_w^T & D_{zw}^T & -\gamma I \end{pmatrix} \begin{pmatrix} N_z & 0 \\ 0 & I \end{pmatrix} < 0 \tag{18}$$

$$\begin{pmatrix} N_w & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} SA + A^TS & SB_w & C_z^T \\ B_w^TS & -\gamma I & D_{zw}^T \\ C_z & D_{zw} & -\gamma I \end{pmatrix} \begin{pmatrix} N_w & 0 \\ 0 & I \end{pmatrix} < 0 \tag{19}$$

$$R = S^{-1} \tag{20}$$

where $N_z$ and $N_w$ define bases of the null spaces of $(B^T, D_z^T)$ and $(C, D_w)$ respectively. Now, comparing Eq. (13) with Eq. (17), one obtains the associations

$$\begin{array}{c} A = A, \quad B_w = -HE^{1/2}, \quad B = \beta^{-1}B, \quad C_z = \begin{bmatrix} C \\ 0 \end{bmatrix}; \\ D_{zw} = \begin{bmatrix} E^{1/2} \\ 0 \end{bmatrix}, \quad D_z = \begin{bmatrix} D \\ I_u \end{bmatrix}, \quad C = C, \quad D_w = E^{1/2} \end{array} \tag{21}$$

Taking into account the size of the matrices and their ranks, and considering the special case $D = 0$, the base of the null space of $\begin{bmatrix} (\beta^{-1})B^T & 0 & I \end{bmatrix}$ is chosen as

$$N_z = \begin{bmatrix} I & 0 \\ 0 & I \\ -\beta^{-1}B^T & 0 \end{bmatrix} \tag{22}$$

and the base of the null space of $\begin{bmatrix} C & I \end{bmatrix}$ is chosen as

$$N_w = \begin{bmatrix} E^{1/2} \\ -E^{1/2}C \end{bmatrix} \tag{23}$$

With the expressions above, necessary conditions for the existence of static parametric $H_\infty$ loop shaping controller are obtained. The $K$ controller is found solving the following LMIs (Prempain and Postlethwaite, 2005), (S. Patra and Ray, 2011a), (Kannan Natesan and Postlethwaite, 2007):

$$\Psi_R + \Omega^T K \Theta_R + \Theta_R^T K \Omega < 0 \tag{24}$$

where

$$\Psi_R = \begin{bmatrix} AR + RA^T & 0_{n\times n} & RC^T & -HE^{1/2} \\ 0_{n\times n} & -\gamma I_n & 0_{n\times n_y} & 0_{n\times n_y} \\ CR & 0_{n_y\times n} & -\gamma I_{n_y} & E^{1/2} \\ -E^{1/2}H^T & 0_{n_y\times n} & E^{1/2} & -\gamma I_{n_y} \end{bmatrix} \tag{25}$$

$$\Omega = \begin{bmatrix} \beta^{-1}B & I_{n\times n_y} & D & 0_{n_y} \end{bmatrix}^T \tag{26}$$

$$\Theta_R = \begin{bmatrix} CR & 0 & 0 & E^{1/2} \end{bmatrix} \tag{27}$$

*Remark 1:* For given values of $\beta > 0$, (15) and (16) give a sufficient condition for synthesis of the static parametric $H_\infty$ loop shaping controller.

## 4. DESIGN OF THE STATIC PARAMETRIC $H_\infty$ LOOP SHAPING CONTROLLERS

To demonstrate the effectiveness of the proposed methodology, four different $\beta$ parameters will be used. The objective of the static parametric $H_\infty$ loop shaping controllers is to solve the problem of tracking reference trajectories, and ensure performance and stability in spite of disturbance and noise.

To eliminate the steady state error, integrators are inserted into at system output for yaw, pitch and roll angle. With insertion of integrators the order of the system increases from 6 to 9. We will select the pre and post-compensators as described in the section above, in order to obtain desired open-loop properties at high and low frequencies.

### 4.1 Selecting Pre and Post-Compensators

The formatting process of the plant is implemented by adding pre-and post-compensators to the plant $G$, such that:

$$G_s = W_2 G W_1 \tag{28}$$

where $W_1$ is the pre-compensator. To select the elements of a diagonal pre-compensator normally means to obtain high gain at low frequencies, roll-off rates of approximately 20 dB/decade at the desired bandwidth(s), with higher rates at high frequencies (Skogestad and I.Postlewaite, 2005). Some trial and error is involved here. $W_2$ is usually chosen as a constant, reflecting the relative importance of the outputs to be controlled and the other measurements being fedback to the controller. Thus, we chose $W_1$, and $W_2$, respectively, as

$$W_1 = \begin{bmatrix} \frac{s+1}{0.1s+1} & 0 & 0 & 0 \\ 0 & \frac{s+1}{0.1s+1} & 0 & 0 \\ 0 & 0 & \frac{s+1}{0.1s+1} & 0 \\ 0 & 0 & 0 & \frac{s+1}{0.1s+1} \end{bmatrix} \tag{29}$$

$$W_2 = diag\begin{bmatrix} 25 & 25 & 25 & 25 & 35 & 35 \end{bmatrix} \tag{30}$$

With these choices the shaped plant has been defined. The frequency response of the shaped plant in comparison to nominal plant is presented in Figure 4.

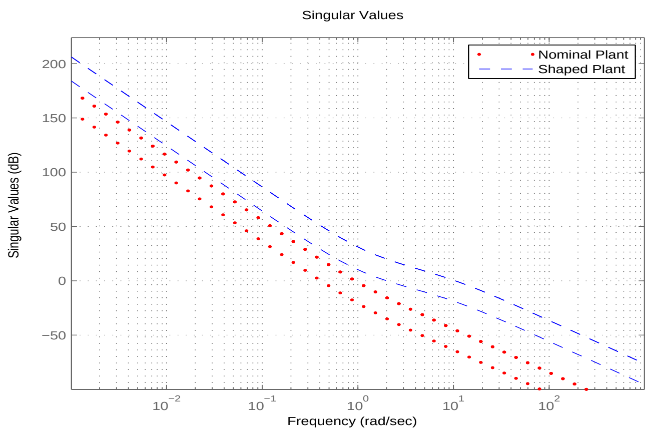


Figure 4. Singular values of plant and shaped plant.

Next, the controller $K$ is synthesized by solving the robust stabilization problem (24) for the shaped plant $G_s$ with a normalized left coprime factorization $G_s = M^{-1}N$ . The feedback controller for the plant $G$ is then $K_\infty = W_1 K W_2$ (Skogestad and I.Postlewaite, 2005). Below we summarize the methodology used.

### 4.2 Design procedure

1. Select $W_1$ and $W_2$ to get a desired open-loop shape and compute the shaped plant $G_s = W_2 G W_1$. Consider ($A$, $B$, $C$, $D$) a realization of $G_s$.
2. Next, determine the matrix $X > 0$ and choice a scalar variable $\beta > 0$ that solve the LMI system (15), (16). Then determine $\gamma$ that satisfy the robust stability margin.
3. If the LMI system (15) and (16) is feasible, then determine the Lyapunov matrix $R > 0$ and $\gamma$ to solve the LMI problem (24).
4. Determine the feedback controller for implementation, $K_\infty = W_1 K W_2$ (Prempain and Postlethwaite, 2005).

## 5. RESULTS AND DISCUSSION

The synthesis procedure described in Section 4 was implemented using the softwares Matlab 7.10.0, SeDuMi and Yalmip (Lofberg, 2004). In Table 2, $\gamma_{opt}$ is shown for the different values of $\beta$. This table allows for a performance comparison of the designed controllers.

Table 2. Static parametric $H_\infty$ loop shaping controller design results using the Theorem 1 applied in 3DOF Hover.

| Controllers | $\beta$ | $\gamma_{opt}$(LMI approach) |
|---|---|---|
| I | 0.5 | 3.06 |
| II | 0.8 | 3.43 |
| III | 1 | 3.70 |
| IV | 1.2 | 3.97 |

In Fig. 5 the step responses of the closed-loop system are presented. With these results, it was possible to further verify and analyze the performance of the static $H_\infty$ loop shaping controllers implemented for the 3DOF Hover. Taking into account the settling time and overshoot obtained with the several controllers designed, it is observed that the controller I has a better performance than the $H_\infty$ loop shaping controller using the conventional formulation (controller III)(Prempain and Postlethwaite, 2005) (Fig. 6).

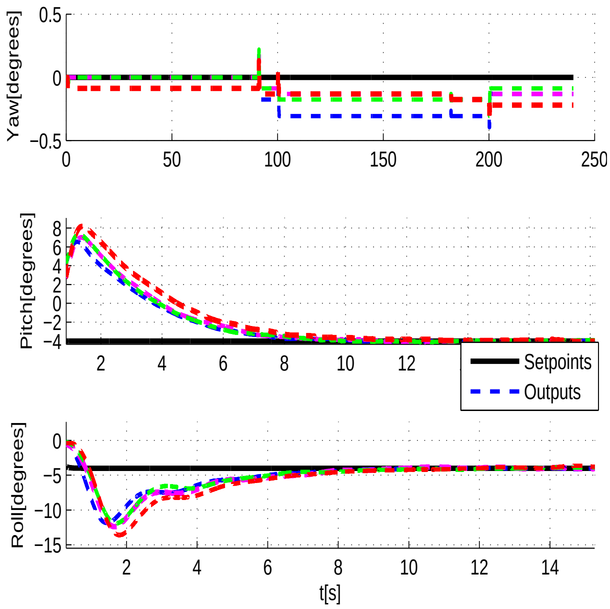


Figure 5. Step response for different values of $\beta$ using static parametric $H_\infty$ loop shaping controllers.

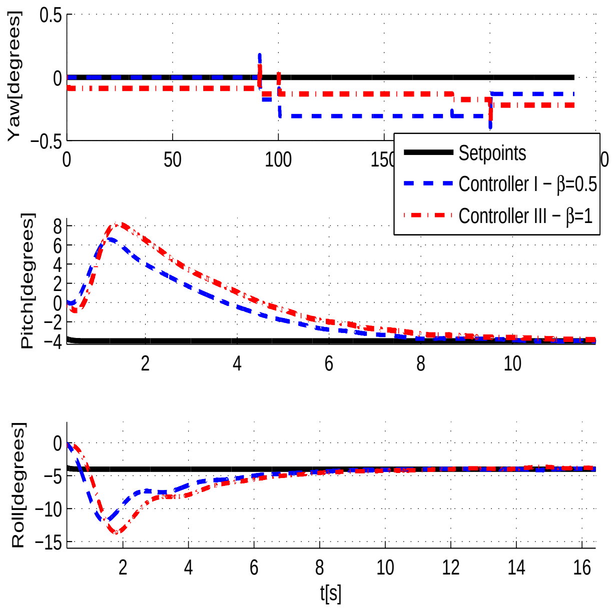


Figure 6. Step response for controllers I and III using static parametric $H_\infty$ loop shaping controller.

Another way to analyze the performance of the controllers, is using the $S$ - $T$ symmetric MIMO gain (GM) and phase (PM) margins, defined in (Prempain and Postlethwaite, 2005)

$$GM = \left[ \frac{1-r_m}{1+r_m}, \quad \frac{1+r_m}{1-r_m} \right] \tag{31}$$

$$PM = \left[ -2\arctan\left(r_m\right), 2\arctan\left(r_m\right) \right] \tag{32}$$

where $1/r_m = \inf_\omega \bar{\sigma}\left(I - L(jw)\right)\left(I + L(jw)\right)^{-1}$ and $L(jw)$ is the open loop frequency response at the plant output. The results are consistent with those obtained previously. Table 3 shows the margins obtained with controllers I and III. These results again show a slight superiority of the controller I.

Table 3. Stability margins of the controllers designed.

| Controllers | Gain margin (GM) | Phase margin (PM) |
|---|---|---|
| I | 2.43 dB | 41.42 |
| III | 2.14 dB | 40.15 |

## 6. CONCLUSION

This paper discusses the design of static $H_\infty$ loop shaping controllers. The controllers are said to be static, but in fact they will contain pre-and pos-compensator dynamics that were used to shape the open-loop plant frequency response. For existence of such controllers a set of solvability conditions is given. The methodology addresses parametric $H_\infty$ loop shaping in a LMI framework. Although this control approach isn't guaranteed to find a static parametric $H_\infty$ loop shaping controller, it has proven to be successful in many plants before and has been successful applied to a 3DOF Hover didactic plant design.

Robustness analysis and experiments illustrate that the controller obtained can be an advantageous alternative to the static compensators. It is clear that better performance could have been achieved with a better Hover model.

## 7. ACKNOWLEDGEMENTS

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## 8. REFERENCES

Cavalca, M.S.M. and Kienitz, K.H., 2009. "Application of TFL/LTR robust control techniques to failure accommodation". In *20th International Congress of Mechanical Engineering*.

Gahinet, P., 1996. "Explicit controller formulas for LMI based $H_\infty$ synthesis". *Automatica*, Vol. 32, pp. 1007–1014.

Gahinet, P. and Apkarian, P., 1994. "A linear matrix inequality approach to $H_\infty$ control". *International Journal of Robust and Nonlinear Control*, Vol. 4, pp. 421–448.

Isidori, A. and Astolfi, A., 1992. "Disturbance attenuation and $H$-infinity- control via mensurement feedback in nonlinear-systems". *IEEE Transactions on Automatic control*, Vol. 37, pp. 1283–1293.

Iwasaki, T. and Skelton, R.E., 1994. "All controllers fo the general $H_\infty$ control problem". *Automatica*, Vol. 30, pp. 1307–1317.

Kannan Natesan, D.W.G. and Postlethwaite, I., 2007. "Design of static $H_\infty$ linear parameter varying controllers for unmanned". *Journal of Guidance, Control and Dynamics*, Vol. 30, pp. 1822–1827.

Lofberg, J., 2004. "Yalmip: A toolbox for modeling and optimization in matlab". *Proc. 2004 IEEE Intelligent symposium on Computacional Aided Control System*, pp. 284–289.

McFarlane, D. and Glover, K., 1992. "A loop shaping design procedure using $H_\infty$ synthesis". *IEEE Transactions on Automatic control*, Vol. 37, pp. 759–769.

Prempain, E. and Postlethwaite, I., 2005. "Static $H_\infty$ loop shaping control of a fly-by-wire helicopter". *Automatica*, Vol. 41, pp. 1517–1528.

Quanser, I., 2009. "3 DOF hover - control manual". Technical report.

S. Boyd, L. El Ghaoui, E.F. and Balakrishnan, V., 1994. *Linear matrix inequalities in system and control theory*. SIAM Studies in applied Mathematics.

S. Patra, S.S. and Ray, G., 2011a. "Design of $H_\infty$ loop-shaping controller for lti system with input saturation: Polytopic gain scheduled". *Journal of Dynamic Systems, Measurement, and Control*, Vol. 133.

S. Patra, S.S. and Ray, G., 2011b. "A linear matrix inequality approach to parametric $H_\infty$ loop shaping control". *Journal of the Franklin Institute*, Vol. 348, pp. 1832–1846.

Skogestad, S. and I.Postlewaite, 2005. *Multivariable feedback control: Analysis and Design*. New York: Wiley.

V.F Montagner, R.C.L. F Oliveira, V.L. and Peres, P., 2005. "LMI approach for $H_\infty$ linear parameter-varying state feedback control". *IEEE Proc. Control Theory and Application*, Vol. 152, pp. 195–201.

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