TOPOLOGICAL OPTIMIZATION OF COMPOSITE AIRCRAFT WING RIB

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Abstract. *Topological optimization gets the best material distribution in a structure, maximizing and/or minimizing parameters of interest. It can be applied to isotropic materials, usually resulting in truss structures. However, it has limitations for use in laminated composite structures. Topometry optimization is a methodology recently developed for commercial use in the GENESIS software. It is a type of sizing optimization where each finite element can be individually designed. With the use of penalization of design variables it is possible to obtain similar results to those of topology optimization. However, topometry optimization can be applied to composite laminates. Using the software GENESIS, both methodologies are applied to the optimization of a rib which is part of an aircraft wing. The topological optimization is applied using an isotropic material while the topometry optimization with penalty factors is applied to the same structure made of composite material. The results are similar and appoint to a truss structure as the optimal rib layout.*

Keywords*: Topology optimization, Topometry optimization, Composite Plates, Wing Rib.*

1. INTRODUCTION

The main purpose of this study is to apply the two methodologies, topology and topometry optimization, to define the layout of composite plates. Using the software GENESIS, one applies the topological optimization for a metal material and the topometry optimization with penalty factors for a composite material. This study is firstly carried out in a simpler problem belonging to the literature in topological optimization. Then, both methodologies are applied to a rib from a central wing box beam.

In this work the rib under study is isolated from the rest of the wing structure finite element model. This makes possible to obtain satisfactory results in the optimization, without the influence of the adjacent structural parts. Only a critical load case is used.

2. DEVELOPMENT

The topology optimization purpose is to determine the best material distribution within a given domain. The topology optimization consists in opening cavities in the structure. The design variables are the finite element material densities, which are supposed to be discrete, converging to 0 or to the nominal value for the material density at the end of the optimization cycles. Elements with zero density are discarded, while elements with nominal density are maintained (Bendsøe and Sigmund, 2004).

The topometry optimization can be considered an evolution of the traditional size optimization and was recently developed in the GENESIS software. It allows each finite element to be sized individually. By using a penalty technique it can achieve similar results to topology optimization (Leiva *et al.*, 2007).

In GENESIS, the topology optimization tool is available only to finite element models of isotropic material. Meanwhile, topometry optimization is only applicable to elements that have cross section properties, such as membrane, plate and shell finite elements. So, the solid elements, which are not associated to section properties cannot be optimized topometrically. However, the topometry optimization can be applied to composite laminate plates and shells.

In the classical topology optimization problem, one finds the material distribution which minimizes the strain energy or compliance, and so maximizes the global stiffness (Bendsøe and Sigmund, 2004). The compliance c is given by $f^T u$, where f and u are respectively the vectors of nodal forces and displacements. The topology optimization problem is the following:

$$min\ c = f^T u \tag{1}$$

s.t. $K\,u = f$

$$\Sigma \rho_e v_e \leq volfrac\ V_o\,, \qquad 0 \leq \rho \leq 1 \tag{2}$$

The equality constraints are the finite element equilibrium equations and the inequality constraint is a volume restriction where V_o is the volume of the initial domain and *volfrac* is the volume fraction, whose value is previously chosen. The design variables are the $0 \leq \rho_e \leq 1$, which are referred as material densities (Bendsøe and Sigmund, 2004).

In the SIMP (*Solid Isotropic Material with Penalization*) formulation one has

$$E(x) = \rho(x)^p E_0, \qquad p > 1 \tag{3}$$

In the above equation, $E(x)$ is the interpolation between material properties 0 and the material elastic modulus E_0. This model requires the use of a penalty factor ($p > 1$) so that intermediate values of density tend to be not used. Usually the value of p is 3, because it provides good results with adequate convergence (Bendsøe and Sigmund, 2004). Thus, it is hoped that the elastic modulus value of each element can converge to $E_e(\rho = 0) = 0$, or $E_e(\rho = 1) = E_0$.

Since in finite element equilibrium equations the element stiffness matrices are assembled with the interpolated (penalized) material elastic modulus it becomes uneconomic to have intermediate values of design variables ρ, since the material volume consumed is proportional to the densities. In other words the stiffness has a cubic relationship with the design variable, while the density, which represents the material available has a linear relationship with the design variable.

The topometric optimization can achieve results very similar to those of topology. For this, the cubic relationship between stiffness and design variable must be set, analogously to the SIMP model. The thickness in this case is a parameter that represents the structural stiffness. The interpolation between 0 and the thickness t_0 is defined by a cubic law, similar to the SIMP formulation for densities (Leiva *et al.*, 2001).

The main ingredients of the topometric formulation are the following:

$$0.001 \leq x_i \leq 1 \tag{4}$$

$$t_i = t_0 x_i^3 \tag{5}$$

$$NSM(x) = \Sigma f(x_i) \tag{6}$$

The design variable x_i is associated to a composite material layer. The *NSM* is a non-structural mass per unit area linear function of the design variables, typical of NASTRAN's type data. A linear relationship between weight and design variables can be defined using the NSM, such that the optimization problem can be made similar to one of topology.

The material density used in this optimization is set to zero so that a more precise linear relationship occurs between the mass and the design variables. When the density is zero, the mass of elements is null. That is, with $\rho = 0$, *mass* = *NSM*(x). Otherwise, with $\rho \neq 0$, *mass* = *element mass* + *NSM*(x).

The Table 1 shows the material model comparison in the problem of topology and topometry.

Table 1. Material model comparison of topology and topometry optimization

	Topology optimization	Topometry optimization
Linear relationship between mass (volume) and the design variable	$V_0 = \Sigma\rho_e v_e$, $0 \leq \rho_e \leq 1$	$NSM(x) = \Sigma f(x_i)$, $0.001 \leq x_i \leq 1$
Cubic relationship between stiffness and the design variable	$E(x) = \rho(x)^3 E_0$	$t_i = x_i^3 t_0$

The problem formulation remains the same as to minimizing the compliance, i.e:

$$\min c = f^T u \tag{7}$$

s.t. $K\,u = f$

But now the volume fraction is applied to the initial mass of the structure, M_o.

$$NSM(x) \leq volfrac\ M_0, \qquad 0.001 \leq x_i \leq 1, \tag{8}$$

The NSM function used in the present work is the following:

$$NSM = \Sigma x_e v_e \qquad (9)$$

The topology and topometry formulations will be respectively applied to isotropic and composite laminated structures.

2.1. Optimization of a clamped plate

The Figure 1 illustrates a finite element model of plate, clamped at the left with nodal forces applied at two central points on the right edge. This simple problem presented in Rozvany (1998) is used for validation and comparison of optimization results.

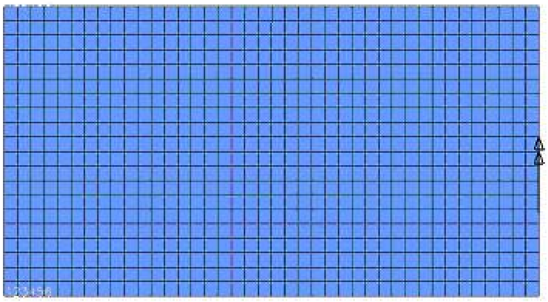


Figure 1. Finite element model of clamped plate

The 200x100mm plate is discretized into 800 elements, each one with a length of 5mm. The plate thickness is 5mm. The Table 2 lists the problem conditions of topology optimization.

Table 2. Problem conditions of topology optimization

Objective function	Minimum strain energy
Constraint	30% of initial mass
Variables	$\rho(x)$

The optimal result for this isotropic plate is shown in Fig. 2, where the remaining elements are the red ones in the last optimization cycle.

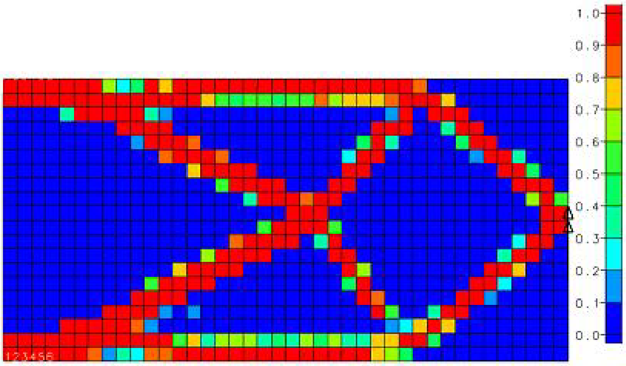


Figure 2. Topology optimization result

The same plate is submitted to topometry optimization, however the material is changed from isotropic to one made of unidirectional fibrous tape layers, having modulus of elasticity in the lamination direction much greater than in the transverse direction. The plate element properties have been configured to an eight-layer laminate $[0/45/-45/90]_s$, as shown in Fig. 3.

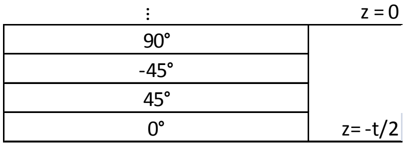


Figure 3. Initial laminate

The optimization parameters are listed in Tab. 3, the variables of interest are the thickness of each layer.

Table 3. Problem conditions of topometry optimization

Objective function	Minimum strain energy
Constraint	30% of initial mass
Variables	$t_i(x)$

The topometry optimization results are shown in Fig. 4, for the directions at 0°, 45°, -45° and 90°.

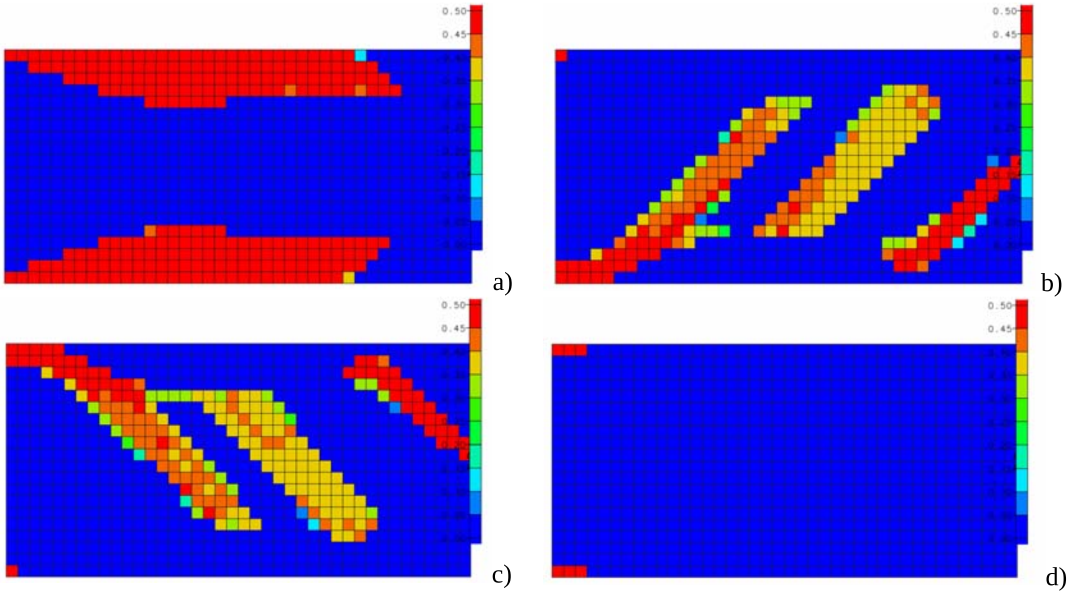


Figure 4. Layers thicknesses distribution: a) Layer 1 – 0°, b) Layer 2 – 45°, c) Layer 3 – -45°, d) Layer 4 – 90°

It may be noted that the layers show a material distribution that follows their lamination directions.
The Figure 5 illustrates the overlap of the eight layers forming the laminate.

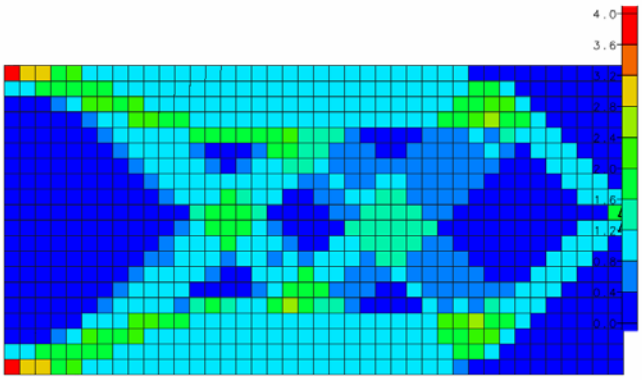


Figure 5. Overlap of layers that forming the laminate

In Fig. 5 a type of truss structure is obtained, very similar to the one designed by topological optimization (Fig. 2). The differences are in the internal angles of the trusses. The trusses in the topological solution are formed in distinct directions to maximize the structural stiffness. The trusses associated to layers in the topometry optimization are formed following layer lamination direction, overlapping in some regions. That is, in the second layer, oriented at 45°, the trusses are formed at 45°, which is the direction of greater stiffness for this layer. Therefore, the results in this type of optimization depend on the direction of the layers available in material data. If the layers were in the directions of ± 60°, one could expect that trusses would appear in these directions to maximize the structural stiffness.

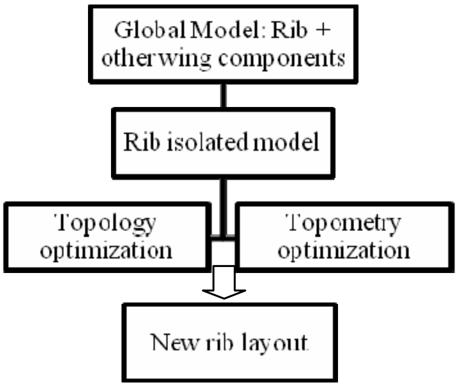
2.2. Optimization of a wing rib

The main components in a wing of an airplane are panels, stringers, spars and the ribs. While the panels, stringers and spars are subject to the main loads, ribs contribute in a smaller portion to the wing stiffness and also support a small portion of the global loads of bending and torsion.

The development of an optimization process of a rib in a wing has great challenges, especially because this component is part of a very redundant structure. The topological optimization is usually applied to the components that have well defined loads, and no flight distributed loads (Krog *et al.,* 2004).

The rib is a secondary structure in the wing. A possibility for rib optimization is to consider the problem locally, and design the rib in such a way to provide sufficient stiffness to support the loads on its boundary. Therefore, in this work, the rib is isolated from the rest of the wing structure.

The following flowchart shows the steps for rib optimization.



A first analysis is done using the global model to determine the forces on rib interface. In Fig. 6 the different colors represents different plate properties.

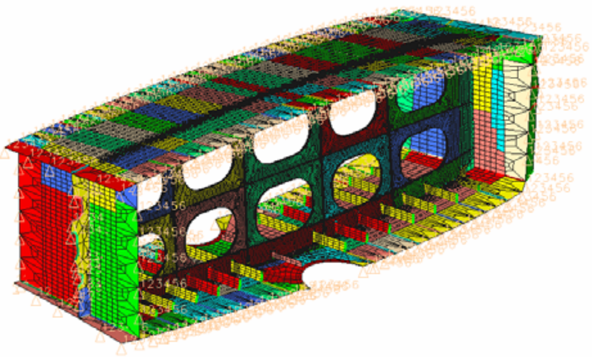


Figure 6. Rib global model with other wing components

The rib global model is loaded with only one selected critical load case and free body forces are obtained due to rib interface with the rest of the structure. Thus, one can simulate the rib isolated model (Fig. 7) with interface loads of the global model. Any difference in the load balance is corrected using the inertia relief resource available in Genesis.

The aluminum rib model can inscribed in a box of approximately 2000x600mm and is discretized into 30288 elements, with an average size of 7mm. The thickness changes along the rib which has a mass of 10.14kg.

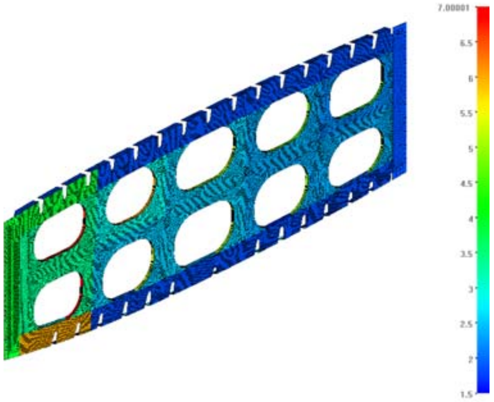


Figure 7. Thicknesses distribution of rib isolated model

A new rib finite element model is set with the same geometry of the original one except that the holes and flanges around them were removed. The result is a flat plate with the loading condition of the original rib, seen in Fig. 8.

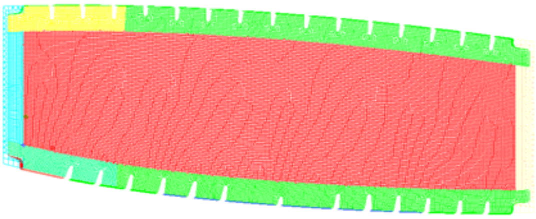


Figure 8. Rib isolated model prepared to optimization

With this model the two optimization techniques, topology and topometry, are used. The rib central region is the initial domain for both methods (Fig. 8), since the frame around this region remains unchanged. The topology optimization is applied to the central region which is of isotropic material. The topometry approach considers the orthotropic layers in the central region.

The problem conditions for topological optimization are listed in Tab. 4.

Table 4. Problem conditions of the rib topology optimization

Objective function	Minimum strain energy
Constraints	30% of initial mass
Variables	$\rho(x)$

The Figure 9 shows the result for the topology optimization. The density is 0 at the blue region, indicating absence of material.

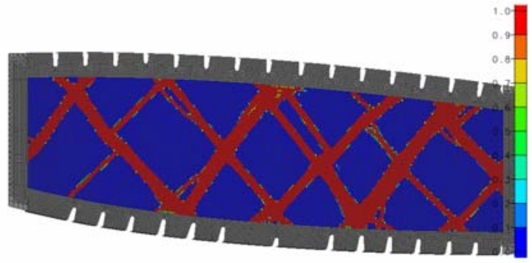


Figura 9. Topology optimization results

The problem conditions of the topometry optimization are listed in Tab. 5.

Table 5. Problem conditions of the rib topometry optimization

Objective function	Minimum strain energy
Constraints	30% of initial mass
Variables	$t_i(x)$

The material is tape layers, with the material axis following the longitudinal rib direction, as in Fig. 10. Lamination positive angles are in a counterclockwise direction and negative angles in clockwise on the figure plane. The material properties have been defined as an 8-layer laminate with configuration $[0/45/-45/90]_s$.

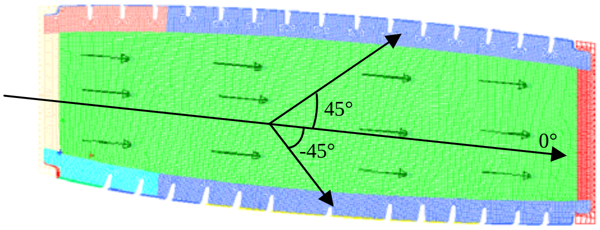


Figure 10. Element material orientation

Figure 11 shows the results obtained for material distribution in the first four layers. The symmetrical layers have exactly the same distribution, and therefore are not shown. There is virtually no material in the layer 1 and layer 4. Layers 2 and 3 show long, narrow stretches of material, which overlap in some points, as shown in Fig. 12.

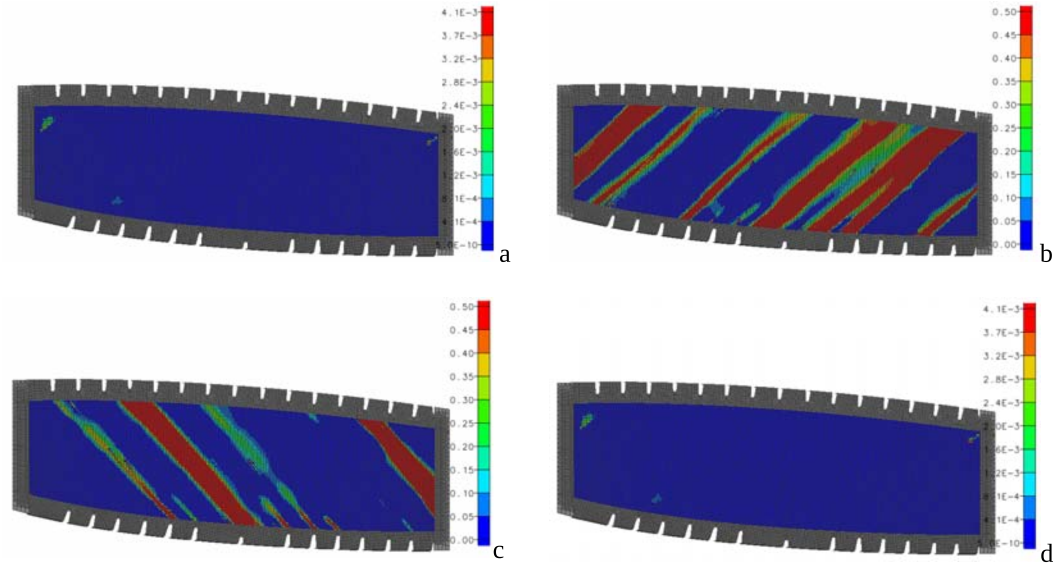


Figure 11. Layers thicknesses distribution: a) Layer1 – 0°, b) Layer 2 – 45°, c) Layer 3 – -45°, d) Layer 4 – 90°

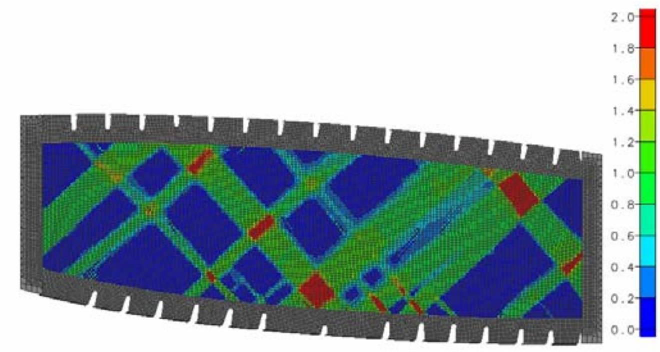


Figure 12. Overlap of layers forming the rib laminate

Looking at the results of Fig. 9 and Fig. 12 one concludes that a truss layout is the optimal solution for both formulations, topology and topometry. Comparing these figures side by side, there are visible similarities between the results. In the Fig. 13 this comparison is made with the assistance of small rectangles indicating the position of trusses. It is possible to identify six trusses aligned to the right, each one positioned at approximately the same region in both methodologies. To the left, there are five trusses in the topology optimization and four trusses in the topometry optimization, also in similar regions.

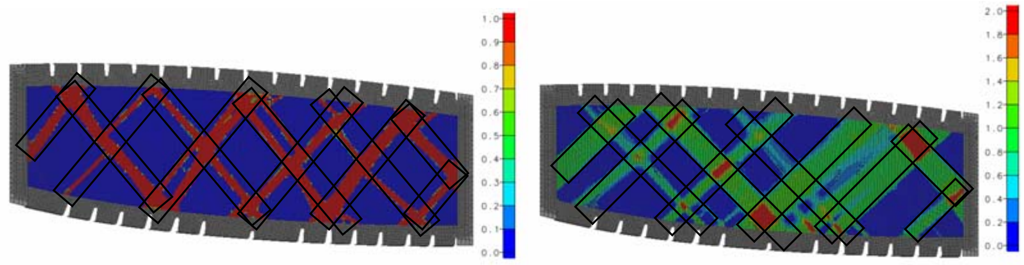


Figure 13. Topology and topometry results comparison

Based on Fig. 13, a new layout for the rib is proposed as shown in Fig. 14, following the positions defined with both optimization results. A small truss is inserted at the left region to maintain the trusses spacing. Now, over this optimal layout any convenient composite laminate sizing optimization scheme can be applied to detail the rib final design, however this will not be explored in the present work.

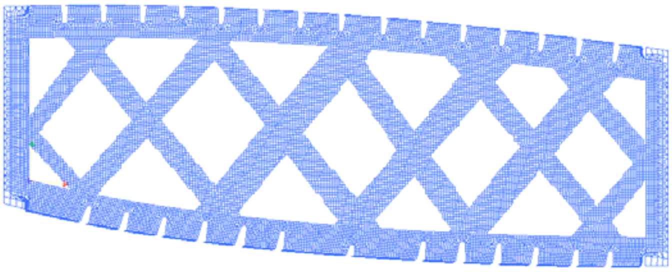


Figura 14. New rib layout

3. CONCLUSION

A study was made applying the techniques available of topology and topometry optimization to the optimal design of a wing rib belonging to an aircraft wing. The rib was isolated from a detailed wing finite element model. It was shown that both methods led to similar optimal layouts and that the topometry optimization technique could deal efficiently with composite laminated material. The results encourage further studies with additional options of fiber orientations and also with composite materials fabrication constraints.

4. ACKNOWLEDGMENT

The authors wish to express their gratitude to VR&D for the license granted of GENESIS and to Fundação Casimiro Montenegro Filho and CAPES.

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