COMPARISON OF TWO DIFFERENT MODAL IDENTIFICATIONS METHODS

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Abstract. *The aim of this paper is to compare two different modal identifications methods: Peak-Amplitude Method (PAM) and Circle-Fit Method (CFM). These methods are often used due to their computational simplicity for estimation of the modal parameters. In order to compare these methods, a linear dynamic system of two degree of freedom is created numerically considering Single-Input Single-Output (SISO) procedure. This system generates a Universal File Format (UFF). The modal properties of the UFF is Known, because, is not originated from an Experimental Modal Analysis (EMA), but rather a created file. Theses modal parameters as natural frequencies and damping factors are extracted from Frequency Response Function (FRF) by both methods. The results of methods are compared each other. Through these comparisons, it can be concluded that both methods found efficiently the modal parameters. However, these methods are limited by the frequency resolution, especially for lightly-damped dynamic systems.*

Keywords*: Circle fit method, peak amplitude method, experimental modal analysis, siso, modal parameters*

1. INTRODUCTION

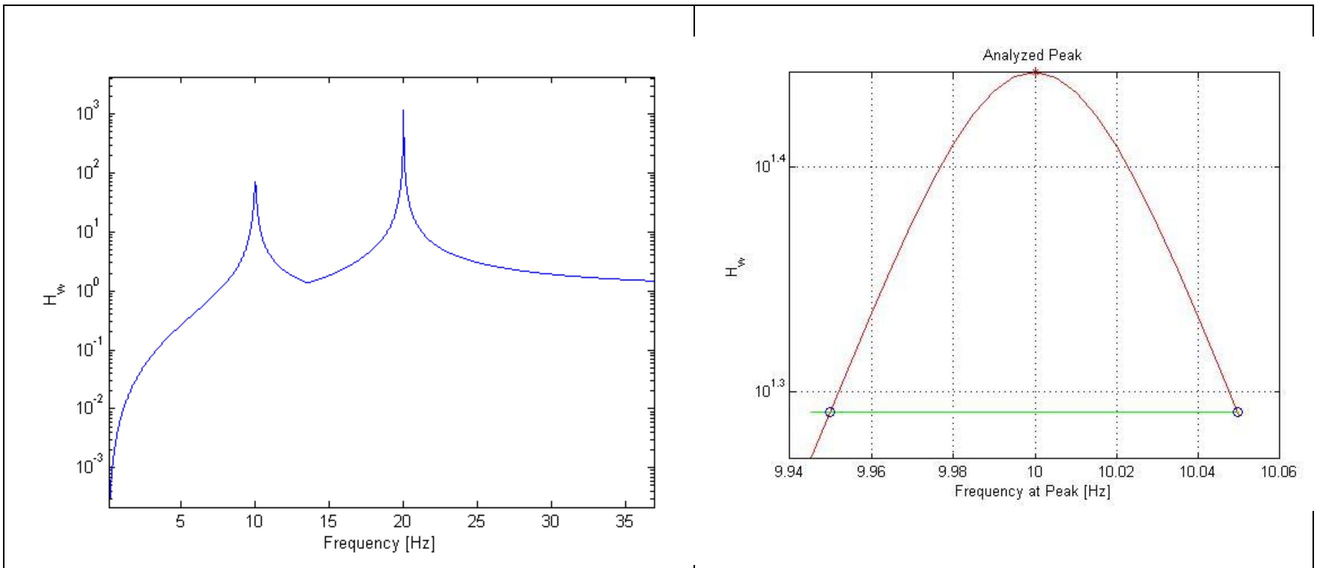
In previously studies, performed by granting scientific initiation, CNPq/PIBIC, takes the implementation of PAM, (Oliveira, 2010), and just after CFM, (Oliveira, 2011), through computational language MATLAB™. These methods are used through Unified Graphical Environment (UGE) that aid activities related aeroelastics analysis. The UGE has been development since 2003 by Divisão de Aerodinâmica at the Instituto de Aeronáutica e Espaço. Among these activities is structural characterization at aircraft studies through numerical models based in Finite Element Method (FEM). The updating of modal parameters is conduct through Ground Vibration Test (GVT). These parameters are extracted of Frequency Function Response (FRF) estimated. One used methodology is submitted the aircraft studied using a unique excitation font and measurement structure response, in acceleration accelerance, in a known point. This methodology is known to “Single Input Single Output” (SISO). Both methods compared uses SISO procedure.

According to studies realized to Ewins (1986) and Maia *et al.* (1997), both methods possess different vantages and disadvantages. However, a notable similarity can be seen between its. The main vantages showed by then are due to fact of that both methods can be implemented in simple’s computers. A common problem occurs when the peaks of FRF are a lot of nearby. Other disadvantage that exist this methods is compromises with frequency resolution of FRF, this resolution have a direct relation with quality modal results. In almost all cases is very difficulty realized assessing of the credibility modal results because don´t know quantity of point went used to represent FRF curve´s.

The main motivation to realized comparison is because both methods are implemented and full operation, due to this, appear necessity of assess functionally specific that each method show. Know specify characteristics of each method can be use more indicated method to input modal data.

1.1 Principle of Operation of the Peak Amplitude Method

The PAM too is known as Peak-Peaking or Peak to Peak. Maia *et al.* (1997) said that is the simplest known method for identifying the modal parameters of a structure. As the names suggested the PAM worked using one mode to time. In Bode diagram at FRF, shown in Fig.1a, each mode correspond the one peak and through selection peak the PAM extracted modal parameters. To realize it, using interpolation procedure, find peak, fall 3dB and draw line parallel ordinate axis to find two points in Bode diagram and realizing news interpolations estimate natural frequency and damping factor.



(a) FRF

(b) Interpolation by CFM

Figure 1. Bode diagram at FRF

1.2 Principle of Operation of the Circle Fit Method

To obtain the natural frequencies, the CFM, approximates a circle geometric relationships between the real and imaginary parts of the FRF in a frequency range close to natural. The CFM uses a several combinations of data points to determinate damping ratio. This advantage is showed in Fig. 2. This picture is by Maia *et al.* (1997).

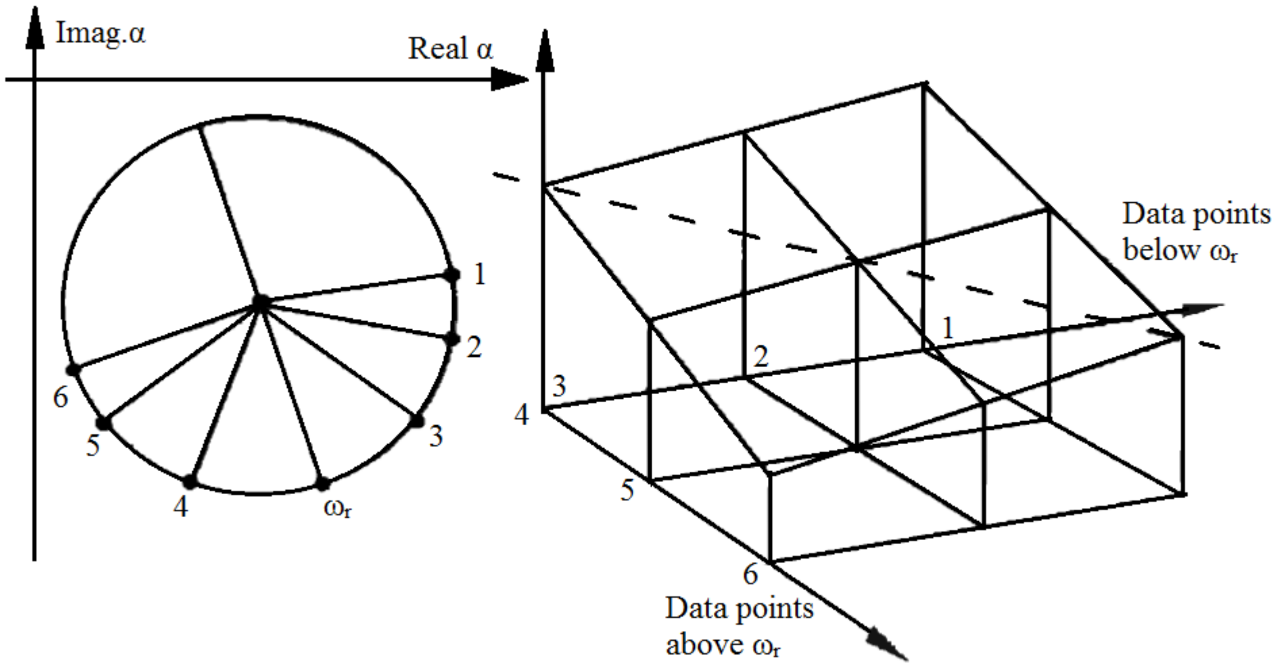


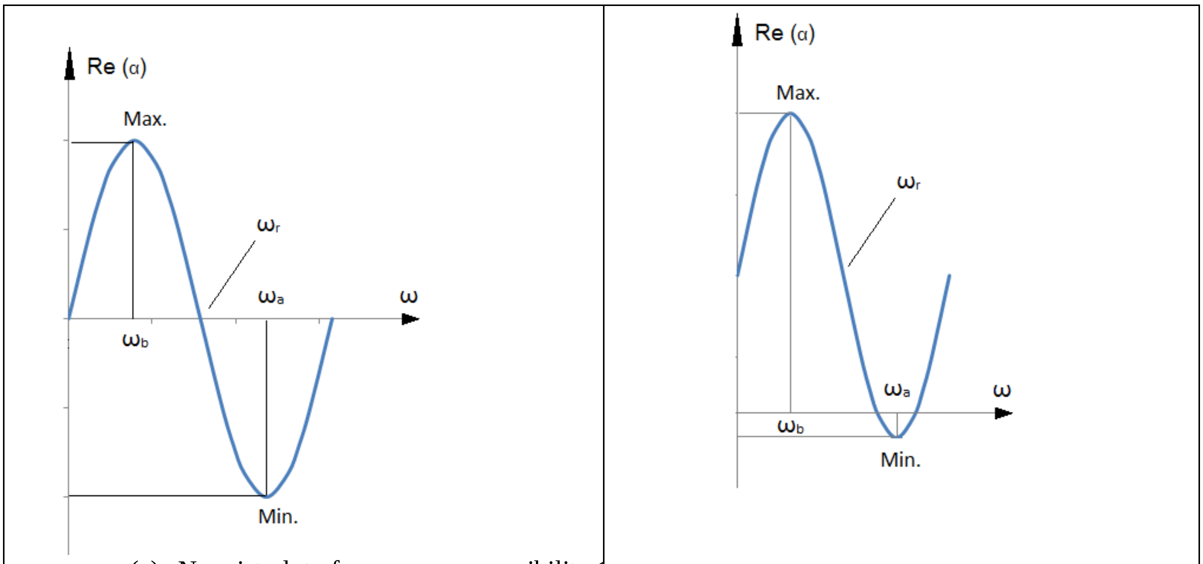
Figure 2. Determination of damping, using several combinations of data points

2. METHODOLOGY

The both methods, CFM and PAM, were implemented in Matlab™. This implementation uses reader routine of data already development in UGE. For tested the capacity of identification of methods, created numerically a linear dynamic system of two degree of freedom. Through this system were generated an FRF. This FRF is originates from Universal File Format (UFF), but the UFF is not originated from an experimental modal analysis but rather a file that was created. In UFF built, were imposed natural frequencies and damping factor in the first two vibration modes, therefore, it is an UFF, where there is certainty of its modal properties. Knowing that the natural frequencies and damping factor can check if they are successfully obtained by CFM and PAM. Purposely was imposed for each mode of UFF a different damping factor. For the second mode, was used a damping factor corresponding to 10% of first mode, therefore, in second mode there are less points to represent the FRF. Like this, can evaluate the performance of both methods for these two different conditions. Thus, it was possible simulate a condition similar to low frequency resolution and verify the ability of methods in handle it.

2.1 Implementation of Circle-Fit Method

The CFM, is implemented using the algorithm showed by Maia *et al.* (1997).
Already CFM extracted dynamic parameters of different way of PAM; these parameters are estimated using geometric relations between real parts and imaginary of FRF. The geometrics relations are achieve for better approximation of one circle around frequency range wish.
Maia *et al.* (1997), in fourth chapter realized an detailed study about CFM, however, Ewins (1986), show that points of frequency resolution more important are between ω_a and ω_b. This set of points between ω_a and ω_b are approximated by a circle and more than six points should be used. According to him, ω_a, in real domain always is maximum and ω_b always is minimum, the two possibilities can be seen in Fig.3.



(a) Nyquist plot of resonance – possibility 1 (b) Nyquist plot of resonance – possibility 2

Figure 3. Real vs. frequency plot for possibilities

Thus, inspiration from this was created a function inside CFM routine that identifies the number of points between ω_a and ω_b, this way, is possible to assess quality of frequency resolution.
The CFM according to Maia *et al.* (1997):

Assuming that an N DOF system with hysteretic damping is represented by equation;

$$\alpha_{jk}(\omega) = \sum_{r=1}^{N} \frac{{}_rC_{jk}}{\omega_r^2 - \omega^2 + i\eta_r\omega_r^2} \quad (1)$$

The hysteretic damping ratio (η_r) and complex modal constant (${}_rC_{jk} = \left(C_r\, e^{i\phi r}\right)_{jk}$) are associated with each mode r. The Contribution of the out-of-range modes is assumption by CFM with being the constant to one particular study, thus, the Eq. (1) can be approximated by;

$$\propto_{jk}(\omega) = \sum_{r=1}^{N} \frac{{}_rC_{jk}}{\omega_r^2 - \omega^2 + i\eta_r\omega_r^2} + D_{jk} \tag{2}$$

The complex constant (${}_rD_{jk}$) associated with mode r is related a simple translation (see Fig.4). The circle is represent by $1/(\omega_r^2 - \omega^2 + i\eta_r\omega_r^2)$. In order words, the Nyquist plot of Eq. (2) is an approach and complete curve will not be exactly circle around each natural frequency.

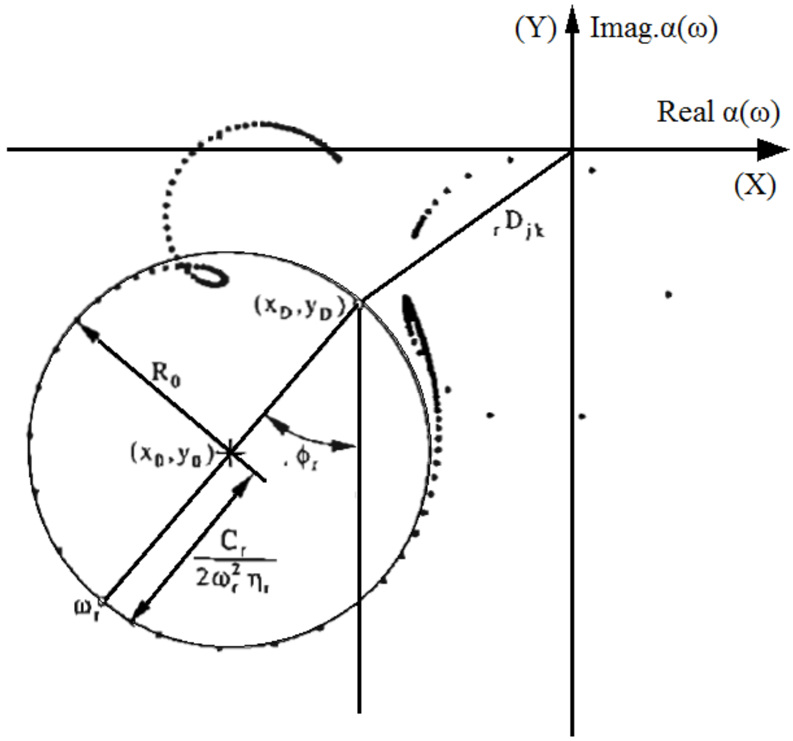


Figure 4. Nyquist plot of the receptance, showing the SDOF circle fit approach

If assumes that data points are in the Argand plane represented by coordinates x_j and y_j, the circle that best fits these data, is minimized by error e_1;

$$e_1 = \sum_{j=1}^{L} \left[R_0 - \sqrt{\left(x_j - x_0\right)^2 + \left(x_j - y_0\right)^2} \right]^2 \tag{3}$$

However, using this error (e_1) is impossible to calculate explicitly x_0, y_0 and R_0, but according to Silva (1988) it can be solved by a new error function (e_2);

$$e_2 = \sum_{j=1}^{L} \left\{ R_0^2 - \left[\left(x_j - x_0\right)^2 + \left(x_j - y_0\right)^2 \right] \right\}^2 \tag{4}$$

Rearranging Eq. (1), can be write

$$e_2 = \sum_{j=1}^{L} \left[c - \left(x_j^2 + ax_j + by_j + y_j^2\right) \right]^2 \tag{5}$$

Whereas

$$a = -2x_o \qquad b = -2y_0 \qquad c = R_0^2 - x_0^2 - y_0^2 \tag{6}$$

Substituting these new parameters (a, b and c) and minimizing the error function (e_2), it comes;

$$\begin{bmatrix} \sum x_j^2 & \sum x_j y_j & -\sum x_j \\ \sum x_j y_j & \sum y_j^2 & \sum y_j \\ \sum x_j & -\sum y_j & L \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} -\left(\sum x_j^3 + \sum x_j y_j^2\right) \\ -\left(\sum y_j^3 + \sum y_j x_j^2\right) \\ \sum x_j^2 + \sum y_j^2 \end{Bmatrix} \tag{7}$$

Trough this expression is possible calculate explicitly the value of the parameters (a, b and c), thus, it obtains x_0, y_0 and R_0 using the relationship of Eq. (6). Thus, a fast and direct calculation has been replaced using an iterative process. However, the error function e_2 does not lead to the same results, if compared when used e_2. Through the simple mathematical manipulation can be seen that if one neglects higher order terms.

$$e_2 \approx 4R_0^2 e_1 \tag{8}$$

Considering the fact that the standard procedure based on e1 is the same that relies on a truncation of higher order terms in a Taylor series expansion, it becomes apparent that the linearised procedure can be reliably used at a very small cost. Extremely good results are obtained with it. With the use such an approach greatly outweigh the inconvenience that introduction of an additional small error. The determination and location of the natural frequency, generally, are based on a frequency spacing technique presented by Klosterman (1971). The phase angle (θ_r) associated with the dynamic response for a determinate mode, can be represented by;

$$\theta_r = \tan^{-1}\{\eta_r/[1-(\omega/\omega_r)^2]\} \tag{9}$$

It may be noted that the equation $d(\omega_r)^2/d\theta_r$ is minimum to $\omega_r = \omega_r$. The result of the minimum of $d(\omega)^2/d\theta$ is the same that the minimum $d(\omega)^2/dy$. Using a set of values with the same experimental data with the same spaced frequency increments can be constructed the following table.

Table 1. Finite difference table

Points	**Frequency**	γ	$\Delta\gamma$	$\Delta^2\gamma$	**...**
A	ω_A	γ_1			
			$\Delta\gamma_1$		
B	ω_B	γ_2		$\Delta^2\gamma_1$	
			$\Delta\gamma_2$		
C	ω_C	γ_3		$\Delta^2\gamma_2$	
			$\Delta\gamma_3$		...
D	ω_D	γ_4		$\Delta^2\gamma_3$	
			$\Delta\gamma_4$		
E	ω_E	γ_5		$\Delta^2\gamma_4$	
			$\Delta\gamma_5$		
F	ω_F	γ_6		$\Delta^2\gamma_5$	
			$\Delta\gamma_6$		
G	ω_G	γ_7			

The values more precise of location of natural frequency are achieved by means of Newton's divided differences. Adopted four know points, two before and two after the natural frequency, and neglecting the residual error comes to Newton's divided differences reduced and simplify. Making the assumption that a minimum of $d(\omega)^2/d\theta$ corresponds to the maximum of $d\theta/d(\omega^2)$ the natural frequency can be obtained by differentiating twice the Newton's divided differences simplified and setting it equal to zero, which gives;

$$\omega_r^2 = \frac{1}{3}\left(\omega_0^2 + \omega_1^2 + \omega_2^2 - \frac{(\theta_0,\theta_1,\theta_2)}{(\theta_0,\theta_1,\theta_2,\theta_3)}\right) \tag{10}$$

The value of θ_r is calculate substituting Eq. (10) in Newton's divided differences. It's very simple technique to implement and leads to very accurate values of the natural frequency and its 'exact' location on the response curve. The accurate is very important because is only apparent when one needs to derive the value of the phase angle (θ_r) of the complex modal constant, calculated by 'drawing' a line joining the natural frequency point ant circle centre, according Fig. (4).

Now, is easy estimate the damping factor (η_r) using two points on the circle, the first corresponding to a frequency below the natural frequency (ω_b) and second corresponding to frequency above the natural frequency (ω_a), thus, the Eq. (9) is write as;

$$\tan\theta_a = \frac{\eta_r}{1-\frac{\omega_a^2}{\omega_r^2}}$$
$$\tan\theta_b = \frac{\eta_r}{1-\frac{\omega_b^2}{\omega_r^2}} \tag{11}$$

Assumption that $\theta_r = \pi/2$ and defining

$$\Delta\theta_a = \frac{\Delta\gamma_a}{2} = \theta_a - \theta_r$$
$$\Delta\theta_b = \frac{\Delta\gamma_b}{2} = \theta_r - \theta_b \tag{12}$$

Simplifying Eq. (11), which gives;

$$\eta_r = \frac{\omega_a^2-\omega_b^2}{\omega_r^2}\frac{1}{\tan(\Delta\theta_\mathrm{a})+\tan(\Delta\theta_\mathrm{b})} \tag{13}$$

2.2 Implementation of Peak-Amplitude Method

The PAM is known mainly due this your simplicity computational. However, others advantages can be observed in Ghulam (2006). Horner (1995) affirm that waveform with a low peak-to-rms ratio is desirable in situations requiring a maximum signal-to-noise ratio. With objective of treat it, he introduces a genetic algorithm-based method for selecting the phases that produces better results. To realize the implementation of the method, adopted Ewins (1986) as reference and due to methodology presented.

PAM according to Ewins (1986) possesses three steps that must be following:

First, individual resonance peaks are detected on the FRF plot and the frequency of maximum response taken as the natural frequency of that mode (ω_r);
Second, the maximum value of the FRF is noted ($|\hat{\alpha}|$) and the frequency bandwidth of the function for a response level of $|\hat{\alpha}|/\sqrt{2}$ is determined ($\Delta\omega$). The two points thus identified as ω_b and ω_a are the 'half-power points'.
Third damping of the mode in question can now be estimated from one of the following formulate.

$$\eta_r = \frac{\omega_a^2-\omega_b^2}{\omega_r^2} \frown \frac{\Delta\omega}{\omega_r} \tag{14}$$

$$\zeta_r = 2\eta_r \tag{15}$$

Last, may now obtain and an estimate for the modal constant of the mode being analyzed by assuming that the total response in this resonant region is attributed to a single term in the general FRF series.

$$|\hat{\alpha}| = \frac{\mathrm{A}_r}{(\omega_r^2\eta_r)} \tag{16}$$
or

$$\mathrm{A}_r = |\hat{\alpha}|\omega_r^2\eta_r \tag{17}$$

$$\mathrm{A}_r = (IMXI + IMNI)\omega_r^2\eta_r \tag{18}$$

3. RESULTS

The data of FRF of dynamic system with two virtual degrees of freedom, collected of UGE are showed in Fig.1. The natural frequencies of virtual dynamic system are compared with the extracted using CFM, as can be seen in Tab. 2. In Figure 6, is showed circular approximation in region of each one of naturals frequencies implemented by CFM in UGE. According to Fig.6, can be seeing that there are density of points more in first mode, due frequency resolution. In second mode the resolution frequency is insufficient to characterize damping by both methods, because number of points in semicircle nearby the natural frequency less of than six. Although, the CFM has provided better results than PAM, it becomes apparent that in CFM also there is a major commitment to the resolution frequency. The data from the FRF of the dynamic system with two degrees of freedom virtual collected by PAM are shown in Fig.5. The natural frequencies of the virtual dynamic system are compared with those extracted using the CFM and PAM, as seen in Tab.2.

Table 2. Comparison results between the virtual and dynamic system acquired by the method CFM and PAM

	Virtual		**Circle Fit Method**		**Peak Amplitude Method**	
Mode	Natural Frequency [Hz]	Damping Factor [%]	Natural Frequency [Hz]	Damping Factor [%]	Natural Frequency [Hz]	Damping Factor [%]
1	10.0	1.00	10.0	1.00	10.0	0.992
2	20.0	0.100	20.0	0.080	20.0	0.079

In Figure 6, shows the circular approach, in each region of the natural frequencies, implemented by CFM at the UGE. Notice that there is a greater density of points in the first mode due to the damping factor. In the second mode the sampling frequency is inadequate to characterize the damping CFM. The number of points in a semicircle, around the natural frequency is less than 6.

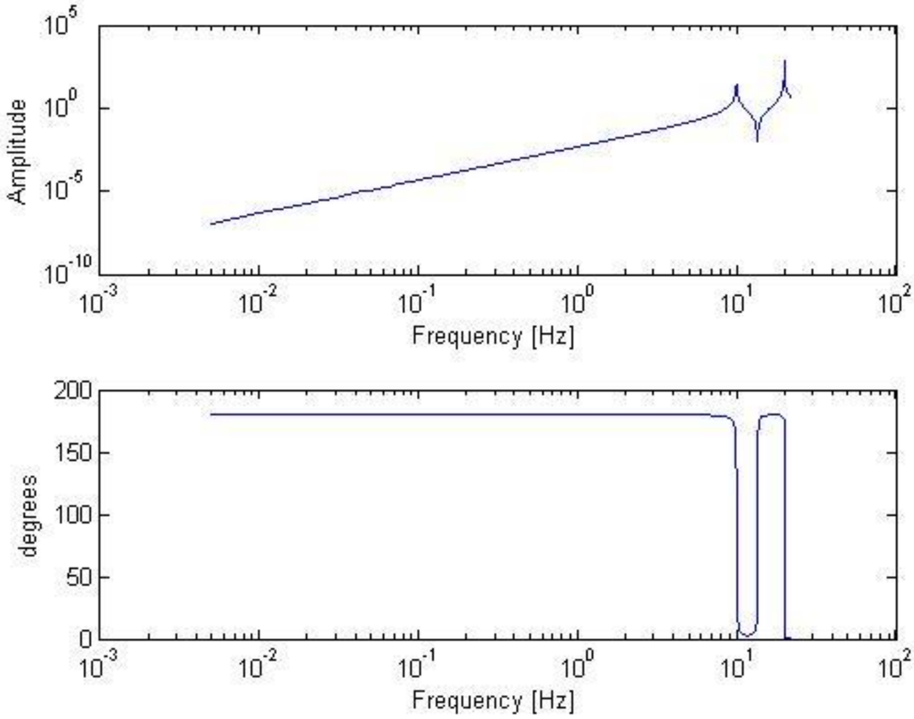
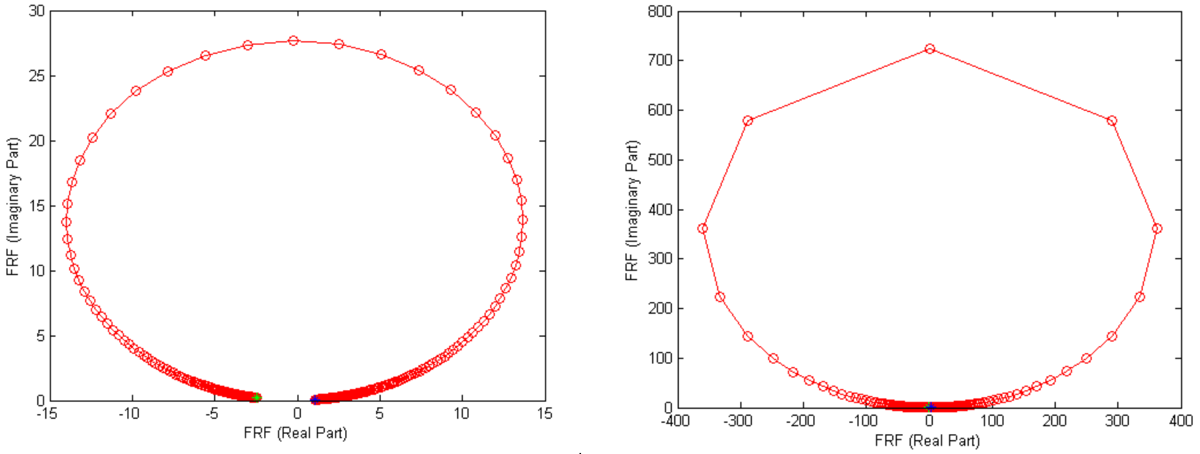


Figure 5. Bode diagram of the dynamic system virtual obtained by PAM



(a) First mode - good resolution frequency

(b) Second mode – poor resolution frequency

Figure 6. Approach circulate around the natural frequencies obtained by CFM

4. CONCLUSION

According to the results we can conclude that the CFM was an increase of tolerance to low frequency resolution in relation to PAM. A notable advantage is noted in the CFM, because the function implemented permit known the quantity of points between with the function cane valuate the representativeness according to the number of dots presented between ω_a and ω_b . However, should know that both methods there are compromises with frequency resolution.

5. ACKNOWLEDGEMENTS

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