

## FLUIDYNAMICS INVESTIGATION OF RECTANGULAR JETS WITH DIFFERENT ASPECT RATIOS

Murilo Godoy Favoretto, [murilofavoretto@gmail.com](mailto:murilofavoretto@gmail.com)

Odenir de Almeida, [odenir@mecanica.ufu.br](mailto:odenir@mecanica.ufu.br)

Universidade Federal de Uberlândia – Laboratório de Mecânica dos Fluidos

**Abstract.** *Non-circular jets have been extensively studied in last years. These jets were identified as one of the simplest and most efficient ways of passive flow control that allows significant improvements of performance in several practical systems at a relatively low cost because they depend only on changes in the nozzle shape. It is realized that non-circular jet configurations have profound advantages versus their circular counterparts in certain aspects. In particular, they can increase the mixing with surrounding fluid. The applications of non-circular jets include improved large and small scale mixing in low and high speed flows, and enhanced combustion efficiency, reducing combustion instabilities and undesired emissions. Additional applications include noise reduction and heat transfer. The main goal of this work was to study the flow dynamics of rectangular jets by investigating the influence of the nozzle aspect ratio on the essential flow characteristics such as spreading rate of the jets. In this paper, numerical simulations of three-dimensional rectangular free jets with different aspect ratios ( $L1/L2$ ), covering the range of 1 up to 5, are performed with the Reynolds-averaged Navier-Stokes equation (RANS) by using a  $k-\epsilon$  cubic turbulence model. All the simulations have been performed with the commercial software CFD++. In all cases, the nozzle exit velocity is taken to be 23 m/s, which is sufficiently low to attend the incompressible condition. The Reynolds numbers covered a range of 30000 to 90000. The numerical results in this work are compared with experimental data available in the literature. The results are shown in terms of contours of mean variables such as mean longitudinal and transversal velocities. Mean streamwise velocity profiles are in good agreement with these experimental data. Results show that as the aspect ratio increases, the velocity decay along the centerline becomes smoother which means that the length of the potential core increases proportionally with aspect ratio.*

**Keywords:** *Fluidynamics, Rectangular Jet, Aspect Ratio, RANS*

### 1. INTRODUCTION

Free shear flows have been extensively studied in the past due to their great significance in practical applications like propulsion and combustion. Since the pioneering work on coherent structures in turbulent flow by Crow and Champagne (1971) and Brown and Roshko (1974), a lot of research has been carried out concerning the mechanism and role of coherent structures in various types of flow. It is generally recognized that the development of coherent structures affects the flow in terms of entrainment of the ambient fluid, the mixing process, noise generation, etc. These finds led to the basic idea that artificial interference on the evolution process of vortices enhances or attenuates the essential flow characteristics. Such interference or control, as noted by Fiedler and Fernholz (1990), can be classified into either active control or passive control. Apart from the classical circular nozzle, utilization of non-circular nozzle such as rectangular nozzle is one of the simplest and most efficient ways of passive control.

It was soon realized that non-circular jet configurations have profound advantages versus their circular counterparts in certain respects. In particular, they can increase the mixing with surrounding fluid. Due to the presence of higher-order instability modes, non-circular jets are more unstable than circular configurations. Stream-wise vorticity created mainly by self-induction of the azimuthal vortex rings increases the mixing with ambient fluid enormously. This research was pioneered by Sforza *et al.* (1966) who conducted measurements for a variety of jet-nozzle shapes. Ho and Gutmark (1987) carried out experiments for a small aspect ratio elliptic jet and concluded that self-induction is the dominating effect that leads to increased spreading. Thorough reviews and summaries of data for different nozzle shapes are available from literature.

Laboratory jet experiments show that as a non-circular jet spreads, its cross-section can evolve through shapes similar to that at the jet exit but with its axis rotated at angles characteristic of the jet geometry, denoted as the axis-switching phenomenon (Ho and Gutmark, 1987; Sfeir, 1979; Krothapalli *et al.*, 1981; Husain and Hussain, 1983; Tsuchiya *et al.*, 1986; Quinn and Militzer, 1988; Toyoda and Hussain, 1989; Hussain and Husain, 1989). The occurrence of axis-switching depends on initial conditions at the jet exit, such as the azimuthal distribution of the momentum thickness and turbulence level (Hussain and Husain, 1989; Koshigoe *et al.*, 1989; Grinstein *et al.*, 1995). Axis-switching is regarded as the main mechanism responsible for the enhanced entrainment properties of non-circular jets relative to those of comparable circular jets.

In circular jets the main mechanism leading to the shear layer growth and formation of small-scale structures is vortex interaction and merging process (Ho and Huerre, 1984). In contrast, in the case of rectangular and elliptic jets the dynamics of self-induced vortex deformation appears to be more important than the vortex merging process. Austin (1992) measured nearly equal mass entrainment values for forced and unforced elliptic jets demonstrating this fact. His

measurements confirmed that production of small-scale structures is related to the vortex deformation process, and thus with axis-switching, and showed that small scales are concentrated within the large-scale vortex cores. Thus it is expected that the characteristic dependence on aspect ratio of vortex self-deformation and axis-switching to have a role in affecting jet mixing. The distance of the jet-width crossover location from the nozzle preceding the first axis-switching is known to grow proportionally with aspect ratio (Krothapalli *et al.*, 1981; Tsuchiya *et al.*, 1986), consistent with the reported growth of the vortex-ring axis-switching periods with aspect ratio (Arms and Hama, 1965; Kiya *et al.*, 1992).

Motivated by experiments findings of the period revealing the rectangular jet capacities in facilitating efficient turbulent combustion systems, in 1995, Miller *et al.* (1995) performed numerical simulations of spatially developing jets from circular and non-circular nozzles at incompressible conditions. Analysis of cross-stream velocities indicated that initially, two vortex pairs are formed with their axes coincident with the z-axis, allowing the jet to adopt an elliptic configuration and to display a recirculation which entrain fluid into the mixing zone along the major axis and to eject it along the minor axis. The effect is to contract the major axis of the jet while simultaneously stretching the minor axis, resulting in axis-switching. In addition, Miller *et al.* (1995) showed that as the aspect ratio is increased, the initial structure of the jet is less preserved. It was seen that although the rectangular jets contains sharp corners and long flat surfaces, which are important factors in facilitating an efficient mixing configuration, its axes switch too far downstream (at a stream-wise distance approximately twice that of the elliptic jet) to cause significant near-field mixing, what makes them relatively inefficient mixers in this region.

In 1999, Zaman (1999) presented an experimental study about the spreading characteristics of jets from several non-circular nozzles, further the effect of tabs - small protrusion placed at the jet nozzle exit that produces a pair of counter-rotating vortices, which can have a profound impact on the spreading of the jet and explain many features of the overall jet evolution (Zaman, 1996) - over the flow. It was shown that for rectangular jets without tabs at subsonic conditions, there is no axis-switching, unlike the rectangular jet with tabs configuration, which presented the phenomenon for both subsonic and supersonic conditions. Furthermore, at supersonic condition, it was shown that when screech occurs, jet spreading increases with all nozzle. Despite the fact that screech is eliminated by tabs, jet spreading for the tab configuration was found to be the largest among all the cases for both subsonic and supersonic conditions. In his results, it was found the jet spreading with the elliptic and rectangular nozzle to be only slightly larger than that with the circular nozzle for aspect ratios up to approximately 10 (for supersonic conditions, it was observed a significantly increase of the spreading even for low aspect ratios). Zaman (1999) defined a threshold for the perimeter stretching to become effective at subsonic conditions. It was expressed in terms of the ratio of hydraulic to equivalent diameter. That ratio needs to be greater than about 2 to enable a noticeable increase in jet spreading.

Although the focus of Zaman's work was mainly on the intermediate and far-field region, and very few entrainment data points were reported in the near field, the suggested poor near-jet entrainment enhancement for subsonic rectangular and elliptic jets was in surprising contrast with previous observations made by Ho and Gutmark (1987) and Hussain and Husain (1989), that the overall spreading of low aspect ratio elliptic jets was clearly larger versus their circular counterparts. Zaman (1999) attributed these apparent disagreements to differences in initial conditions, namely, inherently higher turbulence level, due to higher, although still low-subsonic, Mach numbers in his experiments. In addition, in other reported laboratory experiments (Grinstein *et al.*, 1995), unforced jets exhibited near-field axis-switching even for fairly-turbulent conditions, diverging again from Zaman's experiments.

In order to resolve the apparent disagreements in between the state-of-art laboratory experiments and to recognize and improve the understanding of the near-field entrainment properties of rectangular jets, Grinstein (2001) performed simulations of low aspect ratio rectangular free jets at subsonic conditions. His work was focused to elucidate the dependence of the near-field entrainment properties on: jet initial conditions, the entrainment measurement process itself, and the associated vortex dynamics. It was showed that the far-field entrainment rates of non-circular jets approach asymptotically the values of their circular jets counterparts, with higher far-field values of the entrainment reflecting on higher near-field rates. Analyzing rectangular jets with aspect ratio of  $A=1$ ,  $A=2$  and  $A=3$  at subsonic conditions, significantly larger jet spreading and stream-wise vorticity production was observed in the near-field for  $A=1$  compared to similar cases with  $A=2$  and  $3$ , reflecting the presence of rib vortices pairs aligned with corner regions, rather than single ribs vortices for  $A=2$  and  $3$ , and axis-switching occurrence closer to the jet exit due to significantly smaller characteristic vortex-ring axis-switching times. On the other hand, associated with the larger stream-wise vorticity production in the near-field for  $A=1$ , the vortex rings tend to be more unstable azimuthally and break down closer to the jet exit, and as a consequence, larger jet spreading was observed for  $A=2$  and  $3$  in the far-field.

Recently, Jiang *et al.* (2007) worked out on Large Eddy Simulations of jets from small aspect-ratio rectangular nozzles at incompressible conditions. Analysis of the developing jet process showed that for the jets with larger aspect ratios, vortices originating from the longer side are dominant over that from the shorter side and the jet is characterized by a sequence of vortices from the longer side advancing downstream one after another. It was found that within the potential core, jet entrainment is slight but, in the development region, entrainment rates increase rapidly along the axial direction. It was also shown that, for the conditions of the flow described, the jet with smaller aspect ratio entrains more than that with larger aspect ratio. Analysis of the power spectral density analysis indicated that there is a non-dimensional intrinsic characteristic frequency in the near field of rectangular jets, which is independent of velocity

components, aspect ratios and locations. He observed that when a forced disturbance with frequency equal to the characteristic frequency was imposed at the inlet of the rectangular nozzle, development of the vortical structure in the near-field started earlier, entrainment rate was the highest and velocity decay along the centerline was the strongest.

When it comes to acoustics, rectangular jets can also reduce noise generation compared to their circular counterparts, a fact that holds for subsonic and supersonic cases. Paliath and Morris (2006) conducted a work about noise prediction for an aspect ratio of 3 rectangular nozzles at subsonic conditions, in order to study the effect of a non-circular nozzle shape on the directivity of the far-field noise. It was seen that unlike a circular nozzle, there is an azimuthally variation in the sound pressure levels of the rectangular one. Analysis of the sound pressure level (SPL) showed that at lower frequencies the major axis is slightly quieter (a noise reduction of about 2dB was observed at the spectral peak), while at higher frequencies the noise levels are similar.

Lately, a lot of experimental and numerical results on rectangular jet flows have been reported. Rembold *et al.* (2002) investigated the transition process of a rectangular jet with aspect ratio of 5 by direct numerical simulation (DNS). Abdel-Hameed and Bellan (2002) extended the investigation of rectangular jets to liquid-gas two-phase jets by DNS, and found that the liquid droplet laden gaseous jet exhibited larger entrainment rates than for pure gaseous jets under the same conditions. Despite the above experimental and numerical work, much work is still needed to explore the instantaneous vortex development within a wider rangel, the entrainment rate, and power spectral density (PSD) distributions of rectangular jets.

In this paper, numerical simulations of three-dimensional rectangular free jets with different aspect ratios are performed applying the CFD++ software developed by Metacomp Technologies. The goal is to study and understand the flow dynamics of rectangular jets at incompressible conditions by investigating the influence of the nozzle aspect ratio on the essential flow characteristics. In the future, this work will be extended to compressible conditions.

## 2. JET SIMULATION MODEL

The coordinates system and the jet configurations are shown in Fig. 1. The central plane determined by lines AO and OC is referred to as the major plane, and that by lines BO and OC is called the minor plane. In addition, the z-axis and y-axis in Fig. 1 are called the major and minor axis, respectively.

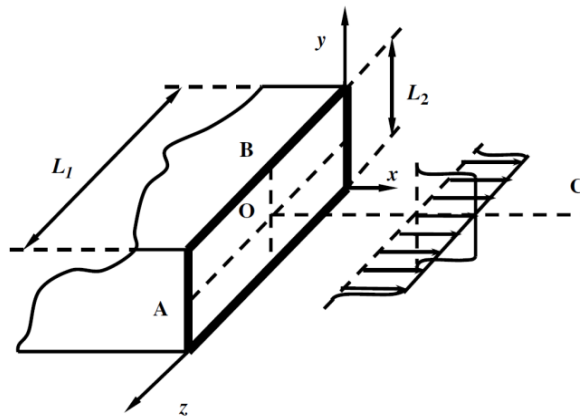


Figure 1. Schematic diagram of the jet

Table 1 presents the geometry of the jet and the Reynolds number (based on the equivalent diameter  $D_e$  and the inlet velocity  $U_0$ ) for the cases investigated. The equivalent diameter  $D_e$  is defined as the diameter of a circular jet having the same initial cross sectional area.

Table 1. Geometry of the rectangular jets.

| Nozzle height<br>$L_2$ (mm) | Nozzle width<br>$L_1$ (mm) | Aspect ratio<br>( $L_1/L_2$ ) | Reynolds<br>Number |
|-----------------------------|----------------------------|-------------------------------|--------------------|
| 22.36                       | 22.36                      | 1                             | 38686              |
| 22.36                       | 44.72                      | 2                             | 54711              |
| 22.36                       | 67.08                      | 3                             | 67007              |
| 22.36                       | 89.44                      | 4                             | 77373              |
| 22.36                       | 111.80                     | 5                             | 86506              |

In order to compare with experimental data, the nozzle dimensions and inflow velocity for the case of  $AR = 5$  are chosen to match the parameters of the experiment of Tsuchiya *et al.* (1986).

In all cases, the nozzle exit velocity is taken to be 23 m/s, which is sufficiently low to attend the incompressible condition. The static temperature and the static pressure in all domain and boundaries are set to 288 K and 101325 Pa, respectively. The computational domains have streamwise lengths of  $30 L_2$  and extend up to  $5 L_1$  and  $5 L_2$  away from the jet axis in the transverse directions “z” and “y”, respectively.

## 2.1. Mesh Refinement

The final axisymmetric computational domain discretization consisted of a block structured mesh with 8 blocks and a total number of grid points ranging from 1392400 in the case of  $AR = 1$  to 7056400 in the case of  $AR = 5$ . The mesh points are concentrated in the regions near the nozzle. A geometric growing law is used to increase the elements in axial and transverse directions. During block transitions, the grid size of one block has the same value of the other. The mesh of each case was designed proportionally to the nozzle dimensions, thus enabling comparisons between the results for each simulation. The figure below shows the mesh refinement for the case of  $AR = 5$ .

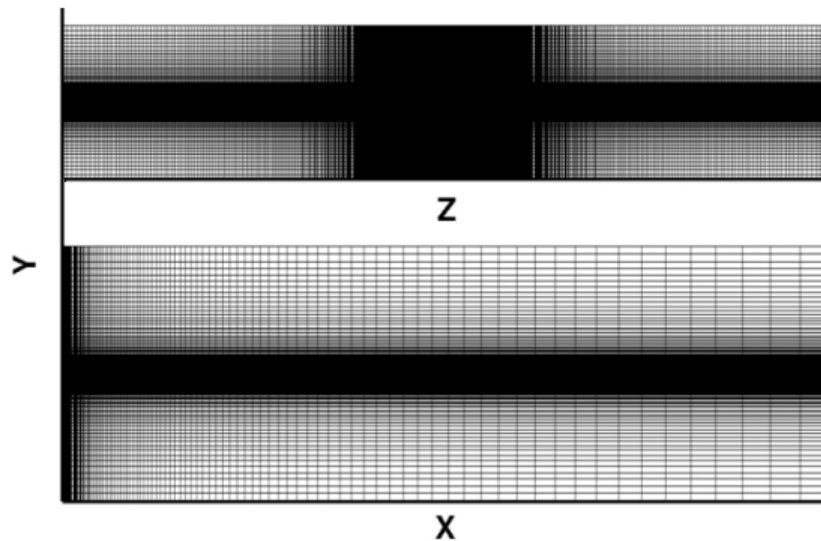
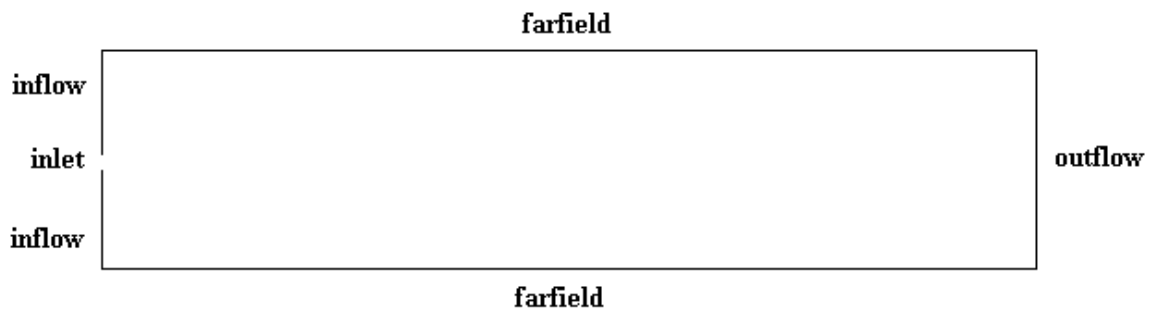


Figure 2. Mesh in 2D view

## 2.2. Boundary Conditions

At the inlet and the inflow, it was assumed a temperature-velocity inflow/pressure outflow condition, with an axial velocity of 23 m/s for the inlet and an axial velocity of 0.001 m/s for the inflow. The velocity normal to the farfield is computed from the interior, and, thus, it determines whether there is inflow or outflow. For the outflow, a simple back pressure condition was set. As depicted earlier, it was fixed all temperatures to 288 K and all pressures to 101325 Pa. Figure 3 summarizes the main boundary condition used in the CFD++ in all simulations in a 2D view.



inlet: Temperature-velocity inflow/pressure outflow  
 inflow: Temperature-velocity inflow/pressure outflow  
 farfield: Pressure temperature inflow/outflow using inside velocity  
 outflow: Simple back pressure

Figure 3. Boundary Conditions

### 2.3. Numerical Scheme

In CFD++ the governing equations were solved with an Incompressible Navier- Stokes/Euler system using preconditioning as the solution methodology. The simulations were performed with the Reynolds-averaged Navier-Stokes equation (RANS) by using a non-linear k-ε closure, named k-ε cubic model. The formulation to obtain Reynolds-stress tensor is defined via a tensorial expansion, cubic in the mean strain and vorticity tensors. The stresses are related to the mean strain and vorticity using the quadratic model of Shih *et al.* (1993) with the cubic extension proposed by Lien and Leschnizer (1996). More details about this turbulence model are given in the work of Goldberg *et. al.* (2000). The formulation for x-direction is presented below:

$$\rho \left[ \frac{\partial}{\partial x} (\overline{u^2}) + \frac{\partial}{\partial y} (\overline{uv}) + \frac{\partial}{\partial z} (\overline{uw}) \right] = -\frac{\partial p}{\partial x} + \left[ \frac{\partial}{\partial x} \left( \mu \frac{\partial \overline{u}}{\partial x} - \rho \overline{u'u'^2} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial \overline{u}}{\partial y} - \rho \overline{u'v'} \right) + \frac{\partial}{\partial z} \left( \mu \frac{\partial \overline{u}}{\partial z} - \rho \overline{u'w'} \right) \right] \quad (1)$$

CFD++ uses upwind formulations, including realizable Riemann solvers and it also solves the mass, momentum and energy equations in a coupled mode. For the Riemann solvers it was set an approximate system velocity of 10 m/s.

### 3. RESULTS

In order to validate the computational method, distributions of mean streamwise velocity “u”, normalized by the local centerline streamwise velocity “um”, along the transverse directions “y” and “z”, normalized by their respective half-velocity widths, for the case of AR=5, are presented in the figures below. The distributions are taken from three different planes:  $x/L_2=1$ ,  $x/L_2=7$  and  $x/L_2=13$ . The results are shown together with the experimental results of Tsuchiya *et al.* (1986).

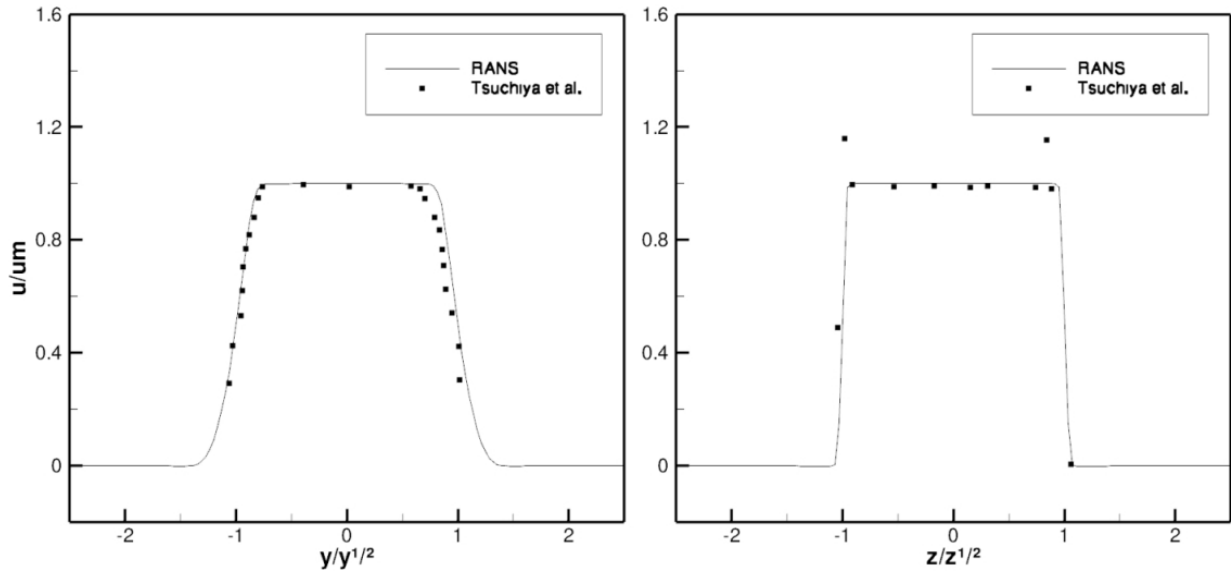


Figure 4. Mean streamwise velocity profile at  $x/L_2=1$

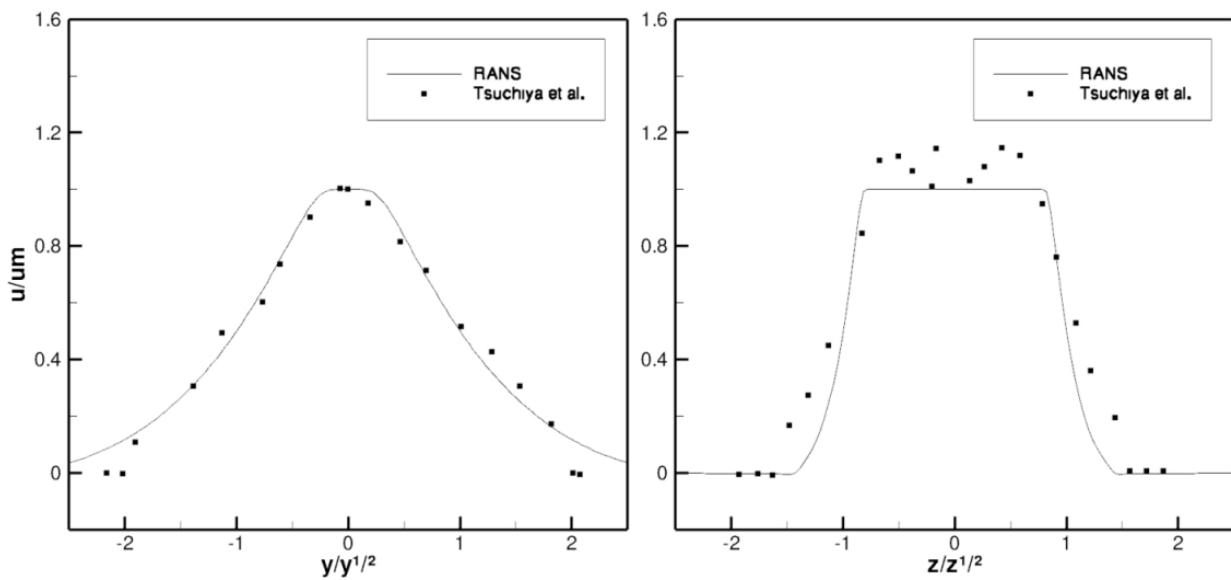


Figure 5. Mean streamwise velocity profile at  $x/L_2=7$

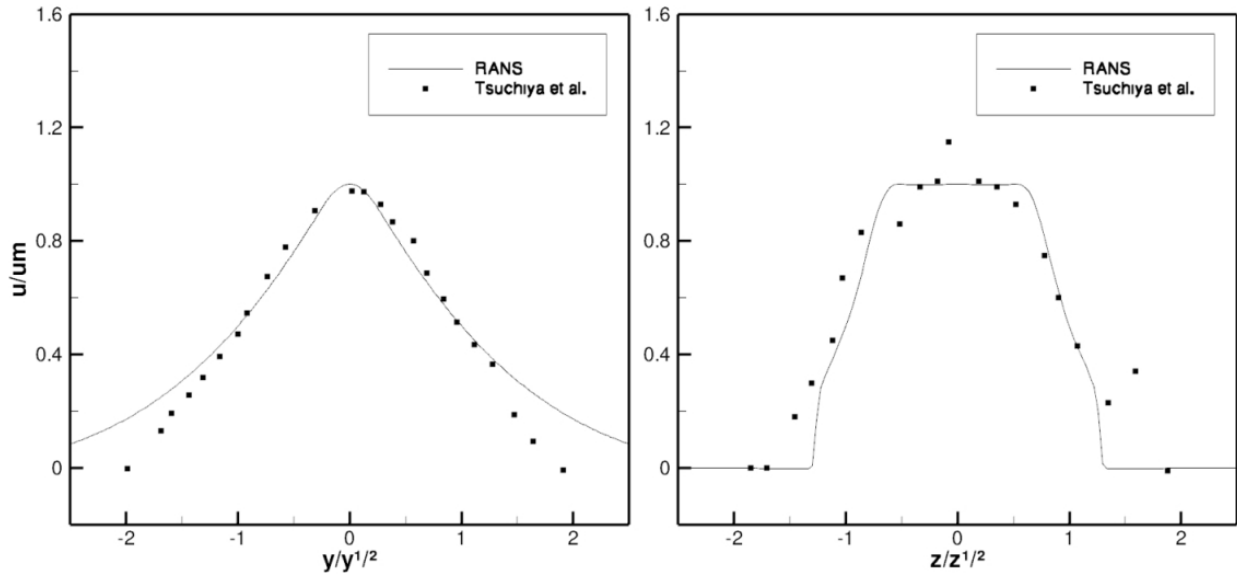


Figure 6. Mean streamwise velocity profile at  $x/L_2=13$

It can be seen that the results are in good agreement with the experimental data, which means that the computational method used is representing well the fluidynamics of the rectangular jet flow. The study of the influence of the nozzle aspect ratio on certain flow characteristics was also made. Figure 7 shows this influence on the velocity decay along the centerline and the length of the potential core. The velocity was normalized by the jet exit velocity  $u_0$  (23 m/s), and the streamwise distance by the minor side  $L_2$ .

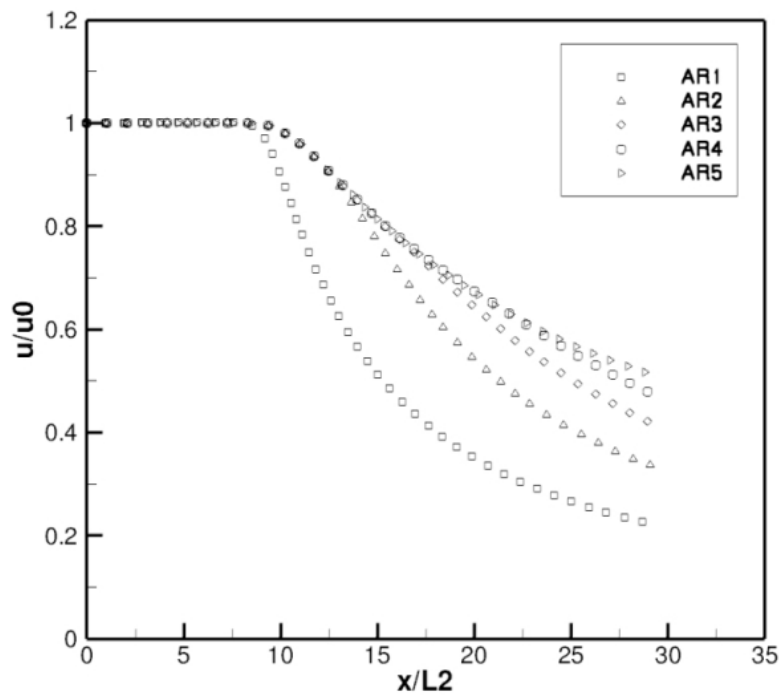


Figure 7. Velocity decay along the centerline for all cases

It is clear that increasing the aspect ratio, the velocity decay becomes smoother and the potential core becomes greater. Next figures present the streamwise velocity fields at cut planes  $z=0$  (minor plane) and  $y=0$  (major plane) for all the five cases. Again, the velocity is normalized by the jet exit velocity  $u_0$ .

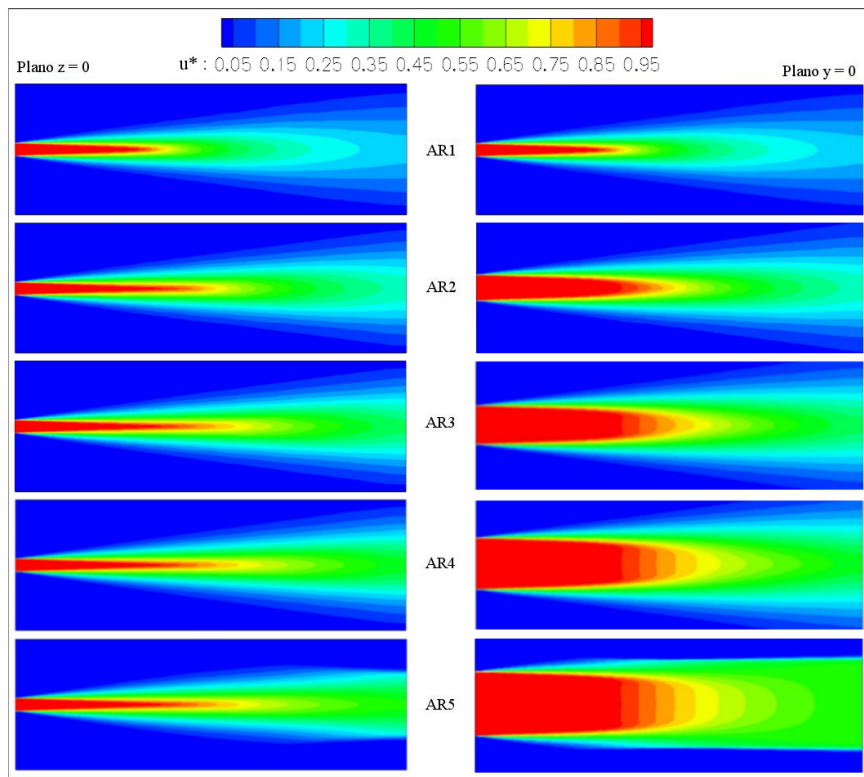
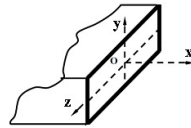


Figure 8. Streamwise velocity fields

Here the growth of the potential core with the aspect ratio becomes clear. Also, a certain inconsistency is observed for the case of AR=5, which led to conclude that it didn't converge very well in the farfield. This problem will be analyzed more carefully in the next studies so that it can be known what caused that non convergence, i.e. the computational domains, boundary conditions and turbulence model.

#### 4. CONCLUSIONS

The main goal of this work was to study the flow dynamics of rectangular jets by investigating the influence of the nozzle aspect ratio on the essential flow characteristics of the jets. The transversal velocities were in good agreement with the experimental data of Tsuchiya et al., which means that the computational method used is representing well the fluidynamics of the rectangular jet flow. The analysis of the velocity decay along the centerline and the streamwise velocity fields showed that increasing the aspect ratio, the velocity decay becomes smoother and the potential core becomes greater.

#### 5. REFERENCES

- Abdel-Hameed, H. and Bellan, J., 2002. "Direct numerical simulations of two-phase laminar jet flows with different cross-section injection geometries". *Phys Fluids* 14 (10), p. 3655-3674.
- Arms, R.J. and Hama, F.R., 1965. "Localized-induction concept on a curved vortex and motion of an elliptic vortex ring". *Phys. Fluids* 8, p. 553-559.
- Austin, T., 1992. "The small scale topology of a 2 : 1 aspect-ratio elliptic jet". PhD thesis, USC, Los Angeles.
- Brown, G.L. and Roshko, A., 1974. "Density effects and large structure in turbulent mixing layers". *J. Fluid Mech.* 64 (4), p. 775-816.
- Crow, S.C. and Champagne, F.H., 1971. "Orderly structure in jet turbulence". *J. Fluid Mech.* 48 (3), p. 547-91.
- Fiedler, H.E. and Fernholz, H.H., 1990. "On management and control of turbulent shear flows". *Prog. Aerospace Sci.* 27, p. 305-387.
- Goldberg, U. *et al.*, 2000. "Hypersonic Flow Predictions Using Linear and Nonlinear Turbulence Closures", *Journal of Aircraft*, Vol 37, No. 4, pp. 671-675.



- Grinstein, F.F., Gutmark, E. and Parr, T.P., 1995. "Near-field dynamics of subsonic, free square jets. A computational and experimental study". *Phys. Fluids* 7, p. 1483-1497.
- Grinstein, F.F., 2001. "Vortex dynamics and entrainment in rectangular free jets". *J. Fluid Mech.* 437, p. 69-101.
- Ho, C.M. and Gutmark, E., 1987. "Vortex induction and mass entrainment in a small-aspect-ratio elliptic jet". *J. Fluid Mech.* 179, p. 383-405.
- Ho, C.M. and Huerre, P., 1984. "Perturbed free shear layers". *Ann. Rev. Fluid Mech.* 16, p. 365.
- Husain, H.S. and Hussain, A.K.M.F., 1983. "Controlled excitation of elliptic jets". *Phys. Fluids* 26, p. 2763.
- Hussain, F. and Husain, H.S., 1989. "Elliptic jets. Part 1. Characteristics of unexcited and excited jets". *J. Fluid Mech.* 208, p. 257-320.
- Jiang, P. *et al.*, 2007. "Frequency characteristics of coherent structures and their excitations in small aspect-ratio rectangular jets using large eddy simulation". *Computers & Fluids* 36 (3), p. 611-621.
- Kiya, M. *et al.*, 1992. "Numerical simulation and flow visualization experiment on deformation of pseudo-elliptic vortex rings". *Fluid Dyn. Res.* 10, p. 117-131.
- Koshigoe, S., Gutmark, E. and Schadow, K., 1989. "Initial development of non-circular jets leading to axis switching". *AIAA J.* 27, p. 411.
- Krothapalli, A., Baganoff, D. and Karamcheti, K., 1981. "On the mixing of rectangular jets". *J. Fluid Mech.* 107, p. 201-220.
- Lien, F. S. and Leschziner, M. A., 1996. "Low-Reynolds number eddy-viscosity modelling based on non-linear stress-strain/ vorticity relations". In: Rodi, W. and Bergeles, G. (eds), *Engineering Turbulence Modelling and Experiments* 3, Elsevier, Amsterdam, pp. 91-100.
- Miller, R.S., Madnia, C.K. and Givi, P., 1995. "Numerical simulation of non-circular jets". *Computers & Fluids* 24 (1), p. 1-25.
- Paliath, U. and Morris, P., 2006. "Prediction of jet noise from rectangular nozzles". *AIAA J.* 618.
- Quinn, W.R. and Militzer, J., 1988. "Experimental and numerical study of a turbulent free square jet". *Phys. Fluids* 31, p. 1017.
- Rembold, B., Adams, N.A. and Kleiser, L., 2002. "Direct numerical simulation of a transitional rectangular jet". *Int. J. Heat Fluid Flow* 23 (5), p. 547-553.
- Sfeir, A.A., 1979. "Investigation of three-dimensional turbulent rectangular jets". *AIAA J.* 17, p. 1055-1060.
- Sforza, P.M., Steiger, M.H. and Tentracoste, N., 1966. "Studies on three-dimensional viscous jets". *AIAA J.* 4 (5), p. 800-805.
- Shih, T. H., Lumley, J. L. and Zhu, J., 1993. "A Realizable Reynolds Stress Algebraic Equation Mode", NASA TM-105993.
- Toyoda, K. and Hussain, F., 1989. "Vortical structures of noncircular jets". In *Proc. Fourth Asian Congress of Fluid Mechanics*, Hong Kong, August 21-25, pp. A117-A127.
- Tsuchiya, Y., Horikoshi, C. and Sato, T., 1986. "On the spread of rectangular jets". *Exps. Fluids* 4, p. 197-204.
- Zaman, K.B.M.Q., 1996. "Axis switching and spreading of an asymmetric jet; the role of coherent structure dynamics". *J. Fluid Mech.* 316, p. 1-27.
- Zaman, K.B.M.Q., 1999. "Spreading characteristics of compressible jets from nozzles of various geometries". *J. Fluid Mech.* 383, p. 197-228.

## 6. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.